Space-Frequency Domain Iterative Method for Modes Analysis of Planar Waveguides

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Abstract **─** We describe a space-frequency domain iterative algorithm to analyze the modes of planar optical waveguides. The one dimensional Maxwell equation was transformed into space-frequency domain by Fourier transform, and became an integral equation which could be solved by an iterative method. For any refractive index profiles, the effective index and mode field distribution are given simultaneously. The numerical result shows that this method is accurate and flexible for planar optical waveguides with any structure.

Index Terms **─** Fourier transform, iterative method, planar waveguide, space-frequency.

I. INTRODUCTION

Planar optical waveguides are the fundamental components in integrated optical circuits and used for optical amplifiers, lasers sensors and other optical devices [1-6]. All the characteristics of planar optical waveguides are based on the analysis of propagating modes which include the propagation constants and field distributions. The modes propagation constants and fields can be obtained by solving Maxwell's equations subject to the appropriate boundary conditions. However, the refractive index profile of planar waveguides is not only step but graded usually, exact solutions are given for only a few class of index profile [7]. For the most planar waveguides, approximate or numerical methods are used to analyze the modes fields. In general, the approximate methods [8,9] such as the method of perturbation and WKB method, have a clear physical insight but are not very accurate; numerical methods [10-16] can give solutions to the desired accuracy but require complex calculation. ATMM [17] is an effective and accurate tool for planar optical waveguides with arbitrary index profile. Unfortunately, ATMM is difficult to expand for two dimensional optical waveguides.

In this paper, a novel iterative algorithm in the space-frequency domain has been introduced to the analysis of planar optical waveguides. The one dimensional wave equation was transformed into space-frequency domain by Fourier transform, and became an integral equation which could be solved by an iterative method. The only complex calculation in the iterative operation is the convolution integral which could be completed by fast Fourier transform (FFT). For any refractive index profiles, the effective index and mode field distribution are given simultaneously. As the test calculation demonstrates that the dispersion curves given by our method agree extremely well with the exact solution.

II. WAVE EQUATIONS IN THE SPACE-FREQUENCY DOMAIN

Considering the TE mode of a planar waveguide, the scalar-wave equation is:

$$\frac{d^2 E_y}{dx^2} + \left(k_0^2 n^2(x) - \beta^2\right) E_y = 0, \tag{1}$$

where k_0 is the free-space wave number, $n(x)$ is the x-dependent refractive index, and β is the propagation constant. It is noticeable that $n(x)$ is given in the form of generalized function which include the boundary conditions expressly. Especially, when $n(x)$ is not a continuous function of x. Given a Fourier transform, the scalar-wave equation (1) in the space-frequency domain is written as:

$$-k^2 E_y(k) + k_0^2 N(k) * E_y(k) - \beta^2 E_y(k) = 0, \tag{2}$$

where k is the space frequency, $E_y(k)$ and $N(k)$ are the Fourier transform of $E_y(x)$ and $n^2(x)$ respectively, $*$ represents convolution integral. For a sample transpose, equation (2) changes into:

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$$\frac{N(k)*E_y(k)}{k^2+\beta^2}=\frac{E_y(k)}{k_0^2}. \quad (3)$$

We define an operator $\hat{A}$:

$$\hat{A}f(k)=\frac{N(k)*f(k)}{k^2+\beta^2},$$

where $f(k)$ is an arbitrary function of k. Thus, equation (3) becomes a normal eigenvalue equation, i.e.:

$$\hat{A}E_y=\lambda E_y, \quad (4)$$

where $\lambda=1/k_0^2$.

For TM modes, the scalar-wave equation is:

$$\frac{d^2H_y}{dx^2}-\frac{d\left(\ln n^2(x)\right)}{dx}\frac{dH_y}{dx}+(k_0^2n^2(x)-\beta^2)H_y=0, \quad (5)$$

where $n(x)$ is the x-dependent refractive index for the whole region including the core and cladding.

As doing for equation (1), in space-frequency domain, equation (5) could be written as a general eigenvalue equation:

$$\hat{A}H_y=\lambda\hat{B}H_y, \quad (6)$$

where $\hat{B}$ is an operator:

$$\hat{B}f(k)=f(k)-\frac{\left(k\boldsymbol{F}\left(\ln n^2(x)\right)\right)*\left(kf(k)\right)}{k^2+\beta^2},$$

$\boldsymbol{F}\left(\ln n^2(x)\right)$ is the Fourier transform of $\ln n^2(x)$.

III. ITERATIVE FORMULA FOR THE SOLUTION

Eigenvalue equations (4) and (6) are the integral equations which can be solved by an iterative method. For the purpose of simplicity, let ψ represent E_y or H_y. We are on the assumption that the eigenvalues:

$$\lambda_1>\lambda_2>\lambda_3>\cdots>\lambda_n,$$

correspond to these eigenvalues, the eigenfunctions are:

$$\psi_1,\psi_2,\psi_3\cdots\psi_n.$$

Choose an initial solver function x_0 which can be represented a:

$$x_0=a_1\psi_1+a_2\psi_2+\cdots+a_n\psi_n+\cdots.$$

For equation (4), we give the iterative formula:

$$x_{m+1}=\alpha_m\hat{A}x_m \qquad m=0,1,2\cdots \quad (7)$$

Thus,

$$x_1=\alpha_0(a_1\lambda_1\psi_1+a_2\lambda_2\psi_2+\cdots+a_n\lambda_n\psi_n),$$

$$x_2=\alpha_1\alpha_0\hat{A}(a_1\lambda_1^2\psi_1+a_2\lambda_2^2\psi_2+\cdots+a_n\lambda_n^2\psi_n),$$

$$x_{m+1}=$$

$$\alpha_0\alpha_1\cdots\alpha_m a_1\lambda_1^m\left(\psi_1+\frac{a_2}{a_1}\left(\frac{\lambda_2}{\lambda_1}\right)^m\psi_2+\cdots+\left(\frac{\lambda_n}{\lambda_1}\right)^m\psi_n\right).(8)$$

Fortunately, as m is increased, except the first one, the other terms in formula (8) converge to zero. This means that when m is large enough, $x_m\to\alpha\psi_1$ is the eigenfunction corresponding to the eigenvalue λ_1 which is the maximum of all the eigenvalues. When you get the maximum of eigenvalues λ_1 and its eigenfunction ψ_1, let $x_i=x_i-\left(x_i^*\cdot\psi_1\right)/\left(\psi_1^*\cdot\psi_1\right)\psi_1$, use the same iterative formula, the second maximum eigenvalues λ_2 will be given.

For equation (6), define another operator $\hat{C}$:

$$\hat{C}f(k)=\frac{\left(k\boldsymbol{F}\left(\ln n^2(x)\right)\right)*\left(kf(k)\right)}{k^2+\beta^2},$$

so the general eigenvalue equation (6) can be written as:

$$\left(\hat{A}+\lambda\hat{C}\right)H_y=\lambda H_y.$$

Like formula (7), the iterative formula is given as:

$$x_{m+1}=\alpha_m(\hat{A}x_m+a\hat{C}x_m) \qquad m=0,1,2\cdots. \quad (9)$$

Searching a when the convergent $\alpha_m a=1$, so the $x_m\to\alpha\psi_1$ is the eigenfunctions corresponding to the eigenvalue $\lambda_1\approx1/\alpha_m$ which is the maximum of all the eigenvalues.

IV. NUMERICAL CALCULATION EXAMPLES

Two numerical calculation examples are given for comparison with theoretical solutions. The first one is step index three layers waveguide, which structure parameters are following: core thickness is $2.0\mu m$, index $n_1=1.50$, cladding index $n_2=1.40$. The dispersion curve of TE_0 mode is given in Fig. 1, and Fig. 2 shows the errors compared with analytical solution. The numerical results conform to the analytical solution very well. Except for very small value of β, the errors are all under 10^{-7}.

At Table 1, the effective index for modes TE_0, TE_1, TM_0 and TM_1 are shown. Contrasted with analytical calculation results, the relative errors are about 10^{-7} for TE modes and 10^{-6} for TM modes.

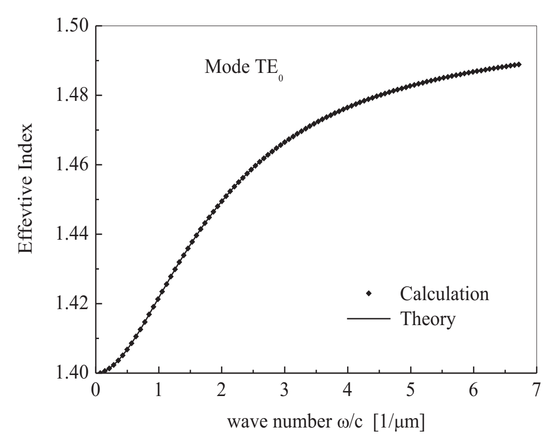

Fig. 1. Dispersion curve of mode TE_0.

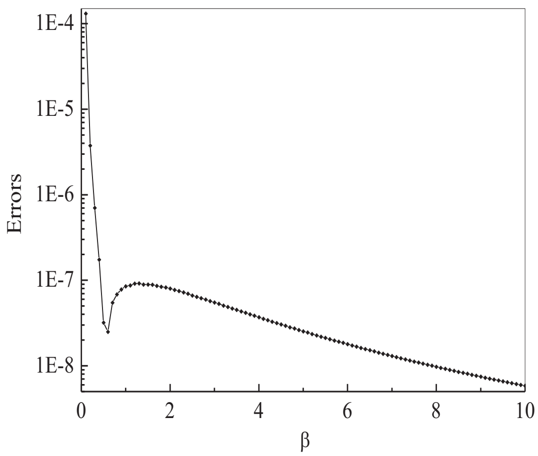

Fig. 2. The errors of dispersion curve for mode TE_0.

Table 1: Effective index of mode TE and TM

	This Work	Analytical	Relative Error
TE_0	1.47429328	1.47429290	2.581×10^{-7}
TE_1	1.41395949	1.41395911	2.735×10^{-7}
TM_0	1.47239343	1.47239162	1.230×10^{-6}
TM_1	1.41221956	1.41221736	1.563×10^{-6}

Figure 3 and Fig. 4 show the field intensity distribution of TE mode and TM mode. It is difficult to distinguish the curve from the theoretical solutions.

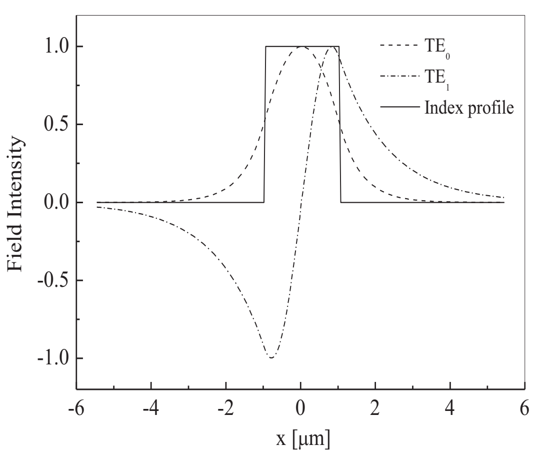

Fig. 3. Field intensity for TE_0 and TE_1 mode in a three layer step index planar waveguide.

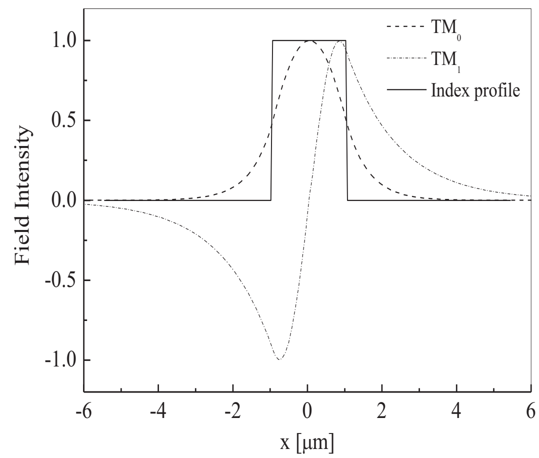

Fig. 4. Field intensity for TM_0 and TM_1 mode in a three layer step index planar waveguide.

Another example is a waveguide with a graded index profile:

$$n^2(x) = n_2^2 + (n_1^2 - n_2^2) / Cosh^2(x/a). \qquad (10)$$

For our calculation, the parameters are taken $n_1 = 1.50$, $n_2 = 1.40$ and $a = 2.0\mu m$. For TE modes, the dispersion relation is [7]:

$$\beta^2 = k_0^2 n_2^2 + \left(\sqrt{k_0^2 (n_1^2 - n_2^2) + \frac{1}{4a^2}} - \frac{1}{a}\left(m + \frac{1}{2} \right) \right)^2 \qquad (11)$$

$m = 0,1,2,\ldots,$

but there is no analytical solution for TM modes. In Table 2, we can find the difference of effective index between our calculation and analytical solver is tiny. Figure 5 shows the dispersion curve of mode TE_0 and Fig. 6 is the errors of this curve compared with theory. For almost all propagation constants β, the errors are less than 10^{-9}. Field intensity distributions of TE and TM modes are given in Fig. 7 and Fig. 8.

Table 2: Effective index for mode TE and TM

	This Work	Analytical	Relative Error
TE_0	1.462148481	1.462148481	-3.60×10^{-11}
TE_1	1.410160571	1.410160572	-2.23×10^{-10}
TM_0	1.460876908	—	—
TM_1	1.410515558	—	—

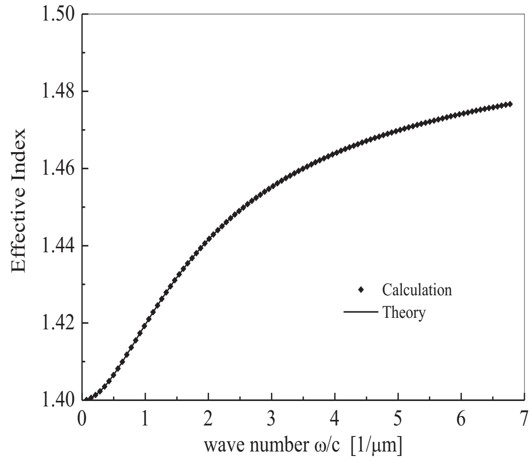

Fig. 5. Dispersion curve of mode TE_0.

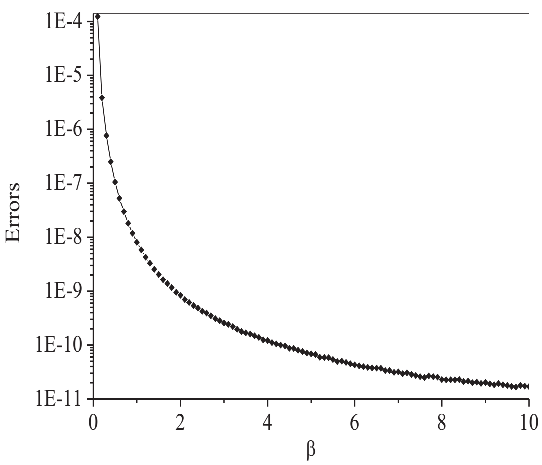

Fig. 6. The errors of dispersion curve for mode TE_0.

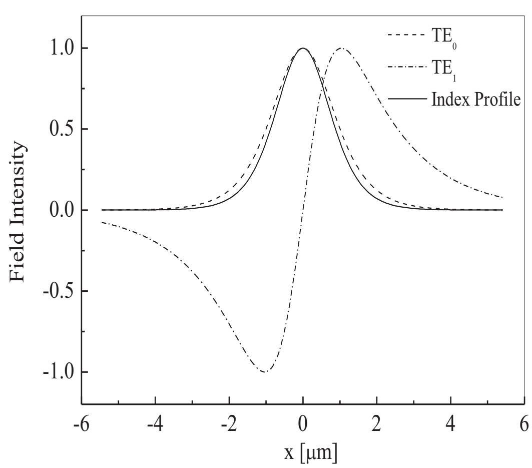

Fig. 7. Field intensity for TE_0 and TE_1 mode in a graded index waveguide.

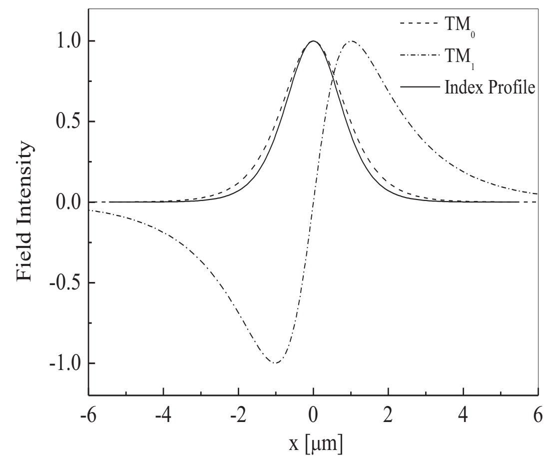

Fig. 8. Field intensity for TM_0 and TM_1 mode in a graded index waveguide.

V. CONCLUSION

In this paper, we find that in space frequency domain the Maxwell's equations for planar optical waveguides become an integral equation. This integral equation is the standard form of eigenvalue problem which can be solved by iterative algorithm. Two numerical calculation examples show that this method has a high accuracy for determining the effective index and mode field distribution simultaneously. Especially, this method is not only used for analysis of arbitrary structure planar optical waveguides, but also can be extended to 2D for optical fibers mode solution.

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REFERENCES

[1] J-E. Broquin, "Glass integrated optics: state of the art and position toward other technologies," *SPIE*, vol. 6475, pp. 647507, 2007.

[2] L. Ke and E. Y. B. Pun, "Modeling and experiments of packaged Er^{3+}–Yb^{3+} co-doped glass waveguide amplifiers," *Opt. Comm.*, vol. 273, no. 2, pp. 413-420, 2007.

[3] S. Yuyama, T. Nakajima, K. Yamashita, and K. Oe, "Solid state organic laser emission at 970 nm from dye-doped fluorinated-polyimide planar waveguides," *Appl. Phys. Lett.*, vol. 93, pp. 023306, 2008.

[4] H. Mukundan, J. Z. Kubicek, A. Holt, J. E. Shively, J. S. Martinez, K. Grace, W. K. Grace, and B. I. Swanson, "Planar optical waveguide-based

biosensor for the quantitative detection of tumor markers," *Sensor. Actuat. B-Chem.*, vol. 138, no. 2, pp. 453-460, 2009.
[5] J. Hu, V. Tarasov, A. Agarwal, L. Kimerling, N. Carlie, L. Petit, and K. Richardson, "Fabrication and testing of planar chalcogenide waveguide integrated microfluidic sensor," *Opt. Express*, vol. 15, no. 5, pp. 2307-2314, 2007.
[6] B. Agnarsson, S. Ingthorsson, T. Gudjonsson, and K. Leosson, "Evanescent wave fluorescence microscopy using symmetric planar waveguides," *Opt. Express*, vol. 17, no. 7, pp. 5075-5082, 2009.
[7] M. Chunsheng and L. Shiyong, *Mode Theory of Optical Waveguides* (in Chinese), Jilin University Press, Changchun, pp. 130-132, 2007.
[8] S. Shouxian, W. Jingyi, and L. Qiao, "Metal-clad graded-index planar optical waveguides: accurate perturbation analysis," *Opt. Comm.*, vol. 90, no. 4-6, pp. 238-240, 1992.
[9] L. Qiao and W. Jingyi, "A refined WKB method for planar waveguides with asymmetric graded index profile," *Opt. Comm.*, vol. 83, no. 1-2, pp. 144-153, 1991.
[10] Z. Anping and S. R. Cvetkovic, "Finite- element analysis of hybrid modes in uniaxial planar waveguides by a simple iterative method," *Opt. Lett.*, vol. 20, no. 2, pp. 139-141, 1995.
[11] E. Anemogiannis, E. N. Glytsis, and T. K. Gaylord, "Determination of guided and leaky modes in lossless and lossy planar multilayer optical waveguides: reflection pole method and wavevector density method," *J. Lightwave Technol.*, vol. 17, no. 5, pp. 929-941, 1999.
[12] R. E. Smith, S. N. Houde-Walter, and G. W. Forbes, "Numerical determination of planar waveguide modes using the analyticity of the dispersion relation," *Opt. Lett.*, vol. 16, no. 17, pp. 1316-1318, 1991.
[13] P. R. Chaudhuri and S. Roy, "Analysis of arbitrary index profile planar optical wave-guides and multilayer nonlinear structures: a simple finite difference algorithm," *Opt. Quant. Electron.*, vol. 39, pp. 221-237, 2007.
[14] A. A. Stratonnikov, A. P. Bogatov, A. E. Drakin, and F. F. Kamenets, "A semianalytical method of mode determination for a multilayer planar optical waveguide," *J. Opt. A: Pure Appl. Opt.*, vol. 4, pp. 535-539, 2002.
[15] K. Tsakmakidis, D. Aryal, and O. Hess, "Accurate modal analysis of 3D dielectric waveguides using the nonstandard FDTD method," *24th Annual Review of Progress in ACES*, pp. 956-961, Niagara Falls, Canada, 2008.
[16] T. A. Tamadan, "A search-and-track algorithm for controlling the number of guided modes of planar optical waveguides with arbitrary refractive index profiles," *ACES Journal*, vol. 26, no. 1, pp. 45-55, 2011.
[17] C. Zhuangqi, J. Yi, S. Qishun, D. Xiaoming, and C. Yingli, "Exact analytical method for planar optical waveguides with arbitrary index profile," *J. Opt. Soc. Am. A*, vol. 16, no. 9, pp. 2209-2212, 1999.

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