Influence of the Simulation Parameters on the Normalized Impedance Derived by the Random Coupling Model Simulation

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Abstract **–** The random coupling model (RCM, introduced by the "chaos group" in the University of Maryland, is found of great use in making statistical predictions of the induced voltages and currents on objects or components within complicated (wave-chaotic) cavities when excited by external high power microwave (HPM) radiation. A key point to applying the RCM to a real system is to generate the normalized cavity impedance, which can be described by the random matrix theory (RMT), from the cavity loss parameter by using random matrix Monte Carlo simulation. The influences of the simulation parameters on the statistics of the generated normalized impedance are presented and discussed in this paper. It's found that the statistics of the normalized impedance only depends on the loss parameter, α, which agrees with the theory. When α increases, the variances of the eigenvalues, the diagonal elements and the off-diagonal elements of the normalized impedance are exponentially damped with different damping factor, which are experimentally verified in the paper.

Index Terms **–** Cavity loss parameter, normalized impedance matrix, random coupling model.

I. INTRODUCTION

The coupling of high power electromagnetic (HPEM) waves with systems such as cars, aircraft, ships, and et. al., has drawn a lot of attention. One problem in treating these problems is the short wave length nature of the radiation. When the wavelength of the radiation becomes much smaller than the enclosure size of the target system, the coupling properties depend in great detail on the size and shape of the enclosure, the geometry of the coupling ports, and on the frequency of the radiation [1]. Furthermore, the electromagnetics quantities within the enclosure will be very sensitive to the enclosure shape, the internal object positions and orientations, the external radiation frequency, and the coupling ports geometry. At present, even with a lot of fast and powerful computers that utilize efficient 3-D numerical simulation, addressing this problem is still a great challenge because of numerous computational time and CPU resources.

To deal with this problem, the "chaos group" at the University of Maryland introduced the random coupling model, which is a statistical model that can characterize the fluctuations of the impedance and scattering matrices of wave-chaotic metallic-enclosed cavity coupled with one port [2] and multiple ports [3]. This model has been extended to make statistical predictions of the induced voltages and currents on components within the enclosure when excited by external short wave-length EM radiation [4].

Considering the short trajectories effects, Hart et. al., show that the statistics of the impedance matrix, $\boldsymbol{Z}_{\text{cav}}$, of a wave-chaotic cavity coupled with N ports, can be divided into two parts that describe the universal and system-specific properties of the cavity as, [5,6]:

$$\boldsymbol{Z} = j\Im[\boldsymbol{Z}_{\text{ave}}] + (\Re[\boldsymbol{Z}_{\text{ave}}])^{1/2}\boldsymbol{z}(\Re[\boldsymbol{Z}_{\text{ave}}])^{1/2}. \quad (1)$$

The matrix, $\boldsymbol{Z}_{\text{cav}}$, is the ensemble average of cavity impedance which describes the system-specific details of the cavity.

The normalized impedance matrix, $\boldsymbol{z}$, can be derived from the random matrix theory and the random plane wave hypothesis [2,3]. The $N\times N$ normalized impedance matrix for the time reversal symmetry (TRS) electromagnetic-wave systems within which the medium is characterized by real, symmetric permittivity and permeability tensors, is shown as:

$$\boldsymbol{z}_N = \frac{1}{j\pi}\sum_{n=1}^{M}\frac{\boldsymbol{w}_N\boldsymbol{w}_N^T}{\left(k^2-k_n^2\right)/\Delta k_n^2 - j\alpha}, \quad (2)$$

where $\boldsymbol{w}_{\text{N}}$ is N dimensional vector whose elements are Gaussian random variables with zero mean and unit

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variance, α is the cavity loss parameter, k is the wave number corresponding to the incoming frequency, k_n is the wavenumber of cavity eigenvalues, Δk_n^2 is the mean spacing of the adjacent eigenvalues and for a 3D cavity $\Delta k_n^2 = 2\pi^2/(kV)$ (V is the cavity volume).

For the TRS system, the probability density function (PDF) of the normalized nearest neighbor eigenfrequency spacing ε, defined as equation (3), follows a certain universal curves which is the well-known Wigner distribution as shown in equation (4), [7]:

$$\varepsilon = \frac{k_{n+1}^2 - k_n^2}{\Delta k_n^2}, \tag{3}$$

$$P(\varepsilon) = \frac{\pi}{2}\varepsilon Exp\left(-\frac{\pi\varepsilon^2}{4}\right). \tag{4}$$

In this paper, 2×2 normalized impedance matrix $\boldsymbol{z}$ is considered. A large ensemble of $\boldsymbol{z}$ is generated based on equation (2) through random matrix Monte Carlo simulation (which will be described in Section II). In Section III and IV, the influence of the simulation parameter M and α on the statistics of $\boldsymbol{z}$ will be discussed respectively. According to the simulation results, the statistics of $\boldsymbol{z}$ only depends on α and the relation between them is experimentally verified in Section V.

II. RANDOM MATRIX MONTE CARLO SIMULATION

According to equation (2), the point to generate the normalized impedance $\boldsymbol{z}$ is to produce M independent normalized eigenvalues $k_n^2/\Delta k_n^2$ with the PDF of nearest spacing ε following the Wigner distribution. An alternative approach given in [7] is to generate an $M \times M$ real symmetric random matrix corresponding to the Gaussian orthogonal ensemble (GOE) in which the diagonal elements are independent Gaussian-distributed with zero mean and unit variance, and the off-diagonal elements have the same distribution with the diagonal elements except for the variance equaling to 0.5.

The distribution of the eigenvalues, λ_M, of the random matrix is shown in Fig. 1 (a), for M=6000 and it's not uniform. By introducing a mapping function $\varsigma(\lambda_M, M)$ (as shown in equation (5)), each λ_M is converted into a new variable λ'_M which is uniformly-distributed in ($-M/2$, $M/2$), as shown in Fig. 1 (b). The PDF of the normalized spacing of λ'_M is plotted in Fig. 2, which agrees well with the Wigner distribution:

$$\lambda'_M = \frac{M}{2\pi}\left[\pi + 2sin^{-1}\left(\frac{\lambda_M}{\sqrt{2M}}\right) + 2\frac{\lambda_M}{\sqrt{2M}}\frac{\sqrt{2M-\lambda_M^2}}{\sqrt{2M}}\right] - \frac{M}{2}. \tag{5}$$

Based on the above approach, it seems that two simulation parameters, M and α, may have influence on the statistical properties of the generated normalized impedance matrix $\boldsymbol{z}$. In this paper, with each given (M, α), 100,000 normalized matrix, $\boldsymbol{z}$, are generated and the eigenvalues, λ_z, of all the $\boldsymbol{z}$ are grouped into one set that contains 200,000 elements. The values of M and α are chosen and shown in Table 1.

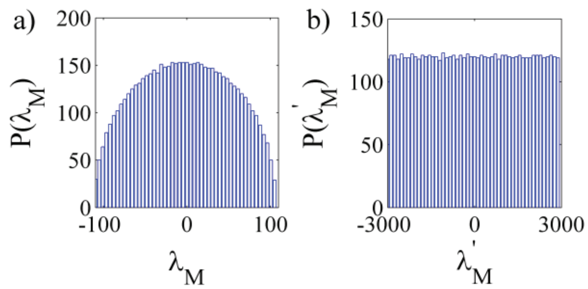

Fig. 1. (a) Distribution of the eigenvalues, λ_M, of 6000×6000 sized random matrix corresponding to GOE, and (b) distribution of the mapped eigenvalue, λ'_M, converted from λ_M shown in (a) through equation (5).

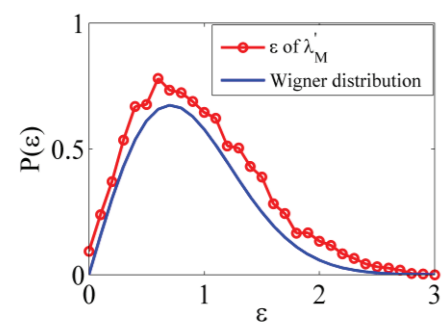

Fig. 2. The PDF of the normalized nearest neighbor eigenfrequency spacing for the mapped eigenvalues λ'_M which agrees with the Wigner distribution.

Table 1: Selections of simulation parameters

M	α
500	0.1-15 in a step of 0.1
1000	0.1-15 in a step of 0.1
6000	1-100 a step of 1

III. VARIATION OF THE STATISTICS OF λ_z FOR DIFFERENT VALUES OF M

Figures 3 to 5 show the PDFs of the real part, Re[λ_z], and the imaginary part, Im[λ_z], of the eigenvalues of the normalized impedance matrix with different M for α equaling to 0.1, 6 and 15 respectively. It can be seen that the value of M doesn't have significant effect on the statistics of the normalized impedance with a given α while α<15. In consideration of the simulation time which consumed most in calculating the eigenvalues of random matrices with high order, M can be chose to 1000.

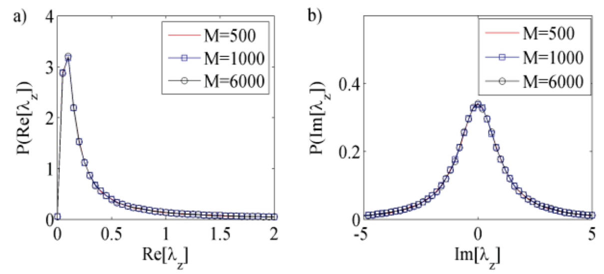

Fig. 3. (a) PDFs of the real part, and (b) the imaginary part of λ_z with different values of M when α equals to 0.1.

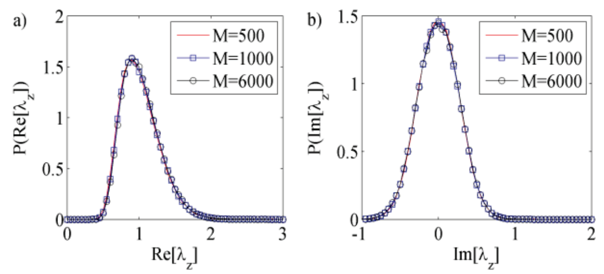

Fig. 4. (a) PDFs of the real part, and (b) the imaginary part of λ_z with different values of M when α equals to 6.

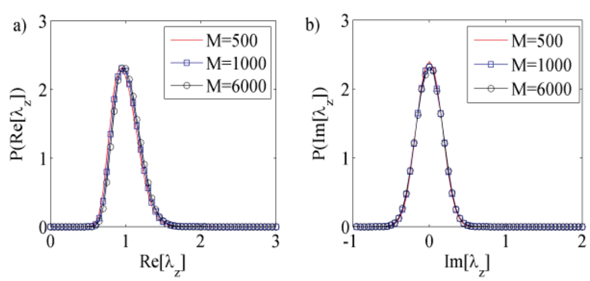

Fig. 5. (a) PDFs of the real part, and (b) the imaginary part of λ_z with different values of M when α equals to 15.

IV. VARIATION OF THE STATISTICS OF λ_z FOR DIFFERENT VALUES OF α

When α=0, the eigenvalues of the normalized impedance matrix are purely imaginary quantities. In this limit, [3] has shown that the eigenvalues are Lorentzian distributed with zero mean and unit width. When α=∞, both the real part and the imaginary part of the eigenvalues will be Gaussian distributed with the mean value equal to 1 and 0, respectively.

The PDFs of (a) the real part and (b) the imaginary part of the grouped eigenvalues of the normalized impedance matrix with different values of α are shown in Fig. 6. It can be seen that as α increase, the PDF of Re[λ_z] evolves from being peaked between Re[λ_z]=0 and Re[λ_z]=1, into a Gaussian-type distribution that peaked at Re[λ_z]=1 for large α, and the PDF of Im[λ_z] begins to sharpen up, developing a Gaussian appearance, which is in good agreement with the description in [7].

The variances of the real part and the imaginary part of the eigenvalues of $\boldsymbol{z}$ are identical and [7] gives the relation between them and the loss parameter which is shown in equation (6):

$$\sigma^2_{\mathrm{Re}[\lambda_z]} = \sigma^2_{\mathrm{Im}[\lambda_z]} = \frac{1}{\pi\alpha}, \quad \text{for } \alpha \gg 1, \tag{6}$$

where, σ^2 is the variance, Re[λ_z] and Im[λ_z] denote the real part and the imaginary part of the eigenvalues respectively.

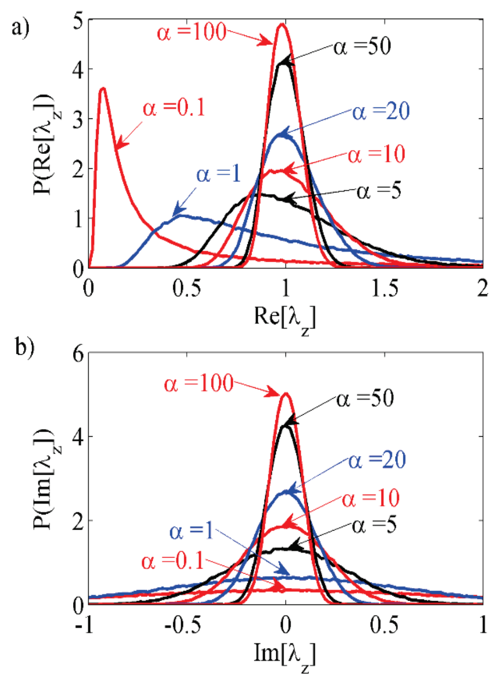

Fig. 6. (a) PDFs of the real part, and (b) the imaginary parts of λ_z with different values of α.

A more accurate result can be derived from Fig. 7, that is:

$$\sigma^2_{\mathrm{Re}[\lambda_z]} = \sigma^2_{\mathrm{Im}[\lambda_z]} = \frac{4}{3\pi\alpha}. \tag{7}$$

Based on Fig. 8, the relation between the variances of the diagonal elements of $\boldsymbol{z}$ and the loss parameter is:

$$\sigma^2_{\mathrm{Re}[z_{jj}]} = \sigma^2_{\mathrm{Im}[z_{jj}]} = \frac{1}{\pi\alpha}, \tag{8}$$

where, the z_{jj} (j=1 or 2) denotes the diagonal element of the normalized impedance matrix $\boldsymbol{z}$.

The relation between the variances of the off-diagonal elements and the loss parameter is shown in Fig. 9 and equation (9), which is in good agreement with that given in [8]:

$$\sigma^2_{\mathrm{Re}[z_{ij}]} = \sigma^2_{\mathrm{Im}[z_{ij}]} = \frac{1}{2\pi\alpha}. \tag{9}$$

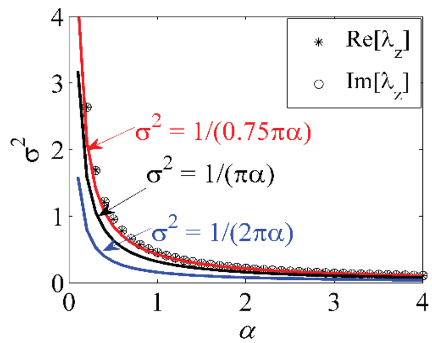

Fig. 7. The variances of real part and imaginary part of the eigenvalues of z vary with increasing α.

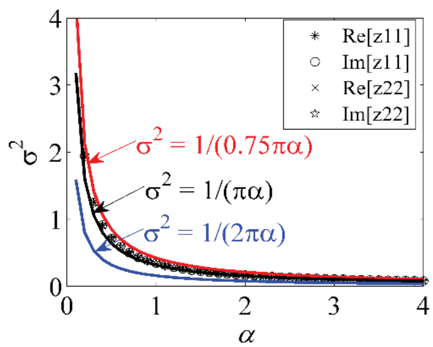

Fig. 8. The variances of real part and imaginary part of the diagonal elements of z vary with increasing α.

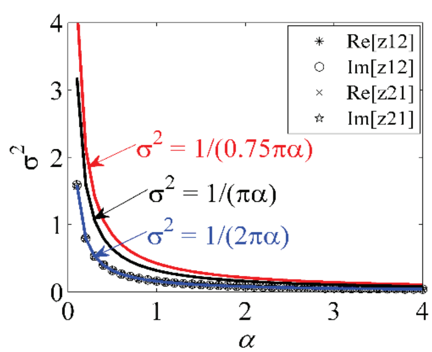

Fig. 9. The variances of real part and imaginary part of the off-diagonal elements of z vary with increasing α.

V. EXPERIMENTAL VERIFICATION

To verify the results given in part III and IV, an experiment is carried out on an empty computer case (40 cm × 40 cm × 20 cm) which is excited by two ports as shown in Fig. 9. The measurement is typically carried by measuring the scattering matrix $\boldsymbol{S}$, which will be transferred to the impedance matrix $\boldsymbol{Z}$, obtained through standard bilinear relationship:

$$\boldsymbol{Z} = \boldsymbol{Z}_0^{1/2}(\boldsymbol{I}+\boldsymbol{S})(\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{Z}_0^{1/2}, \quad (10)$$

where, $\boldsymbol{Z}_0$ is a diagonal matrix of the characteristic impedance of the transmission line connected to the ports of cavities which is 50 Ω in our experiment and I is the unit matrix.

In order to realize a larger ensemble of the cavity impedance matrix for analyzing its statistics, a mode stirrer which comprises of a shaft and two metallic coated, orthogonally oriented blades is employed, as shown in Fig. 10. By rotating the mode stirrer in control of the stepper motor, the orientation of the blades is changed and each orientation corresponds to a different inner configuration. The blades are rotated for 100 different orientations in this paper.

For each configuration, the scattering parameters between two ports are measured by network analyzer from 6 GHz to 7 GHz and the number of points is set to 1000.

Then the normalized impedance matrix can be calculated from the cavity impedance matrix through equation (1). The variances of the eigenvalues, the diagonal elements and the off-diagonal elements are listed in Table 2 with the relevant loss parameters calculated by equations (7), (8) and (9) respectively. For each loss parameter, 100,000, 2×2 normalized impedance matrices are generated by doing Monte Carlo simulation with M=1000.

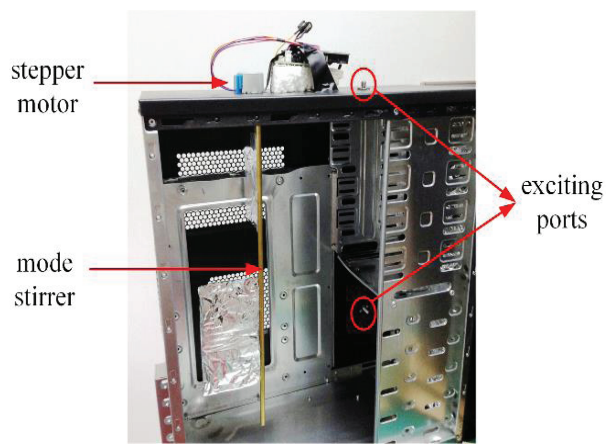

Fig. 10. Experimental setup.

To compare the statistics of these generated normalized impedance matrices with different loss parameters, the PDFs of the eigenvalues of z are chosen to be compared. The PDFs of (a) the real part, and (b) the imaginary part of the eigenvalues of z are shown in Fig. 11. It's obvious that the PDFs of the real and imaginary parts of the eigenvalues of the Monte Carlo simulated normalized impedance with the losses listed in Table 2 are almost identical, which are in good agreement with those derived from the measured data.

Table 2: The variances of the eigenvalues and elements of $\boldsymbol{z}$

$\sigma^2(\times 10^{-2})$	λ_z	z_{11}	z_{22}	z_{12}	z_{21}
Re	6.67	5.66	4.71	2.65	2.49
Im	6.68	5.69	4.70	2.64	2.50
α	6.36	5.61	6.76	6.02	6.38

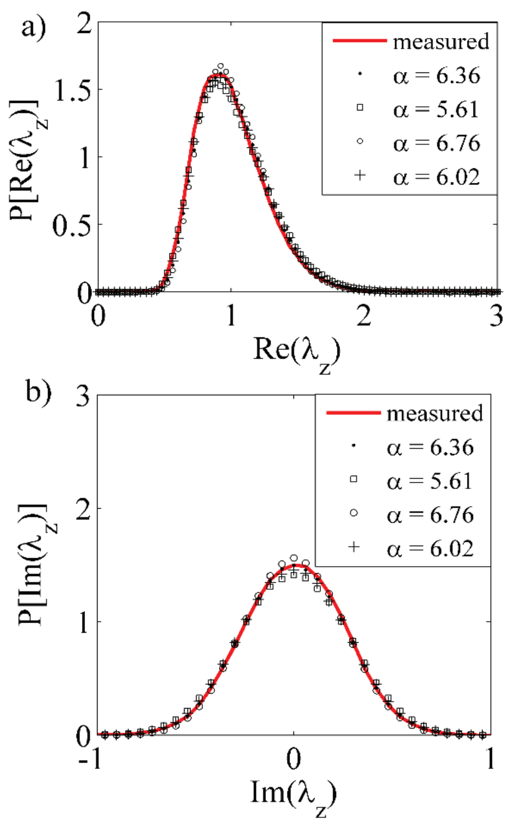

Fig. 11. PDFs of: (a) the real part, and (b) the imaginary part of the eigenvalues of the normalized impedance derived from the measured data (red curve) and those from the Monte Carlo simulated normalized impedance with different loss parameters listed in Table 2.

VI. CONCLUSIONS

It is concluded that, the statistics of the normalized impedance only depends on the cavity loss parameter α, which is in good agreement with the theoretical result. When α increases, the variances of the real part and the imaginary part of the eigenvalues, the diagonal elements and the off-diagonal elements of the normalized impedance matrix are damped as a function of α shown in equations (7), (8) and (9) respectively, which has been experimentally verified.

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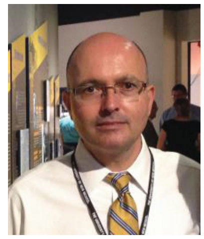

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