# Loop-based Flux Formulation for Three-dimensional Magnetostatic Problems

**Yan-Lin Li [1], Sheng Sun [2], and Zu-Hui Ma [3]**

[1] Department of Electrical and Electronic Engineering
The University of Hong Kong, Hong Kong, China

[2] School of Electric Engineering
University of Electronic Science and Technology of China, Sichuan, China
sunsheng@ieee.org

[3] School of Information Science and Engineering
Yunnan University, Yunnan, China

***Abstract*** **─** In this paper, loop basis functions are introduced to expand the magnetic flux density and the magnetostatic subset of Maxwell's equations are solved in a compact and straightforward manner using finite element method. As linear combinations of div-conforming Schaubert-Wilton-Glisson basis functions in three-dimensional, loop basis functions are inherently divergence-free and originally constructed to represent solenoidal electric current density in the electric field integral equation. Sharing the same physical property with the solenoidal electric current density, the magnetic flux density can also be represented by the loop basis functions and thus, Gauss' law for magnetism is naturally satisfied; which is out of the capability of general Whitney elements. The relationship between the loop basis functions and Whitney elements, as well as the comparison between the proposed method and traditional method pertinent to magnetic vector potential are investigated.

***Index Terms*** **─** Finite element method, flux formulation, loop basis function, magnetostatic problems.

## I. INTRODUCTION

Magnetostatic boundary value problems (BVPs) are generally described by Ampère's law, Gauss' law for magnetism, and corresponding boundary conditions. For complex structures, various numerical methods, including finite element method (FEM), boundary element method (BEM) and finite difference method (FDM) are used to model the flow of magnetostatic fields. Various kinds of formulations are proposed, where the unknowns of the system might be different. As one of most popular methods, the magnetic vector potential **A** was introduced to construct a vector potential formulation, and several gauge conditions were applied to eliminate the nullspace of the resultant matrix system [1–4]. For ringlike current problems, the magnetic field **H** could be obtained from the total scalar potential and reduced scalar potential in different regions [5–7]. Mixed formulations with **H** or **B** being the principle unknown(s) [8–10], were proposed to overcome the computational drawbacks brought about by the aforementioned potential formulations, such as the numerical cancellation and weak enforcement of some physical laws. Although the potential formulations have been well developed in the past few decades, field oriented formulations are still attractive since they work directly with physically meaningful quantities and thus, the implementation is quite straightforward. However, the number of unknowns becomes relatively large because two sets of degrees of freedoms (DoFs) are involved, and specific techniques should be applied to solve the resultant indefinite matrix systems [8–10].

To alleviate the computational burden and complexity of the mixed formulations, one can think in the following ways: consider Gauss' law for magnetism as a gauge condition and incorporate it into Ampère's law, just like the Coulomb gauged vector potential formulation, and thus, only one of the two DoFs is necessary; or expand **B** by certain basis functions such that Gauss' law for magnetism is satisfied automatically and only Ampère's law needs to be solved. The former is unclear because it is difficult to find proper expansion basis functions for **H** or **B** as both the divergence and curl operators will act on it simultaneously; while the later is available, thanks to the successful application of loop basis functions in the electric field integral equation (EFIE) [11–14]. The loop basis functions are linear combinations of the Schaubert-Wilton-Glisson (SWG) basis functions [15] in three-dimensional (3D). The SWG basis functions are divergence-conforming, while loop basis functions are divergence-free, which is consistent with the physical nature of the solenoidal

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current density $\mathbf{J}_{sol}$. Furthermore, the loop basis functions are defined with respect to edges of the geometrical meshes. Hence, the loop representation of $\mathbf{J}_{sol}$ greatly reduces the number of unknowns, in comparison with the SWG representation [14]. As $\mathbf{B}$ shares the same physical property with $\mathbf{J}_{sol}$, the introduction of the loop representation into finite element models pertinent to $\mathbf{B}$ is of great interest and importance.

In this paper, the application of the loop basis functions in finite element modeling is investigated and a novel flux formulation, which works solely with **B**, is proposed for solving 3D magnetostatic problems. By virtue of connection between Whitney elements [4, 16] and SWG basis functions, the space formed by the loop basis functions can be proved to be a subset of Whitney forms, from which one can further conclude that the proposed flux formulation is consistent with the vector potential formulation. In addition, since **H** is not accounted and the loop basis functions are associated to edges of the geometrical mesh, the number of unknowns of the proposed flux formulation is much less than that of the mixed formulations. In other words, the proposed flux formulation alleviates the computational burden and complexity, while retains the virtue of the mixed formulations.

The remainder of this paper is organized as follows. The loop basis functions are constructed and their connection to Whitney elements is demonstrated in Section 2. In Section 3, the proposed flux formulation is derived and compared with the vector potential formulation. In Section 4, numerical examples are presented to verify the accuracy and effectiveness of the proposed flux formulation. Finally, this paper is concluded by an overview of the proposed flux formulation in Section 5.

## II. LOOP BASIS FUNCTIONS FOR 3D FINITE ELEMENT MODELING

In a 3D tetrahedral mesh, the loop basis functions are associated to edges. As shown in Fig. 1, the loop basis function with regard to edge $\vec{e}_{23}$ can be defined as [12]:

$$\boldsymbol{L}_{12}(\mathbf{r}) = \begin{cases} \dfrac{\mathbf{r}_{i2} - \mathbf{r}_{i1}}{V_i}, \ \mathbf{r} \in \Omega_i, \ i = 1,\ 2,\ 3,\ 4 \\ 0, \ \text{otherwise} \end{cases}, \tag{1}$$

with $V_i$ the volume of $\Omega_i$. $\mathbf{r}_{i2}$ and $\mathbf{r}_{i1}$ denote the positions corresponding to the ending and starting vertexes, respectively, of the edge opposite to $\vec{e}_{23}$. It is worthy to note that the loop basis function follows the right hand rule with regard to $\vec{e}_{23}$. $\boldsymbol{L}_{12}(\mathbf{r})$ can be written in the form of the SWG basis functions:

$$\boldsymbol{L}_{12}(\mathbf{r}) = 3\sum_{n=3}^{6} l_n \boldsymbol{f}_{12n}(\mathbf{r}), \tag{2}$$

where $l_n$ is either 1 or −1, indicating a flux flowing out of or into $\Omega_i$, respectively, and the subscripts denote the three vertices of a facet. Meanwhile, the curl of Whitney field element with regard to $\vec{e}_{23}$ can be expressed as [17]:

$$\nabla \times \vec{\omega}_{12}(\mathbf{r}) = \begin{cases} \dfrac{\mathbf{r}_{i2} - \mathbf{r}_{i1}}{3V_i}, \ \mathbf{r} \in \Omega_i, \ i = 1,\ 2,\ 3,\ 4 \\ 0, \ \text{otherwise} \end{cases}, \tag{3}$$

and the linear supposition of Whitney flux elements [16]:

$$\nabla \times \vec{\omega}_{12}(\mathbf{r}) = \sum_{n=3}^{6} l_n \vec{f}_{12n}(\mathbf{r}). \tag{4}$$

From (1), (3) and (4), it is straightforward to find:

$$\boldsymbol{L}_{12}(\mathbf{r}) = 3\nabla \times \vec{\omega}_{12}(\mathbf{r}) = 3\sum_{n=3}^{6} l_n \vec{f}_{12n}(\mathbf{r}), \tag{5}$$

which indicates that the loop basis functions are linear suppositions of Whitney flux elements as well. Furthermore, the 3D loop basis function is apparently divergence-free, i.e.:

$$\nabla \cdot \boldsymbol{L}_{12}(\mathbf{r}) = 3\nabla \cdot \nabla \times \vec{\omega}_{12}(\mathbf{r}) = 0. \tag{6}$$

The above derivation is applicable for every internal edge. For edges at the boundary, half loop basis functions [14] can be defined, which can be considered as full loop basis functions with virtual outside tetrahedra with relative permittivity $\mu_r$ = 1.0. Generally speaking, the loop basis functions include both full loops for the internal edges and half loops for those at the boundary.

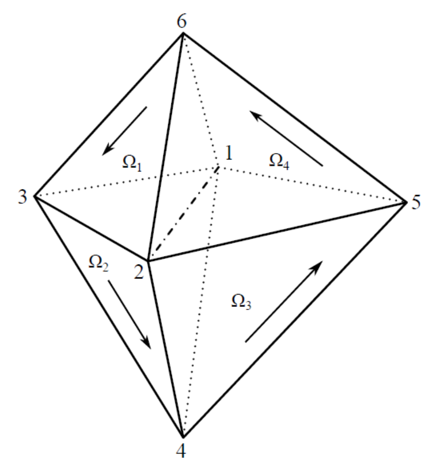


Fig. 1. The loop basis function defined for $\vec{e}_{23}$.

## III. FLUX FORMULATION FOR MAGNETOSTATIC PROBLEMS

### A. Governing equation

Consider a general 3D BVP as shown in Fig. 2. Assume that the structure is inhomogeneously composed of three bodies, $\Omega_0$, $\Omega_1$ and $\Omega_2$, among which $\Omega_0$ is bounded by $\Gamma_D$ (solid line) and $\Gamma_N$ (dash dotted line); $\Omega_1$ and $\Omega_2$ are bounded by $\Gamma_1$ and $\Gamma_2$, respectively. In addition, the structure is excited by an impressed current source **J**. Thus, **B** satisfies the subset Maxwell's equations:

$$\nabla \times \frac{1}{\mu}\mathbf{B} = \mathbf{J}, \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{8}$$

with $\mu$ the magnetic permeability. In addition, two kinds of boundary conditions are imposed on $\Gamma_D$ and $\Gamma_N$, respectively, i.e.:

$$\hat{n} \times \frac{1}{\mu}\mathbf{B}(\mathbf{r}) = \mathbf{K},\ \mathbf{r} \in \Gamma_N, \tag{9}$$

$$\hat{n} \cdot \mathbf{B}(\mathbf{r}) = b,\ \mathbf{r} \in \Gamma_D, \tag{10}$$

where $\hat{n}$ is the unit normal vector to the surface. $b$ and **K** denote the normal component of **B** and surface current, respectively, which are of clear physical meaning. In the vector potential formulation, however, (7), (9) and (10) are rewritten as:

$$\nabla \times \frac{1}{\mu}\nabla \times \mathbf{A} = \mathbf{J}, \tag{11}$$

$$\hat{n} \times \frac{1}{\mu}\nabla \times \mathbf{A} = \mathbf{K},\ \mathbf{r} \in \Gamma_N, \tag{12}$$

$$\hat{n} \times \mathbf{A} = \boldsymbol{\alpha},\ \mathbf{r} \in \Gamma_D, \tag{13}$$

by assuming,

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{14}$$

$$\nabla \cdot \boldsymbol{\alpha} = b, \tag{15}$$

where the selection of $\boldsymbol{\alpha}$ is not evident [18].

Generally speaking, **B** is governed by (7-10), which are called the flux formulation and lead to an over determined system. To make the system solvable, an additional quantity, **H**, **A** or the reduced scalar potential $\varphi$, is introduced into the system for the mixed formulations. Hence, two unknown quantities are involved. Actually, the over determined problem can be solved by reducing the number of equations instead of adding more unknowns. As the loop basis functions are inherently divergence-free, (8) is automatically satisfied and hence, can be discarded if **B** is approximated by them. Similar strategy is applied in the vector potential formulation, where (8) is discarded due to (14).

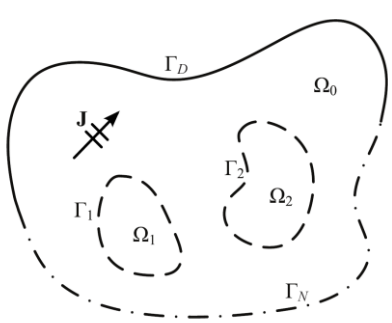


Fig. 2. A general 3D structure excited by an impressed current source **J**.

**B. Finite element discretization**

Expanding **B** with loop basis functions yields:

$$\mathbf{B} = \sum_{n=1}^{N_l} x_n \boldsymbol{L}_n(\mathbf{r}), \tag{16}$$

where $N_l$, the number of the loop basis functions, is equal to the number of edges of the geometrical mesh, which is also the number of expansion elements for the vector potential formulation; $x_n$ is the corresponding unknown coefficient to be determined. Since (8) is automatically satisfied, Galerkin's technique can be directly applied to the BVP governed by (7, 9, 10). Testing (7) with $\vec{\omega}_m$ reads:

$$\int_\Omega \vec{\omega}_m \cdot \left(\nabla \times \frac{1}{\mu}\mathbf{B}\right) d\Omega = \int_\Omega \vec{\omega}_m \cdot \mathbf{J} d\Omega. \tag{17}$$

Integrating the left hand side of (17) by parts yields:

$$LHS = \int_\Omega \frac{1}{\mu}(\nabla \times \vec{\omega}_m) \cdot \mathbf{B} d\Omega - \int_\Gamma \vec{\omega}_m \cdot \left(\frac{1}{\mu}\mathbf{B} \times \hat{n}\right) d\Gamma. \tag{18}$$

For simplicity and without loss of generality, the homogeneous boundary conditions [3, 4], **K** = 0 in (9) and $b$ = 0 in (10), are applied. Substituting (9, 10, 16, 18) into (17), one can obtain:

$$\sum_{n=1}^{N_l} x_n \int_\Omega \frac{1}{\mu} \boldsymbol{L}_m \cdot \boldsymbol{L}_n d\Omega = \int_\Omega \vec{\omega}_m \cdot \mathbf{J} d\Omega. \tag{19}$$

Note that the numbers of unknowns of the vector potential formulation, the mixed formulation (e.g., **H**-**B** formulation), and the proposed flux formulations are $N_e$, $N_e + N_f$ and $N_e$, respectively, where $N_e$ and $N_f$ are the numbers of edges and facets, respectively. Besides, it is interesting to find that the vector potential formulation and the proposed flux formulation are consistent in matrix condition. Specifically, **A** can be expanded by $\vec{\omega}(\mathbf{r})$, i.e.:

$$\mathbf{A} = \sum_{n=1}^{N_e} a_n \vec{\omega}_n(\mathbf{r}). \tag{20}$$

As implied in (5), the vector potential formulation and the proposed flux formulation should have the same solution space, with dimension $N_e - N_n + 1$ ($N_n$ is the number of nodes), which lead to rank deficiency of the matrices. Fast convergence is achieved when the matrix systems are solved using iterative methods [2, 14]. However, the sign of $\nabla \times \vec{\omega}(\mathbf{r})$ in each tetrahedron is determined by the orientation of the tetrahedron [4], while the sign of $\boldsymbol{L}(\mathbf{r})$ in each tetrahedron is determined more straightforwardly by the right hand rule, as shown in Fig. 1.

In sum, the proposed flux formulation is consistent with the vector potential formulation. The former is advantageous in physical interpolation as well as numerical implementation of the boundary conditions over the later. At the same time, as **B** is traditionally expanded by Whitney-2 form (flux space) elements with dimension $N_f$, the number of unknowns can be greatly reduced if loop basis functions, with dimension $N_e$, are applied to expand **B**, since $N_e$ is generally much smaller than $N_f$.

## IV. NUMERICAL VERIFICATION

As shown in Fig. 3, the IEEJ model [19, 20], which is proposed by the Institute of Electrical Engineers in Japan, is investigated to verify the proposed flux formulation. All the dimensions are in *mm*. As the structure is symmetrical, only the portion lying in the first quadrant, instead of the whole domain, is discretized.

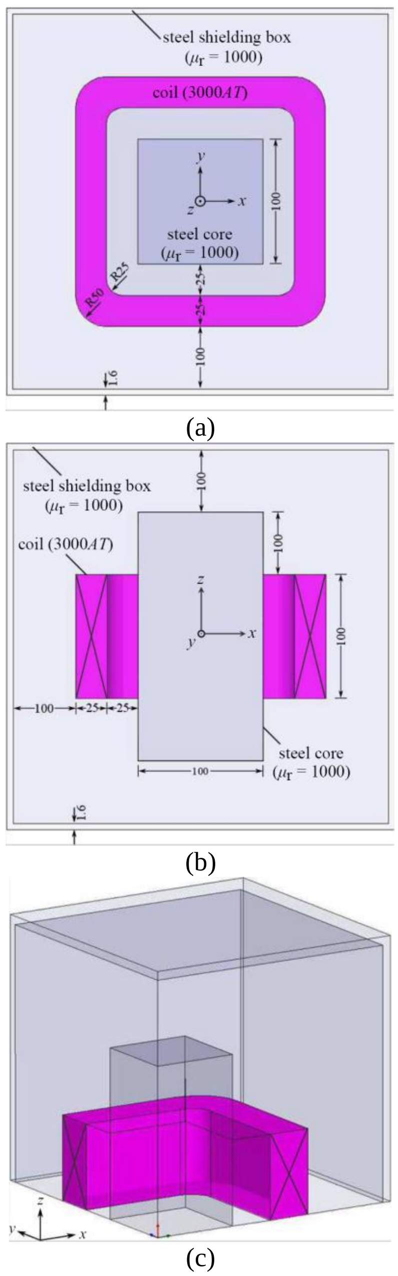


Fig. 3. IEEJ model, which contains a cubic iron core surrounded by a racetrack coil: (a) top view, (b) sectional view, and (c) 3D view of 1/8 domain.

As shown in Fig. 4, the variation of $B_x$ and $B_z$ along *z* axis obtained by the proposed flux formulation is compared with that obtained by the vector potential formulation. Also, |**B**| values at several sample points are listed, in contrast to the measurement [20], in Table 1. From Fig. 4 and Table 1, one can see that the results obtained by the two formulations agree with each other very well. Considerable but acceptable numerical error occurs at point #1, which might be caused by the quality of the mesh. Furthermore, field distributions of **B** are shown in Fig. 5 and the detailed statistics of the computational cost of the numerical methods is listed in Table 2, where Bi-CGSTAB iterative algorithm [21] is used and $10^{-6}$ accuracy is achieved. Obviously, the memory consumptions of the two formulations are almost the same, while the convergence of the proposed flux formulation is a little bit slower than that of the vector potential formulation.

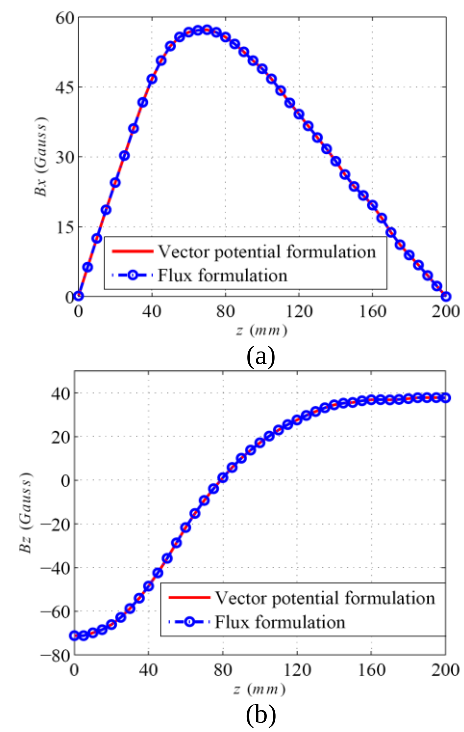


Fig. 4. Variation of (a) *Bx* and (b) *Bz* along *z* axis.

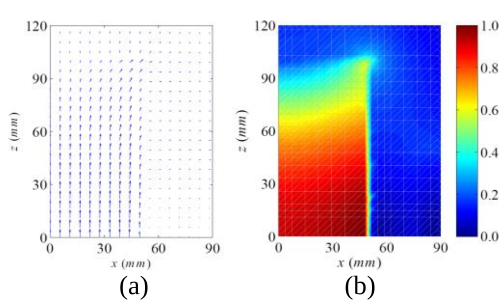


Fig. 5. (a) Vector and (b) magnitude distribution of **B**.

In sum, the proposed flux formulation exhibits a numerical performance as excellent as that of the vector potential formulation, while it is more straightforward since it works with physically meaningful quantity **B**. The application of the loop basis functions in finite element modeling of 3D magnetostatic problems is verified to be accurate and effective. The proposed flux formulation have some potential applications when the governing equations are pertinent to **B**, e.g., the **E**-**B** formulation for dynamic problems [22, 23], where the Whitney flux elements are applied and thus additional effort is needed to enforce the divergence-free condition of **B**. Fortunately, the divergence-free condition of **B** is guaranteed by definition and no more effort is needed if the loop basis functions are applied.

Table 1: Comparison of |**B**| values at sampled points

| Coordinates of Sampled Points (*mm*) | \|**B**\| (*Gauss*) | | |
|---|---|---|---|
| | Vector Potential Formulation | Flux Formulation | Measurement |
| #1 (0, 0, 110) | 254.8656 | 254.8647 | 240.1 |
| #2 (40, 0, 110) | 306.3903 | 306.3896 | 298.1 |
| #3 (40, 40, 110) | 355.6713 | 355.6704 | 355.0 |

Table 2: Computational cost of the numerical methods

| | Number of Unknowns | Matrix Sparsity[1] | Iterative Steps |
|---|---|---|---|
| Vector Potential Formulation | 425428 | $3.49691\times10^{-5}$ | 339 |
| Flux formulation | 425428 | $3.49690\times10^{-5}$ | 365 |

[1] Defined as the ratio of the number of nonzero entries to the number of total entries.

## V. CONCLUSION

The loop basis functions, which are originally proposed to expand solenoidal electric current density in EFIE, are proved to be in Whitney-2 form. They inherit the normal continuity of Whitney facet elements and are divergence-free. They have been applied to expand **B** in the finite element modeling of 3D magnetostatic problems. This implementation makes Gauss' law be satisfied naturally and thus leading to a compact and straightforward flux formulation, which solely works with **B**. This formulation retains the clear physical interpolation of the mixed formulation, while becomes more elegant and compact. At the same time, it can compete with the vector potential formulation in both accuracy and computational cost. This application of the loop basis functions provides a novel perspective to reconsider the BVPs and basis expansion of solenoidal quantities in the realm of FEM.

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**Yan-Lin Li** received the B.Eng. degree in Electronics from Beijing Jiaotong University (BJTU), Beijing, China, in 2007, M.S. degree in Physical Electronics from the University of Chinese Academy of Sciences (UCAS), Beijing, China, in 2010, and Ph.D. degree in Electrical and Electronic Engineering from the University of Hong Kong, Hong Kong, China, in 2015.

His research interests include numerical methods in computational electromagnetics, multi-physics modeling, and wireless power transfer.

**Sheng Sun** received the B.Eng. degree in Information Engineering from the Xi'an Jiaotong University, China, in 2001, and the Ph.D. degree in Electrical and Electronic Engineering from the Nanyang Technological University, Singapore, in 2006.

He was with the Institute of Microelectronics in Singapore (2005-2006), and with the NTU (2006-2008) as a Postdoc Research Fellow. He was also a Humboldt Research Fellow with the Institute of Microwave Techniques at the University of Ulm in Germany (2008-2010), and a Research Assistant Professor at The University of Hong Kong (2010-2015). Since 2015, he has been the Young Thousand Talents Plan Professor at The University of Electronic Science and Technology of China (UESTC). His research interests include electromagnetic theory and computational mathematics, multi-physics, numerical modeling of planar circuits and antennas, microwave passive and active devices, as well as the microwave and millimeter-wave communication systems. He has authored and co-authored one book, over 100 journal and conference publications. He received the Outstanding Reviewer Award for IEEE Microwave and Wireless Components Letters, in 2010. He was an Associate Editor for the IEICE Transactions on Electronics (2010-2014).

Sun was the recipient of the General Assembly Young Scientists Award from the International Union of Radio Science (URSI), in 2014. He also received the Hildegard Maier Research Fellowship of the Alexander Von Humboldt Foundation (Germany), in 2008, and the recipient of the ISAP Young Scientist Travel Grant (Japan), in 2004.

**Zu-Hui Ma** received the B.S. and M.S. degrees in Electromagnetics and Microwave Technology from the University of Electronic Science and Technology of China, and the Ph.D. degree in Electric and Electronic Engineering from the University of Hong Kong, in 2006, 2009 and 2013, respectively. He is now an Associate Professor in the School of Information Science and Engineering of the Yunnan University.

His research interests include numerical methods and fast algorithms in computational electromagnetics.