Arbitrary Shaped Homogeneous Invisible Ground Cloak

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Abstract **─** A general approach is introduced to design an arbitrary shaped invisible ground cloak with homogeneous constitutive parameters. The proposed homogenous ground cloak design method facilitates the fabrication process and achieves a variety of choices for choosing the cloak shape. In this approach, an arbitrary polygonal 2D cloak with any number of sides can be designed by dividing the polygonal cloak cross section into triangular segments and applying the proposed method on each one. The full wave simulations results confirm the idea performance, properly. Finally, a homogeneous ground invisible cloak for an object, which its contour is very similar to the lateral cross section of a car is designed and simulated to validate the capability and generality of the proposed idea.

Index Terms **─** Cloak, coordinate transformation, invisible.

I. INTRODUCTION

Although invisibility used to be deemed as a merely unattainable dream of mankind for years, invisibility cloaks have attracted intense attention, lately [1-6]. The coordinate transformation which is based on form invariant of Maxwell's equations has been widely used to design these invisibility cloaks [7, 8]. Free space and carpet cloaks are two kinds of invisibility cloaks. The free space cloaks hide the objects in free space. Narrow bandwidth and high loss are these cloak serious problems to achieve perfect invisibility which are mainly caused by resonant metamaterials needed for their implementations [9]. The ground cloaks or carpet cloaks are the second group of the invisible cloaks where the cloaked object lies on the flat ground reflects the wave without scattering and the observer perceives a flat ground plane, consequently [10]. Moreover, it has this advantage that its constitutive parameters are not singular and can even be made by isotropic medium [9-11]. The ground cloaks as the second proposed cloak groups have attracted a lot of researcher attentions [9-15]. Also, some demonstrations of the both free space and ground invisible cloaks in different frequency ranges have been presented in [16-19] to show the realized cloaks performances.

The coordinate transformations which are the bases of the cloak design, affect the homogeneity [20-22], order of the anisotropy [22-24] and shape of the cloak [25-28] that are the realization bottlenecks of an invisible cloak. Therefore using a set of proper transformation relations can considerably improve the cloak design approach and facilitates its realization process, consequently.

In many researches, a particular case of transformation relations for a special cloak structure has been used. For example, a triangular ground cloak has been implemented in [12] by using oblique multilayer dielectrics. A 2D triangular ground cloak has been proposed in [13] by using straightforward linear coordinate transformation. A three dimensional ground cloak with conical structure has been discussed in [14] by a rotation of the 2D cloak. An acoustic ground cloak has been proposed in [15] with pyramidal configuration for control of sound propagation and reflection, also.

In this paper, a general approach to extract the coordinate transformations for an arbitrary polygonal shaped ground cloak is proposed which results in homogeneous constitutive parameters. This approach is based on finding the suitable location to place point N, the vertex of a 2D triangular cloak. The simulation results verify the designed cloak performance, also. Afterwards, a simplifying assumption is performed that makes the proposed relations simpler. To show the capability and power of the proposed design method, a ground cloak for a schematic car cross section is designed and simulated. Moreover, homogeneity of the required materials for the proposed cloak facilitates the fabrication process and applications of the proposed cloak.

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II. TRIANGULAR CLOAKS DESIGNING PROCEDURE

Under the coordinate transformations between so-called real and virtual spaces, Maxwell's equations remain unchanged in form, which is referred as form-invariant of Maxwell's equations property [1-6]. In other words, the coordinate transformation is a consequence of form-invariant of Maxwell's equations and provides the constitutive parameters of the transformed medium. The relative permittivity and permeability of the transformed medium can be expressed by $\varepsilon'_r = \varepsilon_r AA^T / \det(A)$ and $\mu'_r = \mu_r AA^T / \det(A)$, respectively where **A** is the Jacobian matrix associated with the transformation established between the above-mentioned spaces, ε_r and μ_r denote the relative permittivity and permeability of the background (host medium). Notice that the structure background is considered as free space in this research.

In the mapping procedure of a virtual space into a real space in a 2D regime, the relations between Cartesian coordinates of two spaces are generally defined in the form of Eq. (1a) and (1b) as:

$$x' = f(x, y), \tag{1a}$$

$$y' = g(x, y), \tag{1b}$$

where *f* and *g* are functions of *x* and *y*, and (x', y') and (x, y) are associated with real and virtual spaces coordinates, respectively. Notice that since the cloak is considered uniformly along the z axis which is the cylindrical axis, it suffices to consider the coordinate transformation in 2D regime and achieves simplicity in the design procedure. One of the critical effective factors in the invisible cloak realization process is the medium homogeneity. Clearly, the homogeneous media is easier to realize rather than the inhomogeneous ones. Therefore, finding the proper relations that result in a homogeneous media facilitates the cloaks realization process, considerably. Since the components of Jacobian matrix **A**, are the first-order partial derivatives ($A_{ij} = \partial X'_i / \partial X_j$), the real space coordinates (x' and y') should be written in terms of a linear combination of virtual space coordinates (x and y) to achieve a transformed homogeneous media. For this purpose, at the first step we assume two linear relations between real and virtual coordinate spaces as:

$$x' = \alpha_1 x + \beta_1 y + \gamma_1, \tag{2a}$$

$$y' = \alpha_2 x + \beta_2 y + \gamma_2, \tag{2b}$$

where $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ are unknown coefficients in these linear relations. On the other hand, to design an invisibility cloak for the triangular bump in a 2D geometry depicted in Fig. 1 (a), the point *N* as the vertex of a triangle shaped cloak should be located in an area with possibility triangle side vertices connection by using straight lines without crossing the ΔAMB sides. In other words, we are searching for a point above the ground (*X* axis) in the exterior area of ΔAMB with possibility to simultaneously connect it with straight lines to side vertices *A* and *B* with no crossing of $\overline{AM}$ and $\overline{BM}$ sides. Obviously, such area is located above the extensions of two sides $\overline{AM}$ and $\overline{BM}$ which is shaded in Fig. 1 (a).

Figure 1 (b) shows the cross section of the triangular bump (ΔAMB) and the ground invisibility cloak consist of two triangular parts, ΔANM and ΔBNM, which are created by finding the suitable region to locate the point *N* and connecting it to vertices of ΔAMB by using straight lines. As it is shown in this figure, the extension of $\overline{NM}$ crosses the line segment $\overline{BA}$ is considered as the coordinate system origin, *O*. Notice that the extensions of $\overline{AM}$ and $\overline{BM}$ sides (dot-dashed lines) are not suitable places for the point *N* to lie on, because it results in a zero area for at least one of the left or right parts of the cloak.

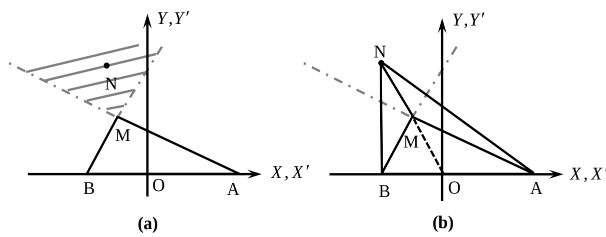


Fig. 1. A lateral view of the design process of an invisibility cloak for a triangular bump. (a) A triangular bump on the flat ground plane. The suitable area for the point *N* is dashed between dot-dashed extensions of the sides. (b) A 2D geometry of the triangular bump covered with a cloak, after choosing the place for *N*. The origin *O* is considered to be positioned at intersection of $\overline{NM}$ and $\overline{BA}$.

By considering of Fig. 1 (b), mapping of $\measuredangle OAN$ vertices, $A(x_A, y_A)$, $N(x_N, y_N)$ and $O(x_O, y_O)$ in the virtual space into ΔMAN vertices, $A(x'_A, y'_A)$, $N(x'_N, y'_N)$ and $M(x'_M, y'_M)$ in the real space, and substituting coordinates of the mentioned points in both spaces in Eqs. (2a) and (2b) result in six linear equations in terms of six unknown aforementioned coefficients. Having this in mind that points *A*, *N* and *B* should have the same coordinates in both virtual and real spaces. Therefore the coefficients associated with the right side part of the cloak (ΔANM) can be obtained as:

$$\alpha_1 = \frac{x_A - x_M}{x_A}, \quad \beta_1 = \frac{x_M (x_N - x_A)}{x_A y_N}, \quad \gamma_1 = x_M, \tag{3a}$$

$$\alpha_2 = -\frac{y_M}{x_A}, \beta_2 = \frac{x_A y_N + \{y_M (x_N - x_A)\}}{x_A y_N}, \gamma_2 = y_M. \tag{3b}$$

In a same way, mapping the ΔOBN vertices, $B(x_B, y_B)$, $N(x_N, y_N)$, and $O(x_O, y_O)$ in the virtual space into the ΔMBN vertices, $B(x'_B, y'_B)$, $N(x'_N, y'_N)$ and $M(x'_M, y'_M)$ in the real space result in the coefficients of linear Eqs. (2a) and (2b) correspond to the left side part of the cloak (ΔBNM) as:

$$\alpha_1 = \frac{x_B - x_M}{x_B}, \quad \beta_1 = \frac{x_M(x_N - x_B)}{x_B y_N}, \quad \gamma_1 = x_M, \tag{4a}$$

$$\alpha_2 = -\frac{y_M}{x_B}, \beta_2 = \frac{x_B y_N + \{y_M(x_N - x_B)\}}{x_B y_N}, \gamma_2 = y_M. \tag{4b}$$

Substituting coefficients of the linear equations (Eqs. 2) with obtained values from Eq. (3) and Eq. (4), results in design of two individually homogeneous media for the right and left side parts of the cloak.

The finite element method (FEM) simulations performed with COMSOL Multiphysics software are used to validate the proposed approach. As shown in Fig. 2, a triangular PEC bump with 0.25 m height and 0.5 m long base, covered with a 0.7 m height triangular invisible cloak is designed and simulated. The working frequency is set as 2 GHz and the simulations are performed under TE polarization of a Gaussian beam incident incoming from left with 45° azimuth angle. The direction of E_z the only component of the electric field depicted in the simulation results is normal to the 2D geometry plane of the problem. Figure 2 (a) illustrates the electric field distribution by the illumination of a perfect reflective flat ground plane with a TE-polarized Gaussian beam electromagnetic wave. Figure 2 (b) demonstrates the response of a triangular perfect electric conductor (PEC) bump lied on the ground to the Gaussian beam incidence. The protrusion of the ground surface performs some irregular scattering in the electric field such that the triangular PEC bump is detectable. Figures 2 (c) and 2 (d) show the performance of ground plane invisible cloak designed by the proposed approach. The electric field distribution of the TE-polarized Gaussian beam on the structure in Figs. 2 (c) and 2 (d) shows that the triangular bump covered with the designed invisible cloaks mimics the response of a flat PEC ground plane. Therefore the designed invisible cloak performs the triangular PEC bump perfectly undetectable. Normalized average scattering power outflows corresponding to Figs. 2 (a)-(d), have been illustrated in Fig. 2 (e), which are in good agreement with the simulations ones.

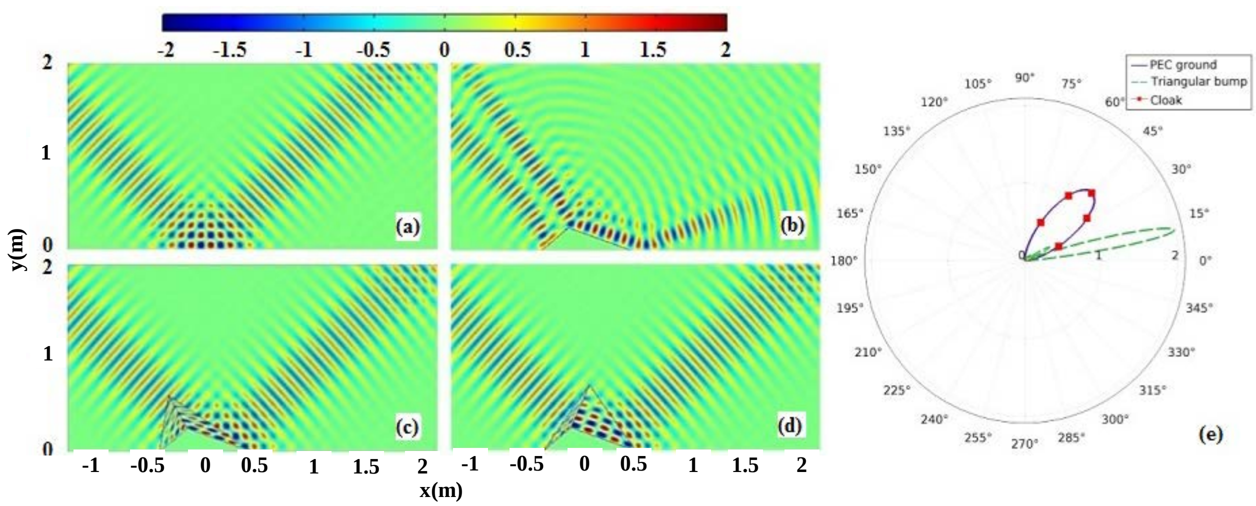


Fig. 2. The electric field distribution of a TE-polarized Gaussian beam incident from the left with *45°* azimuth angle on: (a) flat PEC ground plane, (b) triangular PEC bump with *0.25 m* height and *0.5 m* base, (c) and (d) *0.5 m* height designed cloaks for the triangular bump depicted in Fig. 2 (b), (e) Normalized average power outflows corresponding to Figs. 2 (a)-(d).

As it was mentioned previously, the proposed triangular ground cloak consists of two parts which are homogeneous. Figure 3 represents the constitutive parameters corresponding to the proposed cloak in Fig. 2 (c). Notice that choosing different places for point *N* results in different values for constitutive parameters which can create some options in the realization of the constitutive parameters. The components of the relative permittivity tensor possess a range of -0.85~3.57. It should be noted that ε_{yx} is equal to ε_{xy}, and by considering the 2D geometry of the problem, the rest of the components (ε_{xz} and ε_{yz}) are equal to zero.

According to Fig. 4 (a), choosing the point *N* to lie on a vertical straight line which crosses the vertex M ($x_N = x_M$), results in a set of simpler relations for α, β and γ coefficients for both cloak parts. By connecting vertices B and A to N with $\overline{BN}$ and $\overline{AN}$ line segments, and mapping the points o, A and B into M, A and B, the design process can be simplified as below for the right side part of the cloak:

$$\alpha_1 = 1, \qquad \beta_1 = 0, \qquad \gamma_1 = 0, \tag{5a}$$

$$\alpha_2 = -\frac{y_M}{x_A}, \qquad \beta_2 = \frac{y_N - y_M}{y_N}, \qquad \gamma_2 = y_M. \tag{5b}$$

Also, all coefficients except α_2 can be calculated by using these equations for the left side part of the cloak while α_2 for this side is:

$$\alpha_2 = -\frac{y_M}{x_B}. \tag{6}$$

As is represented in Fig. 4 (b), the triangular PEC bump depicted in Fig. 2 covered with a triangular invisibility cloak designed by using this simplifying assumption, is ideally hided from the incidence of the Gaussian beam of TE-polarized electric field. Although various positions for the point N can be chosen, here N is placed at (0, 0.7). As an inherent part of this approach, both left and right side parts of this cloak have homogeneous constitutive parameters which are very important to facilitate the realization process.

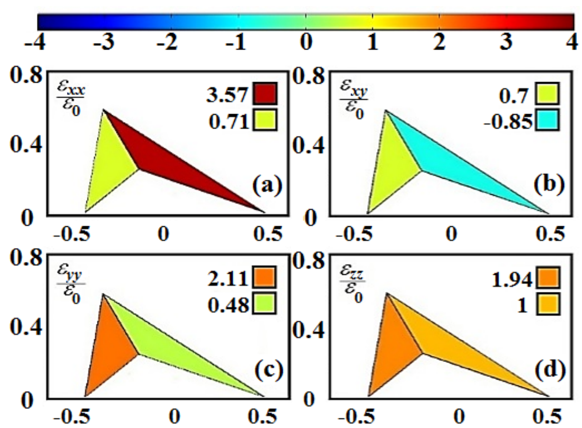
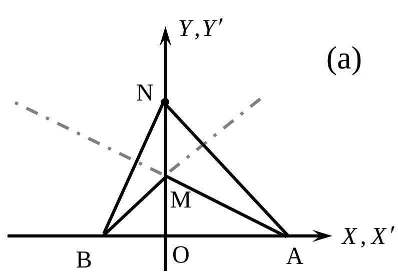


Fig. 3. The relative permittivity tensor components corresponding to the left and right side parts of the designed invisible cloak depicted in Fig. 2 (c).



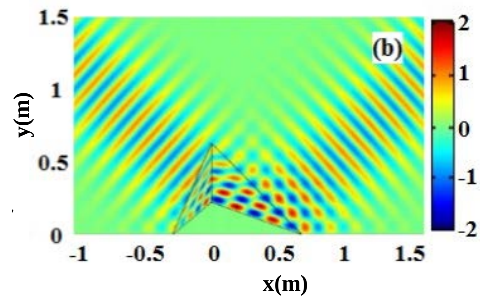


Fig. 4. A lateral view of the invisibility cloak designing procedure corresponding to the particular case in which $x_N = x_M$. (a) 2D design of triangular bump covered with the ground cloak of simpler designing relations. (b) Electric field distribution of a TE-polarized Gaussian beam incident incoming from left at an azimuth angle of *45°* on the cloaked bump at the frequency of 2 GHz.

III. HOMOGENEOUS CLOCK OF ARBITRARY SHAPE

Arbitrary shaped polygonal ground cloaks can be designed by using the proposed approach. For this purpose, the cloak is considered as a set of incomplete triangular segments as shown in Fig. 5. In this figure, a geometric lateral view of the ground cloak designing process is presented for an arbitrary polygonal shaped bump. The presented approach in the previous section can be used to design the segments of the arbitrary polygonal cloak, separately. In this way, a complete triangular cloak is considered in each segment, such that point N is laid on the y axis and $\overline{AB'C'D'E}$ path remains continuous.

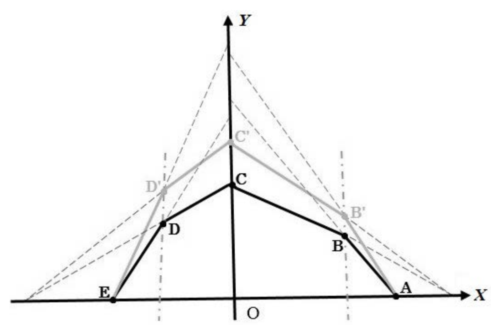


Fig. 5. A lateral view corresponding to design process of an arbitrary shaped polygonal ground plane cloak.

As an example of the cloak design for an arbitrary shaped polygonal bump, we present a 2D ground invisible cloak that its contour is very similar to lateral cross section of a car lied on a flat PEC ground surface in a free space background as shown in Fig. 6 (b). The cross section of the schematic car with 1.5 *m* length and 0.5 *m* height with outer PEC surface is illuminated by an oblique Gaussian beam which caused irregular wave scattering compared with the PEC ground plane as a highly reflective surface in Fig. 6 (a). The FEM simulation results depicted in Figs. 6 (c) and (d) verify that the designed ground cloak perfectly mimics the response of a flat ground plane to the incident wave at two arbitrary frequencies, 2 and 5 GHz, and the car is undetectable, consequently. It should be mentioned that this schematic car designed invisible cloak is consisted of homogeneous parts.

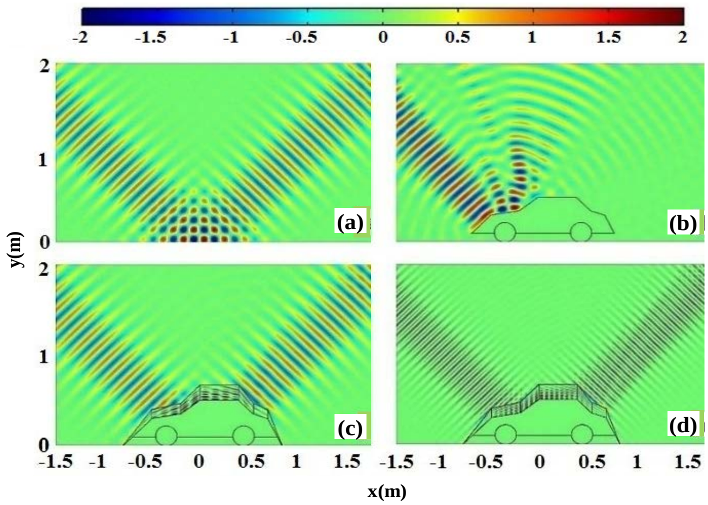


Fig. 6. The electric field distribution of a Gaussian beam incident under TE-polarization incoming from left at an azimuth angle of *45°* on: (a) flat PEC ground plane, (b) contour cross section of a schematic car (0.5 m height and 1.5 m length) with the PEC outer surface, (c) and (d) the 2D designed ground invisibility cloak for the schematic car contour of Fig. 6 (b), at 2 GHz and 5 GHz, respectively.

IV. CONCLUSION

A general approach was introduced to design arbitrary shaped invisible cloaks. By considering the problem geometry, coefficients of a linear system transformation which describes the relations between the real and virtual spaces were obtained. By using this method, the different triangular shape homogeneous cloaks for a triangular bump can be designed. By employing the mentioned relations and using the homogeneous triangular cloak segments, a homogeneous polygonal arbitrary shaped cloak has been achieved. As an example, a 2D homogeneous ground plane invisible cloak for an object where its contour is very close to the cross section of a car was designed and simulated. The FEM simulation results verify that when an oblique Gaussian beam illuminates the cloaked car, it mimics the response of a flat ground plane perfectly and renders the car undetectable, consequently.

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