

The Fourth-Order One-Step Leapfrog HIE-FDTD Method

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Abstract — A new fourth-order one-step leapfrog hybrid implicit-explicit finite-difference time-domain (HIE-FDTD) method has been proposed in this paper. This new method investigates the use of a second-order accurate in time and a fourth-order accurate in space. Because of the utilize of the one-step leapfrog theory, the proposed algorithm not only has the same formulation as that used for the traditional FDTD, but also require only one-step computations. The 2-D formulation of the method is presented and the time stability condition of the method is certified. Simulation results show that the proposed method is 6.8 times faster than the traditional second-order FDTD method and is 2.2 times faster than the second-order HIE-FDTD method, which shows that the proposed method has very high computational efficiency. On the other hand, the proposed method also has less dispersion error by comparing with traditional second-order FDTD method and the second-order HIE-FDTD method.

Index Terms — Computational efficiency, dispersive error, Finite-Difference Time-Domain (FDTD), fourth order, Hybrid Implicit- and Explicit-FDTD (HIE-FDTD), one-step leapfrog.

I. INTRODUCTION

It is well known that the Courant-Friedrich-Levy (CFL) stability condition restricts the applications of the FDTD [1-2] method when it is used to simulate structures where fine mesh needs. In order to remove the CFL limit, many improved method has been developed such as alternating direction implicit FDTD (ADI-FDTD) [3-8], Crank-Nicolson FDTD (CN-FDTD) [9], locally one-dimensional FDTD (LOD-FDTD) [10-13] and hybrid implicit and explicit-FDTD (HIE-FDTD) [14-18]. The HIE-FDTD method is weakly conditionally stable. Its time step size is not determined by fine space discretization, so, the method allows larger time step size than the conventional FDTD method, which is extremely useful for problems with very fine structures in one direction.

However, in the HIE-FDTD method, the cell size could not be larger than 1/10 of the wavelength,

otherwise the numerical dispersion inherent in the classical Yee FDTD algorithm will introduce significant errors. So, when solving the electrically large problems, a large number of cells (10-20 cells per wavelength) are required to decrease the dispersion error, which would increase the computation time inevitably.

To reduce the computation time, two approaches are used in this paper. One is to use more terms in the Taylor series to approximate the spatial derivatives. Fang [19] was the first to present this approach in conjunction with solving Maxwell's equations. He investigated the use of a second-order accurate in time and a fourth-order accurate in space FDTD algorithm, which we denote as the FDTD (2, 4) algorithm [20-24], [5], [6]. The other one is to use the one-step leapfrog method. Cooke [25] firstly presented this approach into the ADI-FDTD method. With the one-step leapfrog, electric field terms arise only on the half time step, while the magnetic field terms arise only on the full time step. Consequently, the one-step leapfrog ADI-FDTD method is much simpler and more efficient than the traditional ADI-FDTD method.

In this paper, a novel fourth-order one-step leapfrog HIE-FDTD is developed. The fourth-order one-step leapfrog HIE-FDTD method not only has the second-order accurate in time and the fourth order accurate in space, but also has the one-step leapfrog schemes. Therefore, since the use of the fourth order accurate in space, the proposed method would provide a higher accuracy than the traditional second-order FDTD method [1-2] and second-order HIE-FDTD method [15]. What's more, the proposed method would spend much less computational time because of the use of one-step leapfrog schemes and the weak stability than the fourth-order FDTD method [24].

Numerical examples in this paper demonstrate that the proposed algorithm has lower dispersion error than the conventional FDTD method even when the cell size is equal to 1/5 of the wavelength in solving some EM problems. For simplicity, the two-dimensional (2-D) fourth-order one-step leapfrog HIE-FDTD update equations are discussed in this paper. The formulations for 3-D fourth-order HIE-FDTD method can be developed

following a similar procedure.

The organization of this paper is as follows. In Section II, the formulation of the proposed algorithm is presented. The stability condition is certified in Section III. The numerical dispersion analysis and numerical results applied to the EM problems are presented in the Sections IV and V.

II. FORMULATION

For simplicity, the two-dimensional TE model with respect to z-direction is considered in this paper. The numerical formulations of the Maxwell's equations in a lossless and isotropic medium are presented as follows:

$$\frac{\partial}{\partial t}(\Psi) = (R)(\Psi). \quad (1)$$

Here, $(\Psi) = (E_x, E_y, H_z)^T$, (R) can be expressed as follows:

$$(R) = \begin{pmatrix} 0 & 0 & (1/\varepsilon)\delta_y \\ 0 & 0 & -(1/\varepsilon)\delta_x \\ (1/\mu)\delta_y & -(1/\mu)\delta_x & 0 \end{pmatrix}, \quad (2)$$

while $\delta_m = \partial/\partial m$, $m = x, y$.

By applying the one-step leapfrog technique [25] and the hybrid implicit-explicit FDTD (HIE-FDTD) [14-18] method, a set of time marching equations are derived as follows.

The space operator (R) is split into two parts, just as $(R) = (R_1) + (R_2)$, where

$$(R_1) = \begin{pmatrix} 0 & 0 & (1/\varepsilon)\delta_y \\ 0 & 0 & -(1/\varepsilon)\delta_x \\ (1/\mu)\delta_y & 0 & 0 \end{pmatrix},$$

$$(R_2) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -(1/\mu)\delta_x & 0 \end{pmatrix}.$$

By using the Peaceman-Rachford theory, Eq. (1) is split into two parts as shown in Eqs. (3) and (4), where time marching over one full time step is broken into two procedures. The first updating procedure involves the advancement from the n th time step to the $n+1/2$ th time step, and in the second procedure, the fields are updated from $n+1/2$ th time step to the $n+1$ th time step.

$$(I - (\Delta t/2)(R_1))\Psi^{n+1/2} = (I + (\Delta t/2)(R_2))\Psi^n. \quad (3)$$

$$(I - (\Delta t/2)(R_2))\Psi^{n+1} = (I + (\Delta t/2)(R_1))\Psi^{n+1/2}. \quad (4)$$

By substituting $[R_1]$ and $[R_2]$ into (3), we obtain

$$E_x^{n+1/2} - (\Delta t/2\varepsilon)\delta_y H_z^{n+1/2} = E_x^n, \quad (5-1)$$

$$E_y^{n+1/2} + (\Delta t/2\varepsilon)\delta_x H_z^{n+1/2} = E_y^n, \quad (5-2)$$

$$-(\Delta t/2\mu)\delta_y E_x^{n+1/2} + H_z^{n+1/2} = -(\Delta t/2\mu)\delta_y E_y^n + H_z^n. \quad (5-3)$$

By substituting $[R_1]$ and $[R_2]$ into (4), we obtain

$$E_x^{n+1} = E_x^{n+1/2} + (\Delta t/2\varepsilon)\delta_y H_z^{n+1/2}, \quad (6-1)$$

$$E_y^{n+1} = E_y^{n+1/2} - (\Delta t/2\varepsilon)\delta_x H_z^{n+1/2}, \quad (6-2)$$

$$(\Delta t/2\mu)\delta_x E_y^{n+1} + H_z^{n+1} = (\Delta t/2\mu)\delta_y E_x^{n+1/2} + H_z^{n+1/2}. \quad (6-3)$$

By replacing n with $n-1$ in (6-1)-(6-3), respectively, we obtain:

$$E_x^n = E_x^{n-1/2} + (\Delta t/2\varepsilon)\delta_y H_z^{n-1/2}, \quad (7-1)$$

$$E_y^n = E_y^{n-1/2} - (\Delta t/2\varepsilon)\delta_x H_z^{n-1/2}, \quad (7-2)$$

$$(\Delta t/2\mu)\delta_x E_y^n + H_z^n = (\Delta t/2\mu)\delta_y E_x^{n-1/2} + H_z^{n-1/2}. \quad (7-3)$$

Substituting (7-3) into (7-1), we have:

$$E_x^n = (1 - (\Delta t^2/4\varepsilon\mu)\delta_y^2)E_x^{n-1/2} + (\Delta t^2/4\varepsilon\mu)\delta_x\delta_y E_y^n + (\Delta t/2\varepsilon)\delta_y H_z^n. \quad (8)$$

By substituting (5-3) and (8) into (5-1), the updating equation for E_x is obtained below:

$$(1 - (\Delta t^2/4\varepsilon\mu)\delta_y^2)E_x^{n+1/2} = (1 - (\Delta t^2/4\varepsilon\mu)\delta_y^2)E_x^{n-1/2} + (\Delta t/2\varepsilon)\delta_y H_z^n. \quad (9)$$

Substituting (7-3) into (7-2), we have:

$$(1 + (\Delta t^2/4\varepsilon\mu)\delta_x^2)E_y^n = E_y^{n-1/2} - (\Delta t/2\mu)\delta_x H_z^n + (\Delta t^2/4\varepsilon\mu)\delta_x\delta_y E_x^{n-1/2}. \quad (10)$$

By substituting (5-3) and (10) into (5-2), the updating equation for E_y is obtained below:

$$E_y^{n+1/2} = E_y^{n-1/2} - (\Delta t/2\varepsilon)\delta_x H_z^n - (\Delta t^2/4\varepsilon\mu)\delta_x\delta_y (E_x^{n+1/2} - E_x^{n-1/2}). \quad (11)$$

By substituting (6-2), (5-2) and (5-3) into (6-3), the updating equation for H_z is obtained below:

$$H_z^{n+1} = H_z^n + (\Delta t/2\mu)(\delta_y E_x^{n+1/2} - \delta_x E_y^{n+1/2}). \quad (12)$$

In Eqs. (9), (11) and (12) it often uses the finite difference to approximate the spatial derivate [21], [23]. For example:

$$\frac{\partial f(x, y, t)}{\partial x} \Big|_{x=i\Delta x}^{n+1/2} \approx \frac{1}{\Delta x} \sum_{l=0}^{N-1} \alpha(l).$$

$$[f|_{i+i+1/2, j}^{n+1/2} - f|_{i-i-1/2, j}^{n+1/2}], \quad (13)$$

where $\alpha(t)$ can be obtained in reference [20],

$$\alpha(t) = \frac{(-1)^t}{2(t+1/t)^2} \cdot \frac{[(2N-1)!!]}{(2N-2-2t)!!} \cdot \frac{[(2N-1)!!]}{(2N+2t)!!}.$$

If N is equal to 1, the order of the one-step leapfrog HIE-FDTD algorithm is equal to 2. When N is equal to 2, the order of the HIE-FDTD algorithm is equal to 4. Then according to the definition of the constant $\alpha(t)$ and by substituting (13) into (9), (11) and (12), the fourth-order one-step leapfrog HIE-FDTD algorithm's equations are obtained as followed.

To further simplify, the auxiliary field variables e and h are introduced as:

$$e_m^{n+1/2} = E_m^{n+1/2} - E_m^{n-1/2}, m = x, y, z, \quad (14-1)$$

$$h_m^{n+1} = H_m^{n+1} - H_m^n, m = x, y, z. \quad (14-2)$$

The final updating equation for E_x of the proposed method is obtained below:

$$\begin{aligned} & (1 + c_1) \mathbf{e}_x \Big|_{i+1/2, j}^{n+1/2} \\ & - c_2 \left(\mathbf{e}_x \Big|_{i+1/2, j+1}^{n+1/2} + \mathbf{e}_x \Big|_{i+1/2, j-1}^{n+1/2} \right) \\ & - c_3 \left(\mathbf{e}_x \Big|_{i+1/2, j+2}^{n+1/2} + \mathbf{e}_x \Big|_{i+1/2, j-2}^{n+1/2} \right) \\ & - c_4 \left(\mathbf{e}_x \Big|_{i+1/2, j+3}^{n+1/2} + \mathbf{e}_x \Big|_{i+1/2, j-3}^{n+1/2} \right) \\ & = c_5 \left(H_z \Big|_{i+1/2, j+1/2}^n - H_z \Big|_{i+1/2, j-1/2}^n \right) \\ & - c_6 \left(H_z \Big|_{i+1/2, j+3/2}^n - H_z \Big|_{i+1/2, j-3/2}^n \right), \end{aligned} \quad (15)$$

where

$$\begin{aligned} c_1 &= 365\Delta t^2 / 576\epsilon\mu\Delta y^2, \\ c_2 &= 87\Delta t^2 / 256\epsilon\mu\Delta y^2, \\ c_3 &= 3\Delta t^2 / 128\epsilon\mu\Delta y^2, \\ c_4 &= \Delta t^2 / 2304\epsilon\mu\Delta y^2, \\ c_5 &= 9\Delta t / 8\epsilon\mu\Delta y, \\ c_6 &= \Delta t / 24\epsilon\mu\Delta y. \end{aligned}$$

The other updating equations of the proposed method can be obtained similarly and are not shown here for simplicity.

III. STABILITY ANALYSIS OF FOURTH ORDER HIE-FDTD METHOD

From Eqs. (3) and (4), we obtain:

$$\begin{aligned} & \Psi^{n+1} \\ & = \left(I - (\Delta t/2)(R_2) \right)^{-1} \cdot \left(I + (\Delta t/2)(R_1) \right) \\ & \cdot \left(I - (\Delta t/2)(R_1) \right)^{-1} \cdot \left(I + (\Delta t/2)(R_2) \right) \Psi^n \\ & = (A)^{-1} (B) (A_1)^{-1} (B_1) \Psi^n, \end{aligned} \quad (16)$$

while

$$\begin{aligned} (A) &= \left(I - (\Delta t/2)(R_2) \right) \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & m_x/2\mu & 1 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} (A_1) &= \left(I - (\Delta t/2)(R_1) \right) \\ &= \begin{pmatrix} 1 & 0 & -m_y/2\epsilon \\ 0 & 1 & m_x/2\epsilon \\ -m_y/2\mu & m_x/2\mu & 1 \end{pmatrix}, \end{aligned}$$

$$(B) = \left(I + (\Delta t/2)(R_1) \right)$$

$$= \begin{pmatrix} 1 & 0 & m_y/2\epsilon \\ 0 & 1 & -m_x/2\epsilon \\ m_y/2\mu & 0 & 1 \end{pmatrix},$$

$$(B_1) = \left(I + (\Delta t/2)(R_2) \right)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_x/2\epsilon & 1 \end{pmatrix},$$

$$m_p = \frac{\Delta t}{\Delta p} \sum_{i=0}^{N-1} \alpha(t) 2\hat{j} \sin\left(\frac{2t+1}{2} k_p \Delta p\right), p = x, y,$$

$$\alpha(0) = \frac{9}{8}, \alpha(1) = -\frac{1}{24}.$$

For simplification, we obtain the formulation of M in (17), shown at the bottom of this page:

$$\begin{aligned} M &= (A)^{-1} (B) (A_1)^{-1} (B_1) = \\ & \begin{pmatrix} \epsilon\mu + \frac{1}{4}m_y^2 & -\frac{1}{2}m_x m_y & \mu m_y \\ -\frac{1}{2}m_x m_y & \epsilon\mu - \frac{1}{4}m_y^2 + \frac{1}{2}m_x^2 & \mu m_y \\ \epsilon m_y + \frac{1}{4\mu}m_x^2 m_y - \epsilon m_x - \frac{1}{2\mu}m_x^3 & \epsilon\mu - \frac{1}{4}m_y^2 + \frac{1}{2}m_x^2 \end{pmatrix} \\ & \times \frac{4}{4\epsilon\mu - m_y^2}. \end{aligned} \quad (17)$$

By substituting (17) into (16), we have $\Psi^{n+1} = (M)\Psi^n$.

By setting the determinant of $[I_{3 \times 3} \xi - M]$ to be zero, as that $|I_{3 \times 3} \xi - M| = 0$. ξ represents the growth factor.

For simplification, the above equation can be represented as follows:

$$\begin{aligned} & \xi^3 - \frac{12\epsilon\mu + 4m_x^2 + m_y^2}{4\epsilon\mu - m_y^2} \xi \\ & + \frac{12\epsilon\mu + 4m_x^2 + m_y^2}{4\epsilon\mu - m_y^2} - 1 = 0. \end{aligned} \quad (18)$$

By solving (18), the values of the growth factor ξ are obtained:

$$\xi_1 = 1,$$

$$\xi_{2,3} = \frac{(8\varepsilon\mu + m_y^2 + 4m_x^2)}{2(4\varepsilon\mu - m_y^2)} \pm \frac{\sqrt{(4\varepsilon\mu + m_y^2 + 2m_x^2)^2 - (4\varepsilon\mu - m_y^2)^2}}{(4\varepsilon\mu - m_y^2)}.$$

In order to satisfy the stability condition during field advancement, the module of growth factor ξ must be less than or equal to 1. It is apparent that when the relation $(4\varepsilon\mu + m_y^2 + 2m_x^2)^2 \leq (4\varepsilon\mu - m_y^2)^2$ is satisfied, the condition $|\xi_{2,3}| = 1$ would be obtained. When $(4\varepsilon\mu + m_y^2 + 2m_x^2)^2 \leq (4\varepsilon\mu - m_y^2)^2$, we can obtain that $-4\varepsilon\mu - m_y^2 - 2m_x^2 \leq 4\varepsilon\mu - m_y^2$, as well as $-m_x^2 \leq 4\varepsilon\mu$.

By substituting the value of m_x , we have:

$$\left(\frac{\Delta t \sum_{t=0}^{N-1} \alpha(t) \sin(((2t+1)/2)k_x \Delta x)}{\Delta x/2} \right)^2$$

$$\leq \frac{\left(\Delta t \sum_{t=0}^{N-1} |\alpha(t)| \right)^2}{(\Delta x/2)^2} \leq 4\varepsilon\mu$$

$$\Rightarrow \Delta t \leq \frac{1}{c \sum_{t=0}^{N-1} |\alpha(t)| \sqrt{(1/\Delta x)^2}},$$

where $c = 1/\sqrt{\varepsilon_0 \mu_0}$ is the speed of light in the vacuum. The values of constant $\alpha(0)$ and $\alpha(1)$ are equal to 9/8 and -1/24, respectively. So through the analysis above, the stability condition of the proposed algorithm is presented as follows:

$$\Delta t \leq \frac{6/7}{c \sqrt{(1/\Delta x)^2}}. \quad (19)$$

IV. NUMERICAL DISPERSION ANALYSIS

We now study the numerical dispersion in the proposed algorithm. Substitute $\zeta = e^{j\omega\Delta t}$ into (1), it can be obtained:

$$(\sin(\omega\Delta t / 2))^2 = \frac{-(m_x^2 + m_y^2)}{4\varepsilon\mu - m_y^2}, \quad (20)$$

while

$$m_x = \frac{\left[\frac{9}{4} \sin\left(\frac{k_x \Delta x}{2}\right) - \frac{1}{12} \sin\left(\frac{3k_x \Delta x}{2}\right) \right]}{-\hat{j} \Delta x / \Delta t},$$

$$m_y = \frac{\left[\frac{9}{4} \sin\left(\frac{k_y \Delta y}{2}\right) - \frac{1}{12} \sin\left(\frac{3k_y \Delta y}{2}\right) \right]}{-\hat{j} \Delta y / \Delta t}.$$

Suppose that a wave propagating at angle θ is in the spherical coordinate system. Then,

$$k_x = k_0 \cos(\theta), \quad k_y = k_0 \sin(\theta),$$

where k_0 is the physical wave number. By substituting them into dispersion relation (20), the wave number $k = \omega/c$ can be obtained, while c is the light speed in the vacuum.

The global phase error:

$$\Phi = \frac{1}{4\pi} \int_0^\pi \left[\frac{k_0 - k(\theta)}{k_0} \right]^2 \sin \theta d\theta. \quad (21)$$

The cell size $\Delta x = \Delta y = \lambda / N$, λ is the operating wavelength and N is the constant.

It can be seen from Fig.1. (a) that when CFLN value is 2, as the increase of the N , the global phase error of the proposed algorithm decreases gradually. Fig.1. (b) shows that when N value is 14, the global phase error of the proposed algorithm increases gradually as the increase of the CFLN.

When the N value becomes bigger, the cell sizes of the proposed algorithm would become smaller so that the algorithm would have much bigger computing accuracy and lower global phase error, which agree well with the Fig.1. (a) When the CFLN value becomes bigger, the time step size of the proposed algorithm would become bigger so that the algorithm have much lower computing accuracy and larger global phase error, which agree well with the Fig.1. (b).

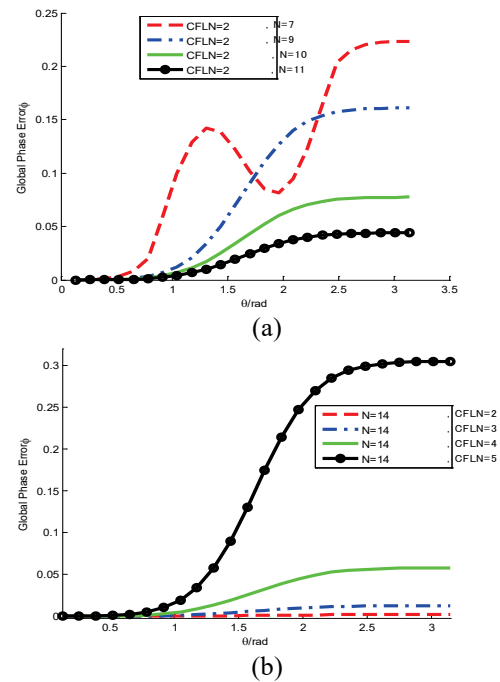


Fig. 1. (a) the relation between the global phase error and constant N , the relation between the global phase error and constant CFLN.

V. NUMERICAL RESULTS

To verify the computational efficiency and the dispersive error of the fourth-order one-step leapfrog HIE-FDTD method, two examples are presented in this section. For comparison, numerical results calculated by using the traditional second-order FDTD algorithm, the fourth-order FDTD algorithm and the second-order HIE-FDTD algorithm are also presented.

(1) Computational Efficiency

In order to verify the validity and computational efficiency of the fourth-order one-step leapfrog HIE-FDTD algorithm, the simulation of the proposed method with pulse source is presented in this sub section.

A 2-D computational domain with the dimension $360\text{ mm} \times 36\text{ mm}$ is shown in Fig. 2 (a). The cell sizes are $\Delta x = 6\text{ mm}$ and $\Delta y = 0.6\text{ mm}$. So the total meshes are 60×60 . 10 cell-thick CPML layers are used to terminate the computational domain. A small current source is placed at the center of the domain. Observation point P_1 is located at the grid (45, 30), as shown in Fig. 2 (a). The time dependence of the source is as follows:

$$P(t) = \cos(2\pi f_0 t) \exp(-4\pi \frac{t-t_0}{\tau}),$$

where $f_0 = 5\text{ GHz}$, $t_0 = 3 \times 10^{-9}\text{ s}$, $\tau = 3 \times 10^{-9}\text{ s}$; thus, the wavelength of the source is about 60 mm .

The electric field values E_y at point P_1 are calculated by using the fourth-order one-step leapfrog HIE-FDTD method and are plotted in Fig. 2 (b). For comparison, the results calculated by using the traditional second-order FDTD method, the fourth-order FDTD method and the second-order HIE-FDTD method are also presented in this figure. The time stability conditions of these four methods are:

$$\Delta t \leq 1 / c \sqrt{(1/\Delta x)^2 + (1/\Delta y)^2} = 1.99\text{ ps} \text{ (second-order FDTD}$$

$$\text{method [1]), } \Delta t \leq (6/70) / c \sqrt{(1/\Delta x)^2 + (1/\Delta y)^2} = 1.71 \text{ (fourth-}$$

$$\text{order FDTD method [10]), } \Delta t \leq 1 / c \sqrt{(1/\Delta x)^2} = 20\text{ ps}$$

$$\text{(second-order HIE-FDTD method [7]) and}$$

$$\Delta t \leq (6/7) / c \sqrt{(1/\Delta x)^2} = 17.14\text{ ps} \text{ (fourth-order one-step}$$

leapfrog HIE-FDTD method), respectively. In this simulation, the time step sizes of the above four algorithms all take the maximum time step size that satisfy their stability conditions.

Figure 2 (b) illustrates that the result of the fourth-order one-step leapfrog HIE-FDTD algorithm is well consistent with these of the second-order FDTD algorithm, the fourth-order FDTD algorithm, and the second-order HIE-FDTD algorithm.

To complete this simulation, the computer costs of these methods are presented in Table 1. Because the fourth-order one-step leapfrog HIE-FDTD algorithm as

well as the second-order HIE-FDTD algorithm can take much larger time step size than the FDTD algorithm, the computer costs of these two methods are much less than the FDTD algorithm. From Table 1, it can see that the proposed method is 6.8 and 14.54 faster than the traditional second-order FDTD method and the fourth-order FDTD method, respectively. What is more, although the maximum time step size of the proposed algorithm is just 6/7 times as that of the second-order HIE-FDTD algorithm, the computer cost of the fourth-order one-step HIE-FDTD algorithm also be less than that of the second-order HIE-FDTD algorithm. It is 2.2 times faster than the second-order HIE-FDTD algorithm. In conclusion, the proposed fourth-order one-step leapfrog HIE-FDTD algorithm can save computational time greatly when solving the fine electromagnetic problems as presented in Table 1.

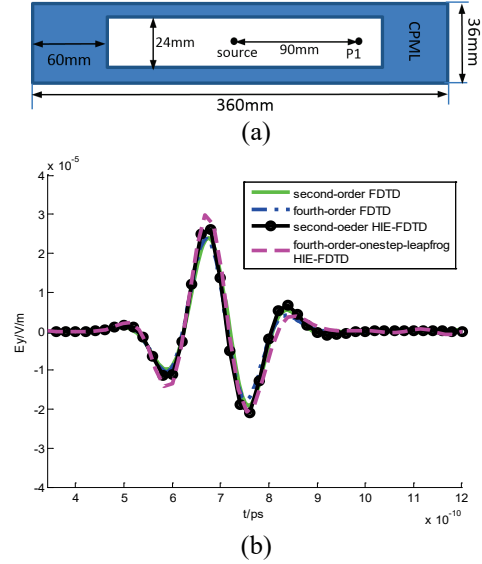


Fig. 2. (a) Free space truncated by CPML, and (b) the electric field values at point P_1 .

Table 1: Computer costs of the FDTD algorithm and HIE-FDTD algorithm

	Δt (ps)	CPU Time (s)
Second-order FDTD	1.990	14.40
Fourth-order FDTD	1.71	30.69
Second-order HIE-FDTD	20.0	4.66
One-step-leapfrog Fourth-order HIE-FDTD	17.14	2.11

(2) Dispersion Error

In order to verify the lower dispersive error of the fourth-order one-step leapfrog HIE-FDTD algorithm,

another numerical example is presented in this subsection. 10-cell-thick CPML layers are used to terminate a 2-D computational domain with the dimensions 360 mm×360 mm. A small current source is placed at the center of the domain. Observation point P_2 is 117 mm far away from source point, as shown in Fig. 3 (a).

The time dependence of the source is as follows:

$$P(t) = \sin(2\pi ft),$$

where $f=10$ GHz, thus the wavelength of the source is 30 mm.

It uses the fourth-order one-step leapfrog HIE-FDTD method, the second-order HIE-FDTD method and the second-order FDTD method to compute the field value E_y at point P_2 , the results are shown in Fig. 3 (b). The cell sizes of the x-direction and y-direction of the fourth-order one-step leapfrog HIE-FDTD algorithm are $\Delta x = \Delta y = 6$ mm, which are equal to 1/5 of the wavelength. For comparison, there are two kinds of cell sizes used in the second-order FDTD algorithm. One is $\Delta x = \Delta y = 3$ mm, which are equal to 1/10 of the wavelength; the other is $\Delta x = \Delta y = 6$ mm, which are equal to the values of the fourth-order one-step leapfrog HIE-FDTD algorithm. The cell sizes of the second-order HIE-FDTD method are also $\Delta x = \Delta y = 6$ mm.

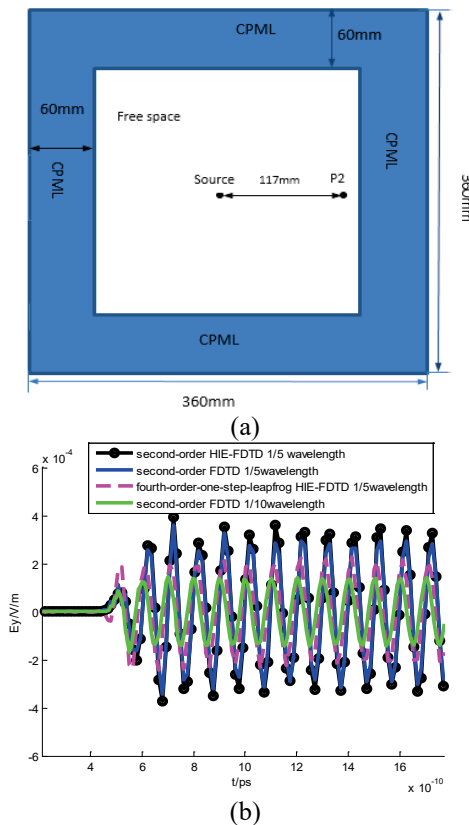


Fig. 3. (a) Free space truncated by CPML, and (b) the electric field values at point P_2 .

It is well known that as the cell sizes increase, the dispersive error of the FDTD method would increase. It means that the accuracy of the FDTD method would decrease as the cell sizes increase, especially when the cell size is larger than 1/10 of the wavelength. So, in Fig. 3 (b), we think the result of the second-order FDTD algorithm whose cell size is equal to 1/10 of the wavelength is accurate. From Fig. 3 (b), it is clear that as the cell sizes increase, the result of the second-order FDTD algorithm and the second-order HIE-FDTD algorithm whose cell size are equal to 1/5 wavelength are not well consistent with the accurate result. However, the result of the fourth-order one-step leapfrog HIE-FDTD algorithm although its cell size is also equal to 1/5 of the wavelength agrees much well with the accurate result. It demonstrates that the fourth-order HIE-FDTD algorithm has much less dispersive error than the second-order FDTD algorithm method and second-order HIE-FDTD method when the cell size increases. Since the cell size could take larger values, the fourth-order one-step leapfrog HIE-FDTD algorithm is more suitable to solve electrically large problems.

VI. CONCLUSION

This paper firstly introduces the fourth-order one-step leapfrog theories into the HIE-FDTD algorithm. It is found that the technique is weakly conditionally stable and supports time step size greater than the CFL limit. The computer cost of the fourth-order one-step leapfrog HIE-FDTD algorithm is much less than the second-order, fourth-order FDTD algorithm and the second-order HIE-FDTD even though the maximum time step size of the proposed algorithm is smaller than that of the second-order HIE-FDTD algorithm. It means that the fourth-order one-step leapfrog HIE-FDTD method has higher computational efficiency than the FDTD algorithm and second-order HIE-FDTD algorithm. Numerical simulations show that the fourth-order one-step leapfrog HIE-FDTD algorithm has higher calculation accuracy and low computational error, even when the cell size is equal to 1/5 of the wavelength. Therefore the fourth-order one-step leapfrog HIE-FDTD algorithm is very suitable to solve problems where both fine and electrically large structures are needed.

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