Full-Wave Analysis of Microstrip Circuits with Reciprocal Matrix Compression Technique

Mengmeng Li [1,2], Chenhui Xia [1], and Rushan Chen [1]

[1] Communication Engineering
Nanjing University of Science and Technology, Nanjing, 210094, China
limengmeng@njust.edu.cn

[2] State Key Laboratory of Millimeter Waves
Najing, 210094, China

Abstract ─ We propose an efficient full-wave simulation method for microstrip circuits with reciprocal multilevel matrix compression method (rMLMCM). The mixed potential integral equation (MPIE) with the layered media Green's function is applied to the exterior layers of the packed interconnects. With quadtree structure, the dense impedance matrix is split up into the "near" and "far" terms according to the admission condition. The "near" term block matrices are full-rank, they are evaluated by the method of moments (MoM) directly. While the "far" term block matrices are low-rank, they are sparse filled and compressed by the rMLMCM. The rMLMCM low-rank approximation precisions of the block impedance matrices with respect to the decomposition thresholds are tested in detail. The current densities on the large-scale interconnects at high frequency are extracted much more effectively with the rMLMCM over the standard rank based methods. Numerical results demonstrate the validity of the proposed method.

Index Terms ─ Integral equation, matrix compression, microstrip circuits.

I. INTRODUCTION

The full wave electromagnetic (EM) simulations of microstrip interconnects are becoming more and more important in electronic packing with the continuing increase of the operating frequency and decrease of the sizes of modern high-speed integrated circuits. The full wave simulations of the high high-density interconnects on printed circuit boards (PCBs) attracted many researches [1]-[3].

The method of moments (MoM) [4] with the layered media Green's functions [5] has been a preferred method for the problem of interconnects in the exterior layers of PCBs. However, the MoM solver leads to the generation of a linear matrix equation $\mathbf{Zx} = \mathbf{V}$, where $\mathbf{Z}$ is a dense matrix. The solution of which requires $O(N^3)$ operations and $O(N^2)$ memory storage when to solve it directly, where N refers to the number of unknowns. The size of the MoM matrix increases so rapidly with the increase of the number of unknowns that the computation will be intractable for the computational capacity.

Therefore a suitable fast method is required to overcome the difficulties. Recently, the kernel independent methods such as IES^3 [6], adaptive cross approximation (ACA) [7], [8], multilevel matrix decomposition algorithm (MLMDA) [9], [10], H-matrix [11], and UV method [12]-[14], are widely researched because they are purely algebraic, kernel function independent and easy to be applied to accelerate the existing MoM codes. The rank-based methods take advantage of the rank-deficient nature of the coupling matrix blocks representing well-separated interactions. The interaction matrix $\mathbf{Z}_{m\times n}$ between two well separated groups can be approximated by the multiplication of matrices $\mathbf{U}_{m\times r}$ and $\mathbf{V}_{r\times n}$. r is the truncation rank of the interaction matrix, which is evaluated by means of predetermined threshold.

For the standard rank based methods [6]-[14], the low rank decomposition will be repeatedly implemented on each pair of far coupling groups, which leads to time and memory consuming process. In [15], a new multilevel matrix compression method (MLMCM) is proposed, where only one receiving matrix $\mathbf{U}$ and radiation matrix $\mathbf{V}$ are defined respectively, and the relative smaller dimension translators are defined between two coupling groups, when coupling with a cluster of its far interaction groups. As a result, the memory requirements and solution time can be saved significantly over the standard methods [6]-[14]. In [16], the radiation matrix is defined as the transpose of the receiving matrix, denoted as reciprocal MLMCM (rMLMCM), to further reduce the low rank approximation time and memory. Then it is employed to compress the near region of the standard multilevel fast multipole algorithm (MLFMA) [18], [19] for the electric field

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integral equation (EFIE) when simulating the high-fidelity multiscale problems.

The contributions of the proposed work are: first, with the rMLMCM, we need to compute the low rank decomposition only once for each groups over the standard low rank method [6]-[14]; second, the rMLMCM proposed for the modeling of perfect electric conductor previously [16], is explored to accelerate the mixed potential integral equation (MPIE) with layered media Green's function. The rMLMCM low rank approximation precisions of the impedance matrices produced by two far coupling groups at different frequencies are first tested and validated in detail. Then the current densities on the large-scale interconnects are extracted with the standard rank based method (e.g., ACA) and the proposed rMLMCM, respectively, to show its advantages.

The remainder of the paper is organized as follows. The proposed algorithm for the simulation of the densely packed interconnects is described in Section II; numerical results in Section III demonstrate the validity of the proposed method. Finally, a brief conclusion is given in Section IV.

II. THEORY

Since the layered media Green's function is used, only the metallic surface of the microstrip circuits has to be meshed as the number of unknowns [5]. The surface of the interconnects are firstly discretized into triangular elements. The induced current on the metallic interconnects can be solved by the MPIE:

$$j\omega u_0 \hat{z}\times[\mathbf{A}(\mathbf{r})+\frac{1}{k_0^2}\nabla\Phi(\mathbf{r})]=\hat{z}\times\mathbf{E}^{inc}(\mathbf{r}), \quad (1)$$

where u_0 is the magnetic permeability, ω is the angular frequency. The vector and scalar potentials can be expressed as:

$$\mathbf{A}(\mathbf{r})=\iint_S \mathbf{G}^A(\mathbf{r},\mathbf{r}')\cdot J(\mathbf{r}')ds', \quad (2)$$

$$\Phi(\mathbf{r})=\iint_S G^q(\mathbf{r},\mathbf{r}')\cdot\nabla'\cdot\mathbf{J}(\mathbf{r}')ds', \quad (3)$$

$\mathbf{G}^A$ is the magnetic vector potential dyadic Green's function due to an electric current source and G^q is the Green's function of the scalar electric potential due to an electric charge. Then the current is expanded with Rao-Wilton-Glisson (RWG) basis functions [17]. The final linear matrix equation can be written as:

$$\mathbf{Zx}=\mathbf{V}, \quad (4)$$

where the elements of the impedance matrix $\mathbf{Z}$ and vector $\mathbf{V}$ are given by:

$$Z_{m,n}=j\omega u_0\iint_{T_m}\iint_{T_n}\mathbf{f}_m(r)\cdot\mathbf{f}_n(r')G^A(r,r')dr'dr + j\omega u_0\iint_{T_m}\iint_{T_n}\frac{1}{k_0^2}\nabla\cdot\mathbf{f}_m(r)\cdot\nabla'\cdot\mathbf{f}_n(r')G^q(r,r')dr'dr, \quad (5)$$

$$V_m=\iint_{T_m}E^{inc}(r)\cdot\mathbf{f}_m(r)dr. \quad (6)$$

It is known that the MoM matrix contains many low-rank sub-blocks which represent the interactions between two well-separated groups [6]-[16]. With this in mind, the rMLMCM starts by grouping the basis functions by the quadtree structure [13], [18], [19]. The grouping of basis functions splits the impedance matrix into a collection of submatrix blocks. In general, the submatrix is full-rank when the observation groups are in the near field of the source group, while the submatrix between them is low-rank when the observation groups are in the far field, the near and far region are defined according to the admission condition following. The rMLMCM is implemented in the case of low-rank submatrix. When accelerating the far part evaluation by the rMLMCM, the matrix-vector product $\mathbf{ZI}$ can be written as:

$$\mathbf{ZI}=\mathbf{Z}_N\mathbf{I}+\mathbf{Z}_F\mathbf{I}, \quad (7)$$

where $\mathbf{Z}_N$ is the near interaction part and $\mathbf{Z}_F$ is the far interaction part of $\mathbf{Z}$, respectively.

The quadtree structure of a densely packed interconnect with 31% interconnect fractional area is shown in Fig. 1. The admissibility condition is a criterion for judging whether a submatrix allows for a low rank approximation. A standard admissibility condition [7] is given as:

$$\mathrm{Min}\{diam(i), diam(j)\}\le\eta dist(i,j), \quad (8)$$

where *diam* and *dist* denote the group size and distance of the center of the two interaction groups respectively, and η is a parameter to control the region of the far coupling groups. In this paper, η is set to be 1. If groups i and j satisfy the admissibility condition, with the proposed MLMCM, the interaction matrix $\mathbf{Z}_{m\times n}$ can be expressed as:

$$[\mathbf{Z}_{i,j}]_{m\times n}=[\mathbf{U}_i]_{m\times r}[\mathbf{D}_{i,j}]_{r\times r}[\mathbf{V}_j]_{r\times n}, \quad (9)$$

where m, n is the number of source and testing basis functions in groups i and j respectively; r is the number of ε-rank with MGS and $r\ll\min(m,n)$.

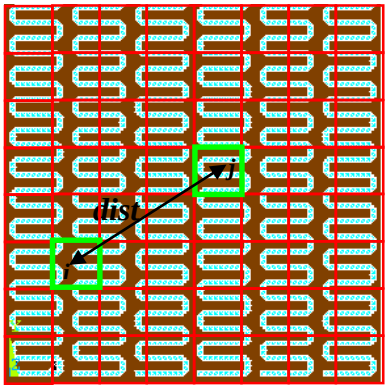


Fig. 1. The quadtree structure of a densely packed interconnect with 31% interconnect fractional area.

To construct the receiving matrix $[\mathbf{U}_i]$ for group i, as shown in Fig. 1, sampling fill the collection of columns of $[\mathbf{Z}_{i,1}, \mathbf{Z}_{i,2}, \mathbf{Z}_{i,3} \cdots]$ produced by group i against its far region groups with ACA, then a modified Gram-Schmidt (MGS) algorithm is used to the columns to get the orthogonal vectors, as the receiving matrix:

$$[\mathbf{U}_i] = \mathrm{MGS}[\mathbf{Z}_{i,1}, \mathbf{Z}_{i,2}, \mathbf{Z}_{i,3} \cdots]. \quad (10)$$

For the radiation matrix $[\mathbf{V}_j]$, it can be constructed repeatedly as in [15] from the collection of the columns $[\mathbf{Z}_{1,i}, \mathbf{Z}_{2,i}, \mathbf{Z}_{3,i} \cdots]^{\mathrm{T}}$. While in this work, we obtained from a more elegant way, it is defined as the transpose of the receiving matrix [16], [20], [21]:

$$[\mathbf{V}_j] = [\mathbf{U}_j]^{\mathrm{T}}. \quad (11)$$

Unlike (9), the receiving matrix now is now defined as:

$$[\mathbf{U}_i] = \mathrm{MGS}[\mathbf{Z}_{i,1}, \mathbf{Z}_{i,2}, \mathbf{Z}_{i,3} \cdots \mathbf{Z}_{1,i}{}^{\mathrm{T}}, \mathbf{Z}_{2,i}{}^{\mathrm{T}}, \mathbf{Z}_{3,i}{}^{\mathrm{T}} \cdots]. \quad (12)$$

For the the planar microstrip packaged interconnects in this work $\mathbf{Z}_{i,j} = \mathbf{Z}_{j,i}{}^{\mathrm{T}}$. As a result, (12) can be rewritten as:

$$[\mathbf{V}_j]^{\mathrm{T}} = [\mathbf{U}_j] = \mathrm{MGS}[\mathbf{Z}_{i,1}, \mathbf{Z}_{i,2}, \mathbf{Z}_{i,3} \cdots]. \quad (13)$$

Finally, the translator $[\mathbf{D}_{i,j}]$ can be defined as:

$$[\mathbf{D}_{i,j}] = [\mathbf{U}_i]^{\dagger}[\mathbf{U}_i][\mathbf{D}_{i,j}][\mathbf{V}_j][\mathbf{V}_j]^{\dagger} = [\mathbf{U}_i]^{\dagger}[\mathbf{Z}_{i,j}][\mathbf{V}_j]^{\dagger}, \quad (14)$$

where $[\bullet]^{\dagger}$ denotes the conjugate transpose; the ACA [7], [8] is used to compress $[\mathbf{Z}_{i,j}]$ this work.

Compared with the standard rank-based methods [6]-[14], for each group, we only construct one radiation matrix, and the truncated rank for the row and column spaces. The translator $[\mathbf{D}_{i,j}]$ with reduced dimension spans, up to a prescribed accuracy, both the row and column spaces of the original interaction matrix $[\mathbf{Z}_{i,j}]$. Compared with the MLMCM employed to simulate the scattering problems in free space [15], only one of the receiving and radiation matrices are stored, lead to about 1/3 time and memory consumption saving for the low rank decomposition. Furthermore, the rMLMCM is error controllable with the ε-rank and the threshold in ACA for multiscale problems [16], which guarantee the accuracy for the densely packed interconnects simulations.

III. NUMERICAL RESULTS

In this section, we show the results of the rMLMCM for the simulation of the densely packed interconnects. The truncation thresholds for the MGS and ACA decomposition are 10^{-4} if not specified, and the double floating point precision is used in the codes to guarantee the accuracy of the simulated results. All the simulations are performed on a personal computer with 2.8 GHz CPU and 8.0 GB RAM.

First, we test the rMLMCM low rank approximation error with respect to the threshold of the MGS and ACA. Without loss of generality, a square 20 mm × 20 mm microstrip patch, with thickness and dielectric constant of the substrates 1mm and 4, respectively, is tested. The simulated frequencies are 30, 15, 7.5, and 3.75 GHz, respectively, and the corresponding selected source/ testing group sizes are 0.5, 0.25, 0.125, and 0.0625 λ. There are 343 and 451 RWG basis functions in the selected source/testing groups. As shown in Fig. 2, we decrease the thresholds in the rMLMCM from 1e-3 to 1e-6, the low rank approximation error (defined as $\|\mathbf{Z}_{\mathrm{MoM}} - \mathbf{UDV}\|_2 / \|\mathbf{Z}_{\mathrm{MoM}}\|_2$, where $\|\cdot\|$ denotes the 2-norm) is decreasing with from 1e-2 to 1e-5, the order of the errors are decreasing in the same speed. This demonstrates the proposed rMLMCM is error controllable for the MPIE equation with layered media Green's function. It should be noted here, we use the predetermined threshold to compute **U**, **D**, and **V**, as a result, the error of the final low rank approximation will be enlarged compared with the predetermined threshold.

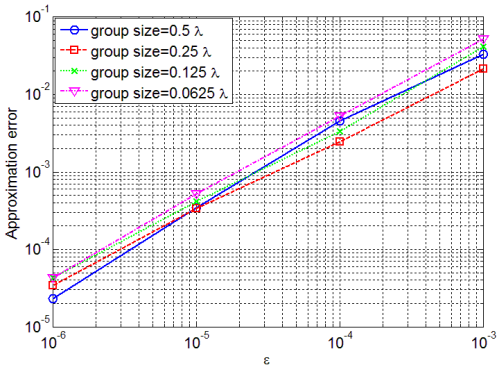
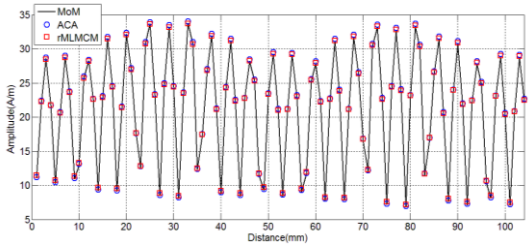


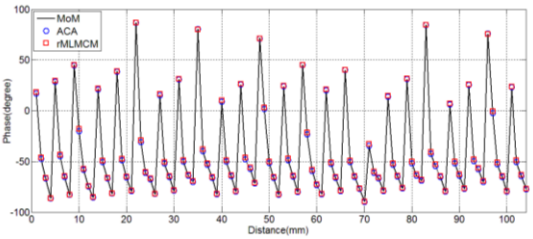
Fig. 2. Validation: the rMLMCM low rank approximation precision for the impedance matrix produced by two selected groups from a square miscrostrip patch with respect to the decomposition thresholds.

Next, a series of 4, 8, 16 and 32 meander lines traces with 31% interconnect fractional area as shown in Fig. 1 are simulated. The traces have a width of 1 mm, the thickness of the substrate is 0.25 mm, dielectric constant is 4 and the simulation frequency is 20 GHz. The numbers of unknowns are 1 628, 6 328, 24 943, and 99 040 respectively. The dimension of the simulated interconnects are 3.1 × 3.3, 6.3 × 6.5, 12.7 × 12.9 and 25.5 × 25.7 wavelengths. The minimum group size of the quadtree structure is 0.4 wavelengths, and the corresponding levels of the MLMCM chosen for the different electrical sizes are 2, 3, 4 and 5 respectively. The interconnects are excited by a delta gap voltage source on the lower end of the third trace. The amplitude and phase of the current densities along the excited trace of the 8 meander lines traces simulated by the MoM, three-level ACA and rMLMCM are plotted in Figs. 3 (a) and (b) respectively. Excellent agreements can be found. The relative error of

the ACA and the rMLMCM in the current densities versus the MoM is defined as $\|\mathbf{J}_{\text{Fast method}} - \mathbf{J}_{\text{MoM}}\|_2 / \|\mathbf{J}_{\text{MoM}}\|_2$. The norm errors are 1.1% and 1.0%, respectively. The amplitude and phase of the current densities along the excited trace of the large scale interconnects with 32 meander lines traces simulated by the five-level ACA and rMLMCM are plotted in Figs. 4 (a) and (b) respectively. Perfect agreements can also be found too. The relative error of the rMLMCM versus the ACA is 1.6%.

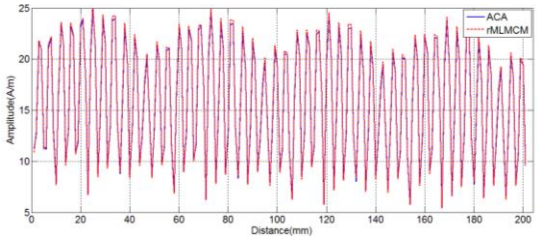


(a)

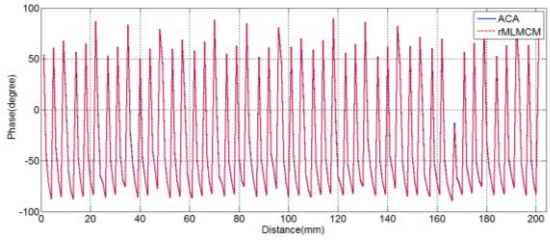


(b)

Fig. 3. The current densities along the excited trace at 20 GHz of the 8 meander lines traces simulated by the MoM, three-level ACA and rMLMCM respectively: (a) amplitude and (b) phase.



(a)



(b)

Fig. 4. The current densities along the excited trace at 20 GHz of the 32 meander lines traces simulated by five-level ACA and rMLMCM respectively: (a) amplitude and (b) phase.

Table 1 lists the memory requirements and CPU time when simulating the series of 4, 8, 16 and 32 meander lines traces by the ACA and the rMLMCM. The memory requirements of the near and far part are presented in column 3, significant reduction can be found in the far part by using the rMLMCM compared with the ACA. Therefore, total memory requirements are decreased significantly of the proposed rMLMCM. The set-up time to construct the low rank approximation matrices **U**, **D**, and **V** in column 4 of the rMLMCM is longer than the ACA, because the constructions of the radiation and translator matrices are based on the ACA low-rank decomposition, which requires additional operations of the matrices. The process can easily be accelerated by parallelization (e.g., OMP [22] or MPI [23]), as the low-ranked decompositions are evaluated independently with respect to each level and each group. Furthermore, this can be compensated by the more efficient matrix-vector product operation. It can be seen the total simulation time from the start to the end of the simulation (containing the near field evaluation, far field low rank approximation, and iterative solution) in column 5 will be reduced significantly, especially for larger number of traces.

Finally, we simulate the S parameters of the miniaturized band-pass microstrip filter, the dimension parameters can be found in Fig. 5, the permittivity and thickness of the substrate is 10.2 and 0.635 mm. The full MoM, two-level ACA, and two-level rMLMCM are employed to simulate the filter, respectively. The simulated frequency band is from 2 to 12 GHz, with discrete frequency points step of 0.25 GHz. Figure 6 plots the simulated S parameters with the two-level ACA, two-level rMLMCM, and measured data in [24]. Good agreement can be found between them. Table 2 list the computation time and memory consumption of the MoM, ACA, and rMLMCM at 6.75 GHz, it can be found with the proposed rMLMCM, both time and memory reduction can be obtained. When for the total 41 frequency points, significant total simulation time will be obtained. In this paper, we did not list the results of the MLFMA, because the MLFMA is Green's function dependent, the approximation of the layered Green's function will be more complex and less efficient when compared with the simulation of the problems in free space [25].

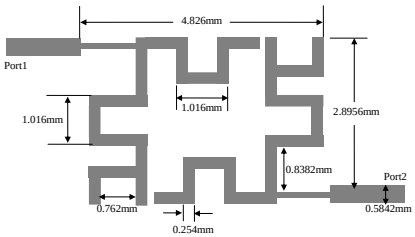
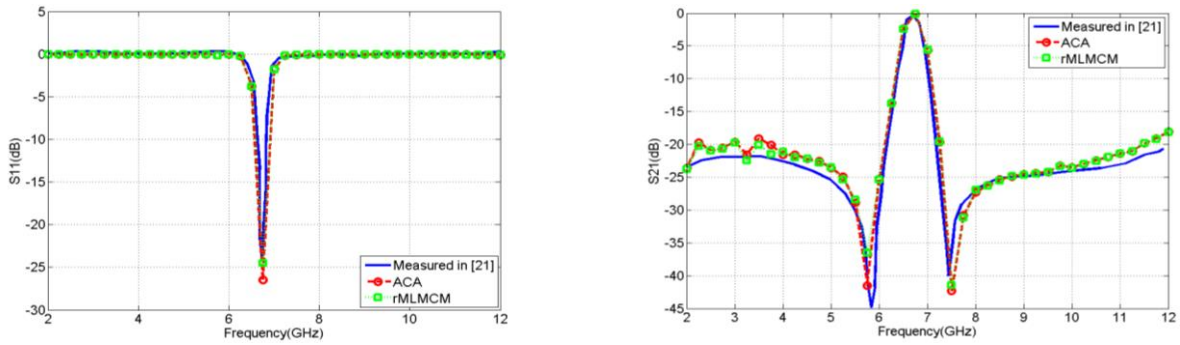


Fig. 5. Parameters of the band-pass microstrip filter.



(a) (b)

Fig. 6. Simulated results of the microstrip filter with two-level ACA and rMLMCM: (a) S11 and (b) S21.

Table 1: The memory and CPU time for the simulation of a series of 4, 8, 16 and 32 meander lines traces with ACA and rMLMCM

		Memory (MB) Near/Far Part	Set-up Time [mm:ss]	Total Time [hh:mm:ss]
4 traces	ACA	4.9/17.7	00:04	00:00:44
	rMLMCM	4.9/4.5	00:04	00:00:27
8 traces	ACA	19.8/129.4	00:31	00:06:15
	rMLMCM	19.8/23.5	00:32	00:03:11
16 traces	ACA	80.1/783.3	03:02	00:26:24
	rMLMCM	80.1/129.5	04:19	00:16:58
32 traces	ACA	320.9/4816.2	18:37	02:19:49
	rMLMCM	320.9/694.8	36:13	01:18:36

Table 2: The memory and CPU time for the simulation of the microstrip filter at 6.75 GHz with MoM, ACA, and rMLMCM

Methods	Frequency (GHz)	Far Memory (MB)	Solution Time [s]	Total Time [s]
MoM	6.75	53	199	347
ACA	6.75	8.3	117	176
rMLMCM	6.75	1.7	81	120

IV. CONCLUSION

In this paper, the rMLMCM is employed for the simulation of the large-scale microstrip circuits. The rMLMCM low-rank approximation precision is discussed in detail. When compared with the standard rank based method (e.g., ACA), the proposed rMLMCM is much more efficient for analyzing the large scale interconnects. Up to 32 meander lines traces with 31% interconnect fractional area are analyzed with moderate memory requirements and CPU time on a personal computer.

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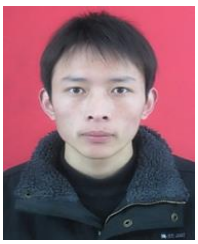

Mengmeng Li received the B.S. (Hons.) degree in Physics from Huaiyin Normal College, Huai'an, China, in 2007, and the Ph.D. degree in Electromagnetic Field and Microwave Technology from Nanjing University of Science and Technology, Nanjing, China, in 2014.

From 2012 to 2014, he carried out fast solver for multiscale simulations as a Visiting Student at the Electronics Department, Politecnico di Torino and Antenna and EMC Laboratory (LACE), Istituto

Superiore Mario Boella (ISMB), Torino, Italy. Since 2014, he has been with the Department of Communication Engineering, Nanjing University of Science and Technology, as an Assistant Professor. His research interests include fast solver algorithms, computational electromagnetic solvers for circuits, signal integrity analysis, and multiscale simulations.

Li was the recipient of the Student Paper Award (top 5 of 260) at the International Conference on Microwave and Millimeter Wave Technology in 2012.

Chenhui Xia received the B.Sc. degree in Electronic Engineering from Taizhou Institute of Science and Techbology in 2015. Now he is pursuing the master degree in Electromagnetic Field and Microwave Technology, His research interest is antenna design and optimization.

Rushan Chen received the B.Sc. and M.Sc. degrees from the Department of Radio Engineering, Southeast University, China, in 1987 and 1990, respectively, and the Ph.D. degree from the Department of Electronic Engineering, City University of Hong Kong, in 2001.

He joined the Department of Electrical Engineering, Nanjing University of Science and Technology (NJUST), China, where he became a Teaching Assistant in 1990 and a Lecturer in 1992. Since September 1996, he has been a Visiting Scholar with the Department of Electronic Engineering, City University of Hong Kong, first as Research Associate, then as a Senior Research Associate in July 1997, a Research Fellow in April 1998, and a Senior Research Fellow in 1999. From June to September 1999, he was also a Visiting Scholar at Montreal University, Canada. In September 1999, he was promoted to Full Professor and Associate Director of the Microwave and Communication Research Center in NJUST, and in 2007, he was appointed Head of the Department of Communication Engineering, NJUST. He was appointed as the Dean in the School of Communication and Information Engineering, Nanjing Post and Communications University in 2009. And in 2011 he was appointed as Vice Dean of the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology. His research interests mainly include microwave/millimeter-wave systems, measurements, antenna, RF-integrated circuits, and computational electromagnetics. He has authored or co-authored more than 200 papers, including over 140 papers in international journals.

Chen received the 1992 Third-Class Science and Technology Advance Prize given by the National Military Industry Department of China, the 1993 Third-Class Science and Technology Advance Prize given by the National Education Committee of China, the 1996 Second-Class Science and Technology Aadvance Prize given by the National Education Committee of China, and the 1999 First-Class Science and Technology Advance Prize given by Jiangsu Province, as well as the 2001 Second-Class Science and Technology Advance Prize. He is the recipient of the Foundation for China Distinguished Young Investigators presented by the National Science Foundation (NSF) of China in 2003. In 2008, he became a Chang-Jiang Professor under the Cheung Kong Scholar Program awarded by the Ministry of Education, China. Besides, he was selected as a Member of Electronic Science and Technology Group by Academic Degree Commission of the State Council in 2009. Chen is a Senior Member of the Chinese Institute of Electronics (CIE), Vice-Presidents of Microwave Society of CIE and IEEE MTT/APS/EMC Nanjing Chapter. He serves as the Reviewer for many technical journals such as IEEE Trans. on AP and MTT, Chinese Physics etc., and now serves as an Associate Editor for the International Journal of Electronics.