The Sinusoidal Ground Electrode: Theory and Case Study Results

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Abstract **─** This paper presents the analytical analysis of the sinusoidal ground electrode commonly used in Portugal by the Portuguese Electric Company. This electrode is easy to install, particularly for two layer soils with rocky bottom layer, and costs much less than it does a strip conductor. Both the theoretical results as well as measurements in the field have shown that the empirical model used by the company leads to large errors. Here the authors propose a new procedure to calculate the grounding resistance for this type of electrode using the average resistance between the wire and strip electrodes, which the calculation is well-established. To avoid heavy computation, the authors also propose the use of simple formulas in order to easily compute the strip resistance. The theoretical results and field measurements are compared and show the validity of the procedure being proposed here.

Index Terms **─** Electrical power distribution, ground electrodes, layered soil, matrix method.

I. INTRODUCTION

The calculation of a ground electrode resistance using a two layer soil model is widely found in the literature. Several methods had been used. Salama et al. have developed formulas for grid in two layers soil using the synthetic-asymptote approach [1]. Berberovic et al. explored the Method of Moments in the calculation of ground resistance by using higher order polynomials approximation in the unknown current distribution [2] and with a variation on that, Sharma and De Four used the Galerkin's Moment Method [3]. The Boundary Element Method is another theoretical tool commonly used by authors such as Columinas et al. [4-6] and Adriano et al [7]. These authors transformed the differential equation that governs the physical phenomenon into an equivalent boundary integral equation. On the other hand, Coa used the Matrix/ Integration Method for calculating the mutual resistance between segments in a homogeneous soil one and in a two layer soil [8], and Ma and Dawalibi used an optimized method of images for multilayer soils [9]. The two layer ground model has been used even in the study of the phenomena of ionization [10]. Mombello et al. developed a method for obtaining the optimized two layer soil parameters using the Least Square Method in order to obtain a two layer resistivity soil curve that fits the actual one [11]. Recently Khan presented the effect of low resistance materials filling in an optimized pit surrounding a rod working in a two layer soil [12]. In general these works have used the theory of images, which implies infinite series for the expanded Green function [2]. However, some of these studies do not compare their work neither with experimental data, nor with data from other references. In this paper, the authors analyse the sinusoidal grounding conductor used in Portugal.

The calculations for the diffusion resistance of the sinusoidal electrode were done using the Matrix Method in homogeneous and two layer soil. The field measurements have also been carried out.

The paper is structured as follows. Section II presents the sinusoidal electrode. Section III briefly introduces the theoretical and empirical models for electrodes in homogeneous and two layer soil. Section IV presents the illustrative results. Section V discusses the obtained results and proposes a new procedure for calculating the sinusoidal electrode resistance comparing the results with field measurements. Finally, Section VI concludes the paper.

II. ELECTRODE PRESENTATION

The sinusoidal grounding electrode is commonly used in Portugal by the Portuguese electric company, in

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soils where the bottom layer is rocky and hence, the voltage reflection coefficient is positive. This type of electrode has its relevance for these kind of soils since the second layer is rocky, making difficult, or even impossible, the rod burying which is the solution adopted in alluvial soils. Its implementation is accomplished manually by the profile of a cupper conductor with section of 35 mm^2 and sinusoidal shape. Its placement is done in trenches of 3, 6 or 10 m long, 0.6 m wide and a maximum depth of 0.8 m. The wire length varies from 10 to 38 m with a section of 35 mm^2, for ground resistances up to 20 Ω. Figure 1 shows a picture of the installation of a sinusoidal electrode.

Fig. 1. Sinusoidal grounding electrode [13].

In the present work, this type of electrode is modeled as a sinusoid, whose amplitude is 0.25 m, almost half the width of the ditch with three maxima per meter, which is nearly 10 m long for an electrode corresponding to a trench of 3 m long. The wire axis was divided into source points whose abscissas are spaced 0.01 m apart. For each generated point source, there is a point at the conductor's surface which shares the same abscissae and ordinates, but whose quota is greater than the point source quota. The difference between the two quotas is the radius of the conductor. A schematic drawing is presented in Fig. 2. The point sources are in parallel position since the conductor is in the horizontal plane.

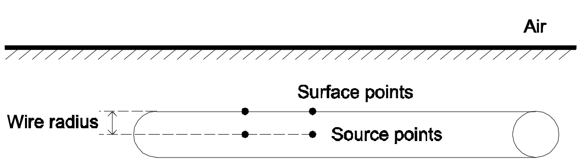

Fig. 2. Source and surface points, for the sinusoidal grounding electrode.

III. THEORETICAL MODEL

A. Homogeneous soil

Fault currents in alternate current power systems, that are harmful to persons and equipment, do not need to be modeled as a time varying phenomenon. Instead, a direct current analysis is sufficient for most applications since the dimensions of a grounding system are much smaller than the penetration depth at industrial frequency as well as for the resistivity of soils commonly encountered [14]. This approach results in a conductive electrostatic model represented by a Laplace equation on the electric potential v,

$$\nabla^2 v(x,y,z) = 0. \tag{1}$$

A current source point at P_f injecting an electric current I_f into a homogeneous soil with resistivity ρ Ωm was considered in (1). Its solution is the potential at any point [14] and is given by:

$$v = \frac{\rho I_f}{4\pi}\left(\frac{1}{r} + \frac{1}{r'}\right), \tag{2}$$

where r and r' are the Cartesian distance between the source point and the point where potential is being calculated and the Cartesian distance between point of interest and source point image, respectively. The expression for the potential presented in (2) is the Laplacian solution, considering Dirichlet open boundaries which can be expressed as:

$$v = q_{i,j} I_f, \tag{3}$$

where q_{ij} is a position factor, depending of point i, with the potential being calculated from source point j and soil resistivity. With n source points, the electrode potential surface in each discretized surface point is, by the superposition principle [14]:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \dots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}. \tag{4}$$

As the potential at the electrode surface is constant, a value to it can be arbitrated:

$$V_1 = V_2 = \cdots = V_n = V_{const}. \tag{5}$$

Equations (4) and (5) become a system of n equations with the n currents points. The total current is the sum of all the source point currents and therefore the resistance of the electrode is found by Ohm's law:

$$R = \frac{V_{const}}{\sum_{k=1}^{n} I_k}. \tag{6}$$

B. Two layer soil

The formulation for a sinusoidal electrode in the upper layer of a two layer soil is almost the same as in the case of homogeneous soil. The sinusoidal electrode axis was discretized again, into source points whose abscissae are spaced 0.01 m apart.

In this case, the expression for the potential is also the Laplacian solution, considering Dirichlet open boundaries. Potential at surface point is given by Equation (3) again, but with a different position factor which is now expressed in the following manner [16]:

$$q_{ij} = \frac{\rho_2}{4\pi}\{G(x_i - x_j, y_i - y_j, z_i - z_j) - k_{12}\sum_{m=0}^{\infty} K^m G[x_i - x_j, y_i - y_j, z_i + z_j - 2(m-1)D] + \sum_{m=1}^{\infty} K^m G(x_i - x_j, y_i - y_j, z_i - z_j - 2mD) - k_{32}\sum_{m=0}^{\infty} K^m G(x_i - x_j, y_i - y_j, z_i + z_j + 2mD) + \sum_{m=1}^{\infty} K^m G(x_i - x_j, y_i - y_j, z_i - z_j + 2mD)\}. \quad (7)$$

The reflection coefficient of voltage is given by:

$$K = k_{12}k_{32}, \quad (8)$$

where $k_{12} = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}$, $k_{32} = \frac{\sigma_3 - \sigma_2}{\sigma_3 + \sigma_2}$, and σ_1 is the bottom layer conductivity, σ_2 the top layer conductivity, σ_3 the air conductivity, ρ_2 the top layer resistivity, G the Green function, k_{12} the voltage reflection between layers 1 and 2, k_{32} the voltage reflection between layers 2 and air (which is -1), and D the top layer thickness. As in the case of homogeneous soil, for each source point was considered a surface point which quota is a radius higher than the source point quota. With n source points, the electrode potential surface, in the each discretized surface point is also given by (4), with the new position factor (7).

C. Empirical model

An empirical model to compute the sinusoidal electrode resistance which has been proposed and adopted by the Portuguese electric company consists of the next simplified scheme [13]:

$$R = \begin{Bmatrix} 0.2\rho \; for \; a \; 3 \; m \; trench \\ 0.15\rho \; for \; a \; 6 \; m \; trench \\ 0.08\rho \; for \; a \; 10 \; m \; trench \end{Bmatrix}. \quad (9)$$

The resistivity used in (9) can be obtained by checking on tables of soil materials, or by an average of a few measurements of the apparent resistivity, using Wenner method.

IV. ILLUSTRATIVE RESULTS

This section presents a comparison between electrode resistances for sinusoidal electrode, strip and wire. Moreover, the average of the theoretical resistance of these two last electrodes is also presented. This comparison is made both for homogeneous and two layer soil considering the top layer with higher or lower resistivity than the bottom layer. In every case a comparison between the obtained solutions with the empirical model is also done. Potentials at surface are also compared in all cases. A comparison for the resistance values obtained with the matrix method and the empirical model will be presented in the next section.

A. Electrode in homogeneous soil

In this case, it was considered a soil with resistivity of 100 Ωm, a sinusoidal electrode buried in a trench with 0.5 m deep and 0.6 m wide. The trench length has been varied from 1 to 10 m. The cable and the sinusoidal electrode have a radius of 3.3 mm. Figure 3 shows the comparing results for a horizontal conductor, a sinusoidal electrode with three maxima per meter, ensuring that the sinusoidal electrode length is accordingly to the trench length as defined by the Portuguese power company, a strip horizontal conductor and for the average resistance between wire and strip. The values of resistance for the horizontal wire and strip were obtained by using Dwight's formulas [15]. Note that, the resistance of the sinusoidal electrode is very close to the average resistance of the wire with horizontal strip and a mean error of 4.7%. In the worst case, the referred average is 11.2% higher than the value of the sinusoidal electrode. Furthermore, the resistance of the sinusoidal electrode gets closer to the strip resistance value when the trench length increases.

For illustrative purposes, the surface potential due to the sinusoidal electrode in a 6 m trench, considering an injected current of 100 A, is presented in Fig. 4.

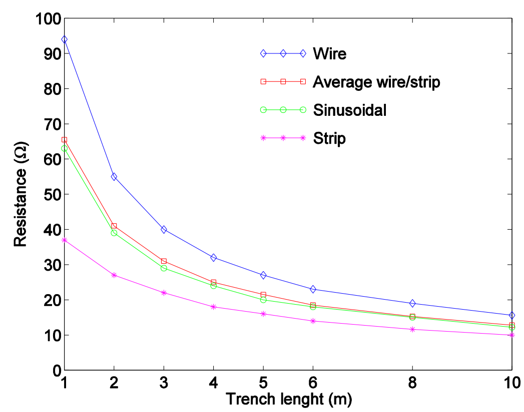

Fig. 3. Comparison between electrodes in homogeneous soil ($K = 0$).

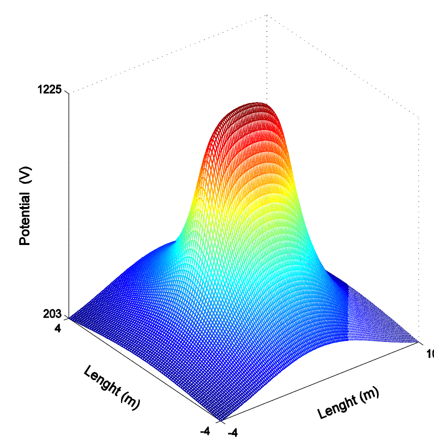

Fig. 4. Surface potential for sinusoidal electrode.

On the other hand, surface potential due to the three types of electrodes, for the same injected current, along axis trench and in a direction perpendicular to the middle of the trench, is presented in Fig. 5. It can easily be seen that the sinusoidal electrode generates a smaller surface potential than the other electrodes.

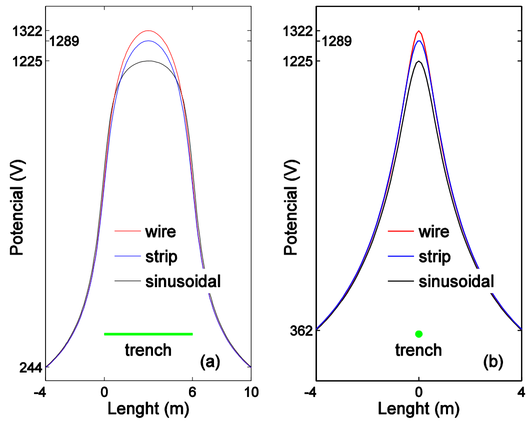

Fig. 5. Surface potential: (a) along axis trench and (b) perpendicular to the middle of the trench.

B. Electrodes in a two layer soil for positive voltage reflection

In this case, it was considered a two layer soil with a positive voltage reflection coefficient. The value of the resistivity of the upper layer, with 1 m thickness, was 100 Ωm and the value of the resistivity of the lower layer was 500 Ωm ($K = 2/3$). For comparison purposes, the buried deep was maintained at 0.5 m.

The resistance of the horizontal wire was calculated by using Tagg's formula [16] and the resistance of the horizontal strip has been calculated by using the moment methods, as shown in the Appendix. The theoretical results for the horizontal conductor, the sinusoidal electrode with three maxima per meter, the strip and for the average resistance between wire and strip are shown in Fig. 6. It can be noticed that the average resistance between strip and wire is very close to the sinusoidal electrode resistance with a 3.4% mean error. In the worst case, the error is 6.5%. Furthermore, as length increases, the resistance of the sinusoidal electrode becomes closer to the strip resistance value.

For illustrative purposes, the surface potential due to the sinusoidal electrode in a 6 m trench, considering an injected current of 100 A, is presented in Fig. 7. Also, surface potential due to the three types of electrodes, for the same injected current, along axis trench as well as in a direction perpendicular to the middle of the trench is shown in Fig. 8. Outside the trench the potentials are almost coincident. The surface voltage perpendicular to the middle of the trench due to the strip electrode is larger, due to a surface leaking current, as expected. The peak voltage produced by the sinusoidal electrode is smaller than those produced by the other electrodes, approximately 5% smaller than the peak produced by the strip electrode.

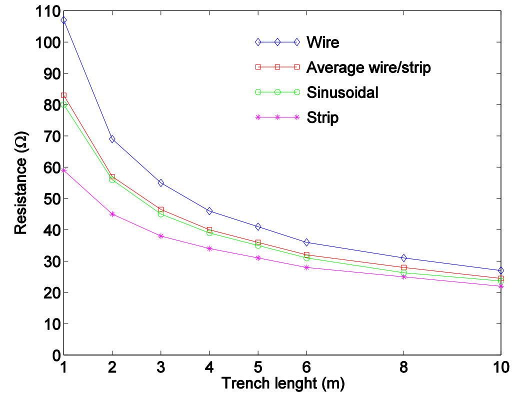

Fig. 6. Comparison between electrodes in a two layer soil ($K = 2/3$).

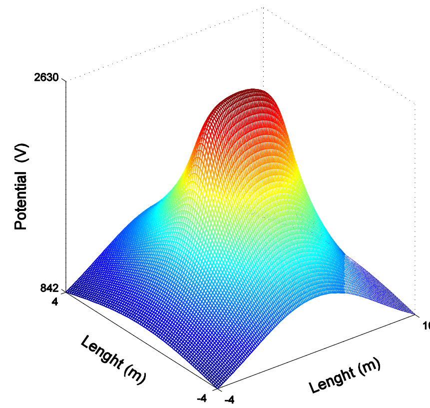

Fig. 7. Surface potential due to sinusoidal electrode.

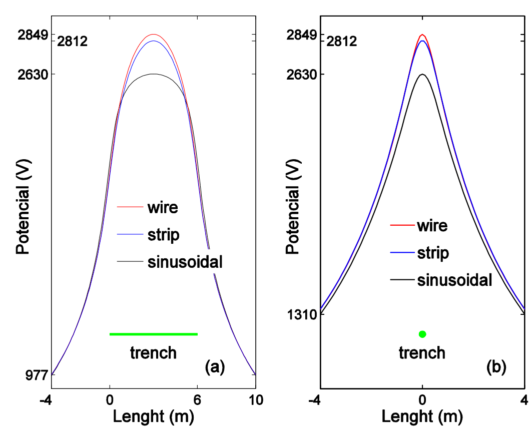

Fig. 8. Surface potential: (a) along axis trench and (b) perpendicular to the middle of the trench.

C. Electrodes in a two layer soil for negative voltage reflection

Switching the values of the layers resistivity considered in the above subsection, the reflection coefficient of voltage becomes negative ($K = -2/3$). Once more, for comparison purposes, the buried deep is maintained at 0.5 m. Also, in this case, the resistance was computed with the methods brought up in previous subsection for all the electrodes considered.

The theoretical results for the horizontal wire, for the sinusoidal electrode with three maxima per meter, for the strip and for the average resistance between wire and strip are all shown in Fig. 9. Again, the average resistance between strip and wire turn out to be close to the sinusoidal electrode resistance and, as trench length increases, the resistance of the sinusoidal electrode becomes closer to the strip resistance. For small trenches, the average resistance between strip and wire can reach 20% above the resistance of the sinusoidal electrode. The mean error of the average is 18%.

A mention deserves to be done here: the electrode under study is not used in this type of soil since it is extremely easy to bury rods which are the most common electrodes for these soils.

Surface potential due to the sinusoidal electrode in a 6 m trench, considering an injected current of 100 A, is shown in Fig. 10.

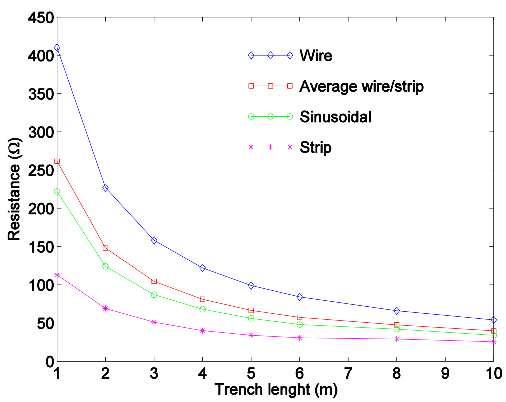

Fig. 9. Comparison between electrodes in a two layer soil ($K = -2/3$).

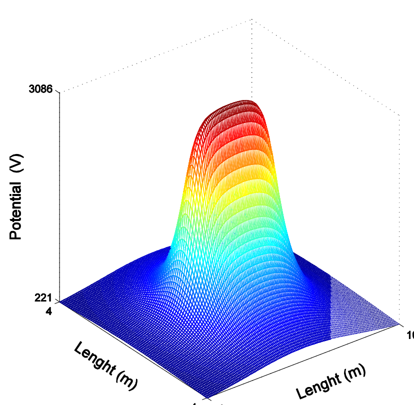

Fig. 10. Surface potential due to sinusoidal electrode.

For the same injected current, surface potential due to the three types of electrodes along axis trench, as well as in a direction perpendicular to the middle of the trench, are both shown in Fig. 11. The potential due to the sinusoidal electrode has the smallest maximum surface voltage, being 5.2% less than for the strip electrode.

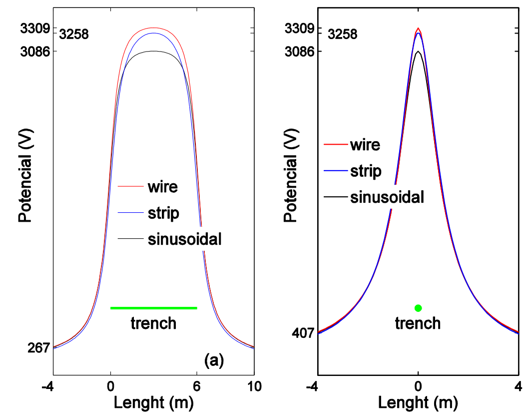

Fig. 11. Surface potential: (a) along axis trench and (b) perpendicular to the middle of the trench.

V. DISCUSSION, PROPOSED APPROACH AND FIELD MEASUREMENTS

A comparison between the resistance values for the sinusoidal electrode obtained by the matrix method and by the empirical model is shown in Fig. 12. In homogeneous soil ($K = 0$) as well as in two layer soil with positive voltage reflection coefficient ($K > 0$), where the bottom layer has higher resistivity, the empirical model produces errors which are significant and values which are smaller than those obtained with the matrix method (not conservative). The sinusoidal electrode is widely used in this type of soil. Only in soils where the bottom layer is less resistive ($K < 0$) than the top layer, due to the water table for instance, the empirical formulas are conservative, but still presenting errors, particularly for 6 and 10 m trench lengths. Note that, relative errors of the empirical model do not necessarily decrease with growing trench length. With respect to the surface potential, the sinusoidal electrode produces a smaller maximum voltage than the remaining electrodes.

Concerning the cost of the sinusoidal electrode, it is about 15% of the cost of the steel sheet commonly used in Portugal. The horizontal cable costs around 3 times less than sinusoidal electrode due to the greater length of the latter. The cost of cable is 6.70 euros per meter, whereas sinusoidal length with 3 maxima per meter stands at 21.90 € per meter of trench. The strip with dimensions of 1x0.5x0.003 m costs roughly 146 €. Considering the comparison between the resistance values of the three types of electrodes and their costs, one reaches the conclusion that the sinusoidal electrode is an excellent choice.

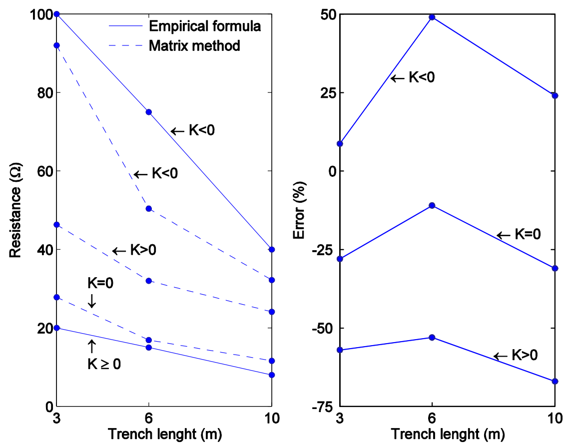

Fig. 12. Calculated values of the sinusoidal electrode grounding resistance with empirical formulas and matrix method and their corresponding errors.

Once the empirical model is not the best choice for computing the sinusoidal electrode resistance, the authors propose a new procedure for that calculation, based on the average resistance between wire and strip, which leads to better results as it has been previously shown. This choice is a good option since the mean error is less than 5% for homogeneous and two layer soil with more resistive bottom. When the bottom layer is the water table, the mean error reaches 18%. These errors are always conservative with respect to the resistance value given by the matrix method, as it can be seen in Fig. 3, Fig. 6 and Fig. 9.

The proposed calculations have been performed accurately and based in widely known numerical methods that demand a computer program with specific software requirements. With the purpose to provide the field engineer with a tool for calculating the strip resistance in a two layer soil, it is proposed the adoption of the simple formula [1]. This formula is used here for computing the strip resistance at small burring depth:

$$R = \rho_2 \left[\frac{1}{\sqrt{2\pi A}} - \frac{\ln\left(\frac{2\rho_2}{\rho_2+\rho_1}\right)}{2\pi(D+h_0)} \right], \qquad (10)$$

where A is the area of the strip and,

$$h_0 = \sqrt{\frac{A}{2\pi}}\ \ln\left(\frac{2\rho_2}{\rho_2+\rho_1}\right)\ \left(\frac{\rho_2}{\rho_2-\rho_1}\right).$$

The proposed procedure to estimate the sinusoidal electrode resistance is: (i) determine the strip resistance by using formula (10); (ii) determine the wire resistance by using the Tagg formula; and (iii) compute the estimated sinusoidal electrode resistance as being equal to the average of the values obtained in previous steps.

Now considering the most important case, in which the bottom layer is more resistive (such as for example a rocky layer), the average between the strip resistance, calculated by using (10), and horizontal wire resistance is presented in Fig. 13. The results then obtained are a conservative approach to the sinusoidal electrode with a maximum relative error of 27%, 22% and 19%, respectively for a 10 m, 6 m and 3 m trenches.

This simple formula and the proposed procedure to use the average value between strip and wire is a much better approach than expressions which are given by the empirical model, whose errors, in spite of not being conservative, are also of -67%, -53% and -57%, respectively for a 10 m, 6 m and 3 m trenches, as can be seen in Fig. 12.

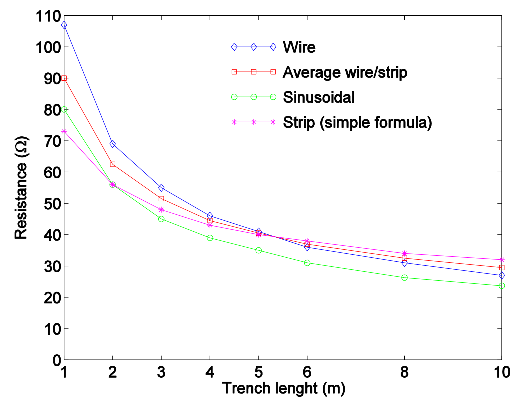

Fig. 13. Comparison between the grounding resistance in a two layer soil ($K = 2/3$) when applying the simple formula for the strip.

For soils with a less resistive bottom layer (K<0), were this type of electrode is not generally used, the calculation of the strip resistance by using (10) leads to average resistance between strip and wire with values greater than the previous case (K>0), but still conservative however.

Next, experimental data are presented and compared with the theoretical values for two layer soil, where the water table produces the bottom layer and a horizontal discontinuity plane between layers. Tests with three electrodes have been performed in a 1 m trench, with a depth of 0.3 m and sandy soil. This type of soil can be modelled as a two layer soil. The resistivity of the upper layer is 2400 Ωm with 1.5 m thickness, whereas the bottom layer has a resistivity of 443 Ωm due to the presence of water. This type of soil has been chosen in order to guarantee a discontinuity plane (or with separation between layers) and levelled with the soil's surface. Nevertheless, the type of soil considered is geologically homogeneous and the water table generates the bottom layer with less resistivity. A linear extrapolation of the empirical model coefficients, found in Equation (9), would bring a calculation for a 1 m trench of the kind $R = 0.23\rho$ and that yield the value 560 Ω for a sinusoidal electrode with 1 m length. This represents a considerable error when compared to the measured value. The results are shown in Table 1.

The lack of a perfect contact between the strip bottom surface and the soil, with the presence of air bags, can explain the unexpected error of -44% for the theoretical value obtained with the method of moments. The value obtained by using (10) which should be bigger than the previous one, presents a small error however.

If the wire and strip resistances were computed with errors about -35%, -30%, -20%, -10% and 0%, the proposed method would give results having errors of -21%, -15%, -2%, +10% and +22%, respectively. For the experimental case reported and using input data with errors about -20%, the proposed method has given results having errors of approximately -2%. The estimation of wire and strip resistances with values close to the measured ones makes the average resistance to have a higher error, with an expected value around 20%, as mentioned in Section IV.C for this type of soil. It should be pointed out that the proposed average for the sinusoidal resistance is better than the empirical method, which has in this case an error of -68%. That error is much higher than the maximum absolute error of approximately 20% obtained with the proposed method, using estimated wire and strip resistances with errors from -35% to 0%.

The adoption of the new procedure introduced here based on the average values between the theoretical resistances of the wire and the strip turned out as an excellent choice to estimate the field resistance of the sinusoidal electrode.

Table 1: Measured and calculated values

Electrode	Ground Resistance (Ω)/Error (%)					
	Measured	Calculated/Error to Measured	Eq. (10)/Error to Measured	Empirical Model/Error to Measured	Average Wire–Strip/Error to Measured: Wire Calculated and Strip Calculated	Average Wire–Strip/Error to Measured: Wire Calculated and Strip Eq. (10)
Wire	2660	2167/-19	–	–	–	–
Strip	1566	870/-44	1235/-21	–	–	–
Sinusoidal	1735	1364/-21	–	560/-67.7	1518/-12.5	1701/-2

VI. CONCLUSION

A new procedure to calculate the grounding resistance of the sinusoidal electrode was proposed and validated here, particularly for two layer soils with rocky bottom layer (positive voltage reflection). The errors are minor and quite acceptable when compared with those obtained through the empirical model.

The use of a simple formula to calculate the strip resistance may increase the errors. Nevertheless, they still remain smaller than those obtained with the empirical model in the case of positive voltage reflection.

The field measurements agree with theoretical results, except in strip electrode due to airbags that have increased the measured value. Even in this worst case, the proposed procedure produces errors which are much smaller (-12% comparing with -67.7% of the empirical model).

In all cases considered, the surface potential due to the sinusoidal electrode was also analyzed and have shown peak voltages smaller than the remaining electrodes tested.

The sinusoidal electrode is easy to install with a cost much lower than the strip electrode and with better grounding resistance than the wire electrode. The proposed procedure allows obtaining much better accuracy in the grounding resistance estimation for the sinusoidal electrode.

APPENDIX

Here it was considered a linear source of current in the upper layer, x-oriented, centered at $P_F(x_F, y_F, z_F)$ of length $2L$, injecting the current I_F into the soil. Assuming a constant current density leaving the electrode, its value is $I_F/2L$. The current leaving the infinitesimal length dx_L is $I_F dx_L/2L$. By using Equation (3), the infinitesimal voltage caused by this infinitesimal segment at a point of the upper layer is:

$$dV = q_{i,j} \frac{I_F}{2L} dx_L, \tag{A1}$$

where $q_{i,j}$ is given by Equation (7). By integrating from $x_F - L$ to $x_F + L$ an expression for the potential generated by a linear current source in the upper layer is then obtained:

$$V_{22}(x,y,z) = \frac{\rho_2 I_F}{8\pi L} \left[F(x-x_F, y-y_F, z-z_F, L) - K_{12}\sum_{m=0}^{\infty} K^m F(x-x_F, y-y_F, z+z_F-2(m-1)D, L) + \sum_{m=1}^{\infty} K^m F(x-x_F, y-y_F, z-z_F-2mD, L) - K_{32}\sum_{m=0}^{\infty} K^m F(x-x_F, y-y_F, z+z_F+2mD, L) + \sum_{m=1}^{\infty} K^m F(x-x_F, y-y_F, z-z_F+2mD, L)\right], \tag{A2}$$

where F, the indefinite integral of Green function [17], is given by:

$$F(t,u,v,L) = ln\frac{t+L+\sqrt{(t+L)^2+u^2+v^2}}{t-L+\sqrt{(t-L)^2+u^2+v^2}}. \tag{A3}$$

The strip was modeled using a set of linear conductors whose diameter is the thickness of the strip with axes separated by the diameter of their corresponding conductors.

The average potential over the length of a passive conductor, caused by a current which leaves an active conductor, must be calculated in order to create a similar system described by (4) and (5). The term q_{ij} is the mutual resistance between the segments. Since *2L* is the length of the active segment and *2C* is the length of the passive segment also *x-oriented* and centered at (x_c, y_c, z_c), the expression for the electric potential in the passive conductor is the well-known average formula:

$$V_{xx}(x_c, y_c, z_c) = \frac{1}{2C}\int_{x_c-C}^{x_c+C} V_{22}(x_c, y_c, z_c)dx_c. \quad \text{(A4)}$$

The result of the integral itself is identical to the expression in brackets in (A2) with function *H* replacing function *F*:

$$H(t,u,v,L,C) = N \cdot ln\left|N + \sqrt{N^2+u^2+v^2}\right| - \sqrt{N^2+u^2+v^2} - O \cdot ln\left|O + \sqrt{O^2+u^2+v^2}\right| + \sqrt{O^2+u^2+v^2} - P \cdot ln\left|P + \sqrt{P^2+u^2+v^2}\right| + \sqrt{P^2+u^2+v^2} + Q \cdot ln\left|Q + \sqrt{Q^2+u^2+v^2}\right| - \sqrt{Q^2+u^2+v^2}, \quad \text{(A5)}$$

where *N=t+L+C*, *O=t+L-C*, *P=t-L+C* and *Q=t-L-C*.

For the self-resistance of a segment, its radius must be used in the second parameter, thus avoiding zeros in the calculation of the logarithmic function.

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