Stochastic Radiation Model for Cable Bundle with Random Wires

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Abstract **─** Cable bundle is often the main radiation structure due to its length in automotive electrical or electronic systems. Random wire positions in a cable bundle is a challenge for the modeling in perspective of Electromagnetic Compatibility (EMC). This work addresses the uncertainty property of a cable bundle due to its random wire positions, through a stochastic-model approach. Random wire position distributions in a bundle adopt Gaussian norm. A *spline* interpolate function is used to improve the continuity of wires along the bundle. To calculate the common-mode (CM) current on the bundle, the composed non-uniform wires are modeled by cascaded uniform segments or Chebyshev Expansion Method based smooth lines. Further CM current based bundle radiation is calculated using electric-dipole model. Proposed modeling methodology is assessed by comparing CM current and radiation predictions versus measurement data and theoretical results. Predictions agree well with measurements especially in statistics.

Index Terms **─** Cable bundle, Electromagnetic Compatibility (EMC), Common-Mode (CM) current, radiation, statistics, stochastic-model.

I. INTRODUCTION

In the ALSE method according to CISPR 25 [1], the long cable bundle is often the dominant radiation structure. To predict radiated emissions from this structure at early EMC design stage, a reasonable radiation model is necessary. However, in a real automotive bundle the variability of the wire positions along the bundle lead to difference in final radiation measurements. Modeling a highly-precise cable bundle is difficult due to its random wire position. Therefore, a stochastic model is promising to simulate this uncertainty. Recent works provided a variety of methods to model stochastic characteristics of a bundle, which can be divided to two main groups, according to different approximation approaches for non-uniform transmission lines [2]. The first group method is based on MTL theory to approximate a cable bundle with cascaded uniform segments. "Monte Carlo" method [3] is introduced to divide a cable bundle into non-uniform segments of which 2-D cross sections are identical, but the positions of the wires are randomly interchanged from segment to segment. This model produces geometrical conflicts due to the uncorrelated sequence of cross sections. The "Random Mid-point Displacement" [4] and "Random Displacement Spline Interpolation" [5] are employed to improve model precision. The second group method approximates the non-uniform transmission line by specific numerical method, for example "Taylor Series Expansion Method" [6], "Chebyshev Expansion Method" [7] and "Green Function Expansion Method" [8]. The common feature of these methods is to approximate non-uniform transmission lines through smooth specific functions. However, the computation time is often long due to the applied high order approximation smooth functions. This work aims to investigate the influence to radiated emissions from random wire positions in a bundle. The wire position distribution with statistical manner is analyzed. Then, *spline* interpolation function is used to smooth position function of wires in a bundle. Cascaded approximation from the first group methodology and Chebyshev expansion approximation from the second group methodology are both applied to model a non-uniform transmission line. In essence, a stochastic cable bundle model is a set of deterministic non-uniform MTLs (Multiconductor Transmission Line) characterized with different TL (Transmission Line) parameters. Each deterministic MTL model could derive a deterministic solution of Common-Mode (CM) current distribution. Thereby, the stochastic radiation results can be calculated from a set of CM currents from non-uniform MTL models, which have different TL characteristic impedances and propagation constants.

II. APPROXIMATION METHODS FOR NON-UNIFORM TRANSMISSION LINES

In the MTLs, voltage and current on each wire are dependent on the other wires due to the coupling effects

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between wires. Considering termination boundary conditions, a generalized two-port network representation for a MTL can be shown in Fig. 1.

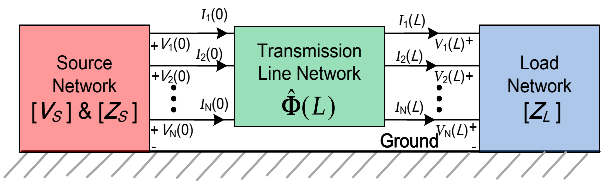


Fig. 1. Characterization of multiconductor transmission line (N+1-wire) as a generalized two-port.

Being analogy with two-wire transmission line, $N\times 1$-dimension vectors $\boldsymbol{V_S}$ contain the effects of the independent voltage and current sources in the source network. While the $N\times N$-dimension $\boldsymbol{Z_S}$ and $\boldsymbol{Z_L}$ matrices contain the effects of the impedance in the terminal networks. Chain-parameter matrix, which can be extended to represent (N+1)-wire MTL, is defined by [2]:

$$\left[\hat{\boldsymbol{\Phi}}(L)\right]=\begin{bmatrix}\cosh(\sqrt{\boldsymbol{Z}'\boldsymbol{Y}'}L) & -\boldsymbol{Z}_C\sinh(\sqrt{\boldsymbol{Y}'\boldsymbol{Z}'}L)\\ -\boldsymbol{Y}_C\sinh(\sqrt{\boldsymbol{Z}'\boldsymbol{Y}'}L) & \cosh(\sqrt{\boldsymbol{Y}'\boldsymbol{Z}'}L)\end{bmatrix}.\tag{1}$$

Here $\boldsymbol{Z}'$ and $\boldsymbol{Y}'$ indicate per-unit-length impedance matrix and admittance matrix respectively; the characteristic impedance $\boldsymbol{Z_C}$ and admittance matrices $\boldsymbol{Y_C}$ can be derived from trough diagonalizing $\boldsymbol{Z}'\boldsymbol{Y}'$ and $\boldsymbol{Y}'\boldsymbol{Z}'$ simultaneously [9]. In order to achieve the final stochastic model for a cable bundle, the first step is to model non-uniform transmission line. Cascaded approximation and Chebyshev expansion approximation are both used to solve telegraphic equations of MTL.

A. Cascaded approximation

Non-uniform TLs indicate that the cross-sectional dimensions or the wires positions vary along the wire axis. In this work the latter is only considered, which means the proposed stochastic model is based on the assumption the cross-sectional dimension of the bundle is identical. Considering real automotive bundles, this assumption is reasonable. Due to the variability of wire positions, the per-unit-length impedance $\boldsymbol{Z}'$ and admittance matrix $\boldsymbol{Y}'$ of MTL become functions of position z, denoted by $\boldsymbol{Z}'(z)$ and $\boldsymbol{Y}'(z)$. Therefore the MTL with non-uniform TLs can be described by non-constant coefficient differential equations:

$$\frac{\partial}{\partial z}\boldsymbol{V}(z)+\boldsymbol{Z}'(z)\boldsymbol{I}(z)=0,\tag{2}$$

$$\frac{\partial}{\partial z}\boldsymbol{I}(z)+\boldsymbol{Y}'(z)\boldsymbol{V}(z)=0,\tag{3}$$

$\boldsymbol{V}(z)$ and $\boldsymbol{I}(z)$ are wire voltage and current vectors on the position z. Differential equations above is of great difficulty on the solution in mathematics, due to the nonlinearity property. Alternatively a set of short uniform sections could be a simple solution to represent the entire non-uniform MTL. This approach neglects the interaction between each two sections, but enough precision could be achieved through sufficient divisions of lines. Chain-parameter $\hat{\Phi}(L)$ can associate the two port quantities of MTL as shown in Fig. 3:

$$\begin{bmatrix}\boldsymbol{V}(L)\\ \boldsymbol{I}(L)\end{bmatrix}=\left[\hat{\boldsymbol{\Phi}}(L)\right]\begin{bmatrix}\boldsymbol{V}(0)\\ \boldsymbol{I}(0)\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12}\\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{V}(0)\\ \boldsymbol{I}(0)\end{bmatrix}.\tag{4}$$

According to cascaded principle [2], the total chain-parameter $\hat{\boldsymbol{\Phi}}(L)$ for entire wires can be calculated as the product of the chain-parameter of the individual uniform segments:

$$\begin{aligned}\hat{\boldsymbol{\Phi}}\ (L)&=\hat{\boldsymbol{\Phi}}_1(\Delta z_1)\times\cdots\times\hat{\boldsymbol{\Phi}}_i(\Delta z_i)\times\cdots\times\hat{\boldsymbol{\Phi}}_n(\Delta z_n)\\ &=\prod_{i=1}^{n}\hat{\boldsymbol{\Phi}}_i(\Delta z_i)\end{aligned}\ .\tag{5}$$

Figure 2 illustrates the basic principle of breaking the non-uniform MTL into a cascade of sections, each of which can be modeled approximately as a uniform segment characterized by a chain-parameter matrix $\hat{\boldsymbol{\Phi}}_i(\Delta z_i)$. Due to each chain-parameter matrix associating voltages and currents of corresponding two ports, we can extract voltages and currents at interior points when MTL incorporated with terminal constraints at bundle ends. For example, the voltages and currents at the left port of the second subsection can be calculated from the terminal voltages and currents as:

$$\begin{bmatrix}\mathbf{V}(z_2)\\ \mathbf{I}(z_2)\end{bmatrix}=\hat{\Phi}_1(\Delta z_1)\times\hat{\Phi}_2(\Delta z_2)\begin{bmatrix}\mathbf{V}(0)\\ \mathbf{I}(0)\end{bmatrix}.\tag{6}$$

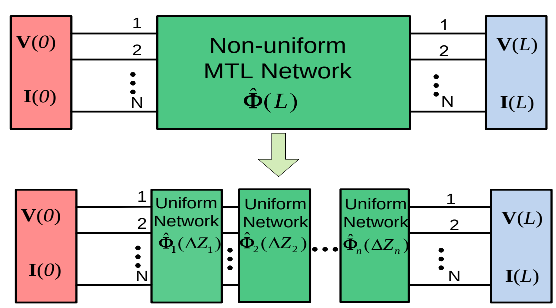


Fig. 2. Cascade approximation of entire chain-parameter.

B. Chebyshev expansion approximation

Compared with cascaded approximation method, Chebyshev expansion method [7] provides different approach to model the non-uniform MTL. This method directly solve nonlinear differential equations of MTL in (2) and (3). For example, $\boldsymbol{V}(z)$ and $\boldsymbol{I}(z)$ are defined by column-vectors of wire voltages and currents in a N-wire MTL:

$$\boldsymbol{V}(z)=\left[V_1(z),V_2(z),\cdots,V_i(z),\cdots,V_N(z)\right]^t,\tag{7}$$

$$\boldsymbol{I}(z)=\left[I_1(z),I_2(z),\cdots,I_i(z),\cdots,I_N(z)\right]^t, \tag{8}$$

$\boldsymbol{Z'}(z) = [Z_{i,j}(z)]$ and $\boldsymbol{Y'}(z) = [Y_{i,j}(z)]$ are the $N\times N$ per-unit-length impedance and admittance matrices. From these expressions, transmission line variables are functions of position z. In mathematics, any piecewise smooth and continuous function $F(z)$ can be expanded into an infinite series of Chebyshev polynomial functions $[T_k(z) = \cos(k\cos^{-1}z)]$ [10]:

$$F(z)=\frac{1}{2}F^{(0)}+\sum_{k=1}^{\infty}F^{(k)}T_k(z). \tag{9}$$

Thereby transmission line variables $V_i(z)$, $I_i(z)$, $Z_{i,j}(z)$ and $Y_{i,j}(z)$ can be approximated by Chebyshev series with (M+1) elements, respectively:

$$\begin{aligned} V_i(z) &\approx \frac{1}{2}V_i^{(0)}+\sum_{k=1}^{M}V_i^{(k)}T_k(z) \\ I_i(z) &\approx \frac{1}{2}I_i^{(0)}+\sum_{k=1}^{M}I_i^{(k)}T_k(z) \\ Z_{i,j}(z) &\approx \frac{1}{2}Z_{i,j}^{(0)}+\sum_{k=1}^{2M}Z_{i,j}^{(k)}T_k(z) \\ Y_{i,j}(z) &\approx \frac{1}{2}Y_{i,j}^{(0)}+\sum_{k=1}^{2M}Y_{i,j}^{(k)}T_k(z) \end{aligned}, \tag{10}$$

where $V_i^{(k)}$, $I_i^{(k)}$, $Z_{i,j}^{(k)}$, $Y_{i,j}^{(k)}$ are Chebyshev coefficients. Using the vector notations the Chebyshev coefficients for wire voltage and current are defined by:

$$\boldsymbol{V}_i=[\frac{1}{2}V_i^{(0)},V_i^{(1)},V_i^{(2)},\cdots,V_i^{(\mathrm{M})}]^t, \tag{11}$$

$$\boldsymbol{I}_i=[\frac{1}{2}I_i^{(0)},I_i^{(1)},I_i^{(2)},\cdots,I_i^{(\mathrm{M})}]^t. \tag{12}$$

Substituting voltage and current quantities by (11)-(12), and implementing the orthogonal property and the multiplication rule of Chebyshev polynomial functions $T_k(z)$, non-constant coefficient differential Equation (2) can be transformed into [7]:

$$\begin{bmatrix} 0_M & Q_M \\ 0 & 0_M^t \end{bmatrix}\boldsymbol{V}_i+\sum_{k=1}^{N}[Z_{i,k}]\boldsymbol{I}_k=0, \tag{13}$$

where $[Z_{i,k}]$, ($k = 1, 2, \ldots, N$) are (M+1)×(M+1) matrices with $(p,q)^{\text{th}}$ element defined below:

$$[Z_{i,k}]_{p,q}=\frac{1}{2}[Z_{i,k}^{(|p-q|)}+Z_{i,k}^{(|p+q-2|)}]. \tag{14}$$

And 0_M is an M×1 column vector with all zero elements and $\boldsymbol{Q}_M$ is an M×M matrix, which is defined in [7]. Assembling (13) for all $k = 1, 2, \ldots, N$, we can obtain the matrix formulation for the N-conductor system:

$$\hat{\mathbf{Q}}\hat{\mathbf{V}}+\hat{\mathbf{Z}}\hat{\mathbf{I}}=\mathbf{0}, \tag{15}$$

where

$$\begin{aligned} \hat{\mathbf{V}} &= [\boldsymbol{V}_1,\boldsymbol{V}_2,\cdots,\boldsymbol{V}_j,\cdots,\boldsymbol{V}_N]^t \\ \hat{\mathbf{I}} &= [\boldsymbol{I}_1,\boldsymbol{I}_2,\cdots,\boldsymbol{I}_j,\cdots,\boldsymbol{I}_N]^t \end{aligned}, \tag{16}$$

$\hat{\mathbf{V}}$ and $\hat{\mathbf{I}}$ are N(M+1)×1 column based on wire voltage and current (7) and (8). $\hat{\mathbf{Q}}$ consists of N-block of (M+1)×(M+1) Q-matrix:

$$\hat{\mathbf{Q}}=\begin{bmatrix} \mathbf{Q} & & & \\ & \mathbf{Q} & & \\ & & \ddots & \\ & & & \mathbf{Q} \end{bmatrix}_{[N(M+1)]\times[N(M+1)]} \quad \left(\mathbf{Q}=\begin{bmatrix} 0_M & Q_M \\ 0 & 0_M^t \end{bmatrix}\right). \tag{17}$$

$\hat{\mathbf{Z}}$ is constructed from the $[Z_{i,k}]$ matrix defined in (14):

$$\hat{\mathbf{Z}}=\begin{bmatrix} [Z_{1,1}] & [Z_{1,2}] & \cdots & [Z_{1,N}] \\ [Z_{2,1}] & [Z_{2,2}] & \cdots & [Z_{2,N}] \\ \vdots & \vdots & \ddots & \vdots \\ [Z_{N,1}] & [Z_{N,2}] & \cdots & [Z_{N,N}] \end{bmatrix}. \tag{18}$$

Similarly, the transmission-line Equation (3) also can be transformed into following formulation:

$$\hat{\mathbf{Q}}\hat{\mathbf{I}}+\hat{\mathbf{Y}}\hat{\mathbf{V}}=\mathbf{0}. \tag{19}$$

Therefore, the non-linear differential Equations (2) and (3) of MTL can be approximated by (15) and (19) through Chebyshev expansion. Considering the incorporated termination conditions of the transmission lines network, as shown in Fig. 1, the $\mathbf{V}_S$ is the source matrix for N-conductor MTL, $\boldsymbol{Z}_S$ and $\boldsymbol{Z}_L$ are the impedance matrix of source and load terminations, we can obtain (20) through combining with (15) and (19). By solving equation system (20), Chebyshev coefficients $\hat{\mathbf{V}}$ and $\hat{\mathbf{I}}$ in (16) can be obtained, which can be further used to calculate the wire voltages and currents via (10):

$$\begin{bmatrix} \hat{\mathbf{Y}} & \hat{\mathbf{Q}} \\ \hat{\mathbf{Q}} & \hat{\mathbf{Z}} \\ \hat{\mathbf{E}}_\mathbf{n} & \mathbf{Z}_S\hat{\mathbf{E}}_\mathbf{n} \\ \left|\hat{\mathbf{E}}_\mathbf{n}\right| & -\mathbf{Z}_L\left|\hat{\mathbf{E}}_\mathbf{n}\right| \end{bmatrix}\begin{bmatrix} \hat{\mathbf{V}} \\ \hat{\mathbf{I}} \end{bmatrix}=\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{V}_S \\ \mathbf{0} \end{bmatrix}. \tag{20}$$

Here, $\hat{\mathbf{E}}_\mathbf{n}$ is N×[N(M+1)] diagonal block matrix, of which each block is given by [7].

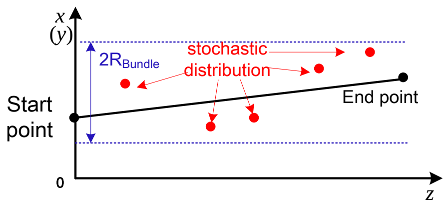
III. STOCHASTIC RADIATION MODEL OF CABLE BUNDLE

Stochastic model of a cable bundle, in essence, produces a set of deterministic non-uniform transmission lines randomly. The section above provides two different approximation approaches for a non-uniform transmission lines. This section discusses the wire position distribution norm via statistical manner, and proposes a determination method for the needed transmission line parameters in simulations.

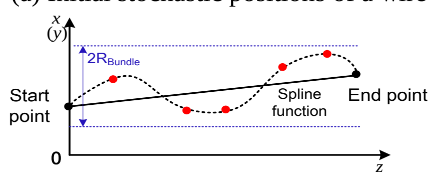
A. Stochastic distribution of wire position

The wires positions in a cable bundle are usually difficult to be determined. However the wire beginning and the wire end connected to terminations are definite. Moreover, initial wire positions in a cable bundle are distributed stochastically, but they are limited by the radius of cable bundle, as shown in Fig. 3 (a).

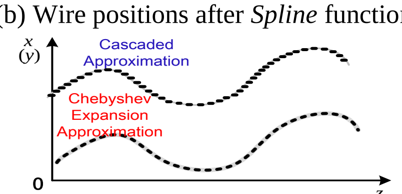
Here, z is the wire axial coordination; x and y are the cross-sectional coordination. In mathematics, a deterministic position distribution can be produced according "Gaussian" law instead of an absolute random position. After determining initial positions, other more positions can be interpolated through *Spline* function, which can improve the continuity of final constructed wire model [5], as shown in Fig. 3 (b). When the coordinates of variable wire positions are determined, cascaded method can further approximates the wire as a set of uniform sub-segments; or Chebyshev expansion method provides other solution to fit non-uniform transmission lines through smooth Chebyshev polynomial functions, as shown in Fig. 3 (c).



(a) Initial stochastic positions of a wire



(b) Wire positions after *Spline* function



(c) Cascaded or Chebyshev expansion approximation for a wire

Fig. 3. Process illustration for modeling random wires position in a cable bundle.

B. Per-unit-length parameters

Varying wire positions in a bundle lead to non-constant of impedance and admittance matrix in telegraphic Equations (2) and (3). Numeric methods such as Finite Element Method (FEM) could acquire these parameters accurately. However, the computation time is a challenge, due to the cable bundle possibly consisting of dozens of wires for each deterministic non-uniform MTL case. Thereby, a simple approach for the TL-parameters determination is necessary. When the diameters of all the wires inside the bundle are the same and 2-D cross section of the bundle is invariant along the axial direction, the evaluation of $\boldsymbol{Z}'(z)$&$\boldsymbol{Y}'(z)$ matrices only needs to be performed once. And then these determined $\boldsymbol{Z}'(z)$&$\boldsymbol{Y}'(z)$ matrices can be used as basis for evaluating new impedance and admittance matrices for other segments with different wire positions. Figure 4 gives the 'wire method', which illustrates how to obtain new $\boldsymbol{Z}'(z)$&$\boldsymbol{Y}'(z)$ matrices from the known impedance and admittance information [11].

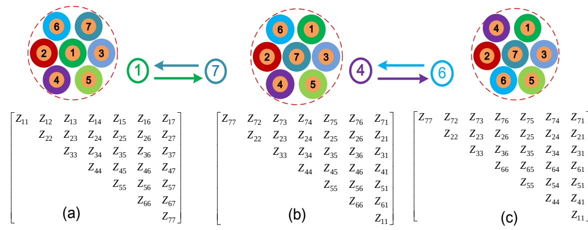


Fig. 4. Per-unit-length parameter matrix interexchange with 'wire method'.

For implementing this method, we first determine the original coordinates of wires center in a cable bundle with known $\boldsymbol{Z}'(z)$&$\boldsymbol{Y}'(z)$ matrices. Then "Gaussian" law is used to produce stochastic coordinates of wire center positions. Through comparing the distance between new coordinate of wire center to original wire centers, we choose the original wire center, of which the distance is shortest to the new coordinate, as the new position of the wire.

C. Common-mode current and radiation model

After determining the parameters matrices of the cable bundle, telegraphic Equations (2) and (3) of non-uniform MTL can be solved equivalently by 'Cascaded approximation method' or directly through 'Chebyshev expansion method' for each deterministic case in stochastic analysis. Subsequently CM current at each segment, the sum of currents on the wires in this segment, can be calculated. And then each segment with known CM current can predict the radiated emission according to electric dipole model [12]. For example the y-component field is given by:

$$H_y^d = \frac{-IdL \cdot x}{4\pi r}\beta_0^2 (j\frac{1}{\beta_0 r} + \frac{1}{\beta_0^2 r^2})e^{-j\beta_0 r}, \tag{21}$$

$$E_y^d = \frac{IdL \cdot zy}{4\pi r^2}\eta_0 \beta_0^2 (j\frac{1}{\beta_0 r} + \frac{3}{\beta_0^2 r^2} - j\frac{3}{\beta_0^3 r^3})e^{-j\beta_0 r}, \tag{22}$$

where r is the distance from the dipole to the observation point P; ε_0 is the dielectric constant of the vacuum; dL is dipole length; I is the current through the modeled segment; η_0 (377 Ω) is the wave impedance in the vacuum; β_0 ($2\pi/\lambda$) is the electromagnetic wave phase constant in the vacuum. Radiation from a cable bundle can be calculated by superposition, as shown in Fig. 5. Therefore, stochastic radiation model of a cable bundle should include a set of deterministic radiation models with different CM current distribution.

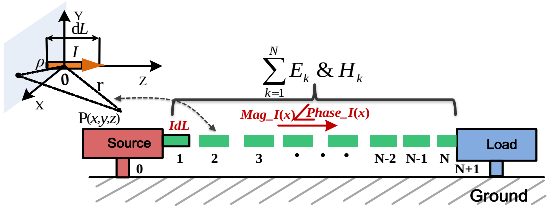


Fig. 5. Radiation from a cable bundle due to CM current.

IV. RESULTS AND EXPERIMENTS

In order to verify the proposed cascaded method and Chebyshev expansion method for modeling a non-uniform transmission line, a MTL model with analytic solution is investigated firstly, of which transmission line parameters are parabolic functions with respect to the position. Moreover, the stochastic characteristics of a real cable bundle with random wires are evaluated through simulation and measurement.

A. Parabolic multiconductor transmission line

MTL with terminations can be shown in Fig. 6 (left).

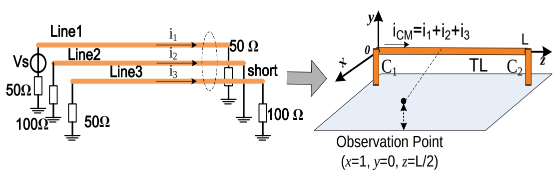


Fig. 6. Parabolic MTL with terminations (left) and its common-mode radiation model (right).

The parabolic transmission line parameters are characterized by the functions with position z:

$$\begin{aligned} \mathbf{Z}'(z, j\omega) &= (1+az)^2\left[j\omega\mathbf{L}+\mathbf{R}r(\omega)\right] \\ \mathbf{Y}'(z, j\omega) &= (1+az)^2\left[j\omega\mathbf{C}+\mathbf{G}\right] \end{aligned}, \qquad (23)$$

where

$$\mathbf{L}=\frac{1}{3600}\begin{bmatrix} 87 & 25 & 23 \\ 25 & 85 & 25 \\ 23 & 25 & 87 \end{bmatrix}\mu H/cm \qquad \mathbf{R}=diag\left[\frac{5}{12},\frac{5}{12},\frac{5}{12}\right]\Omega/cm$$

$$\mathbf{C}=\frac{1}{324}\begin{bmatrix} 68 & -40 & -13 \\ -40 & 95 & -40 \\ -13 & -40 & 68 \end{bmatrix}pF/cm \quad \mathbf{G}=diag\left[\frac{3}{5.12e^6},\frac{3}{5.12e^6},\frac{3}{5.12e^6}\right](\Omega\cdot cm)^{-1}. \qquad (24)$$

We analyze a general parabolic coupled transmission lines of the tapering factor a = 0.1, line length L = 2 m and $r(\omega) = r_0 + r_1\sqrt{\omega}$ for simulation of the skin-effect [13]. In cascade approximation method, the line is divided into 100 short segments. While in Chebyshev expansion method, the transmission line parameter matrices $\boldsymbol{Z}'(z, j\omega)$&$\boldsymbol{Y}'(z, j\omega)$ in (23) are expanded into 30 Chebyshev polynomial functions of position z at each frequency point according to (9). Since the high-order Chebyshev polynomial function might bring computation efficiency problem, cascaded approximation for non-uniform MTL takes more advantages when the bundle divided into enough short segments. Figure 7 depicts current at the middle position on line-3 by cascaded method, Chebyshev expansion method and analytic method from 1 MHz to 500 MHz, respectively. The analytic solution is referred to [14].

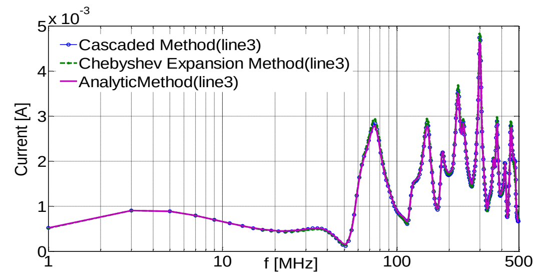


Fig. 7. Current at the middle position on line-3.

Currents on the line-3 at 200 MHz from these three methods are compared in Fig. 8. After obtaining currents on each line, CM current can be calculated by summing these line currents. Then, the radiation from the transmission lines can be evaluated with a set of short dipoles as shown in Fig. 5. Electric field in y-direction at the observation point (1 meter distant to the cable bundle) is further calculated as shown in Fig. 9. It can be seen that the results from proposed methods can match well with the analytic method.

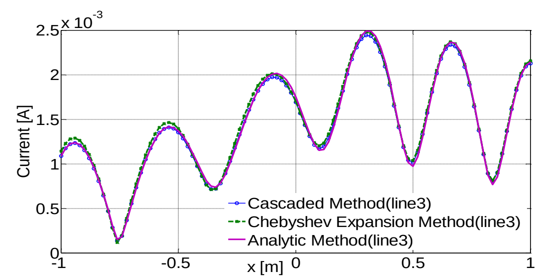


Fig. 8. Current of 200 MHz on line-3.

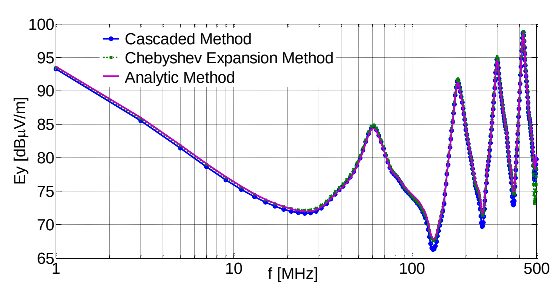


Fig. 9. Radiated emission at the observation point from parabolic transmission lines.

B. Cable bundle with random wires

In this section, CM current distribution and radiated emissions from a real cable bundle with seven random wires are investigated. Average height of the cable bundle to the ground plate is 5 cm. Diameter of each wire in the bundle is 1.2 mm, and the conductor diameter is about 0.8 mm. The detailed geometry of 2D cross section of bundle and experimental setup are shown in Fig. 10.

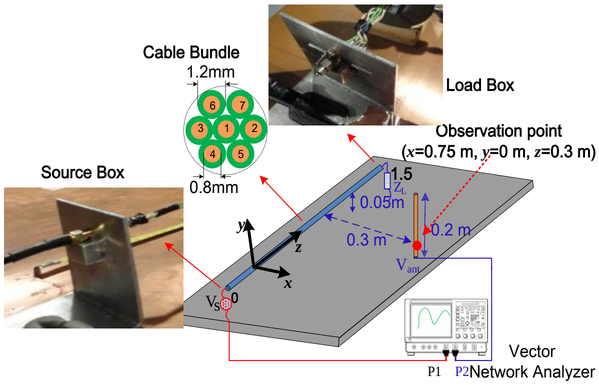
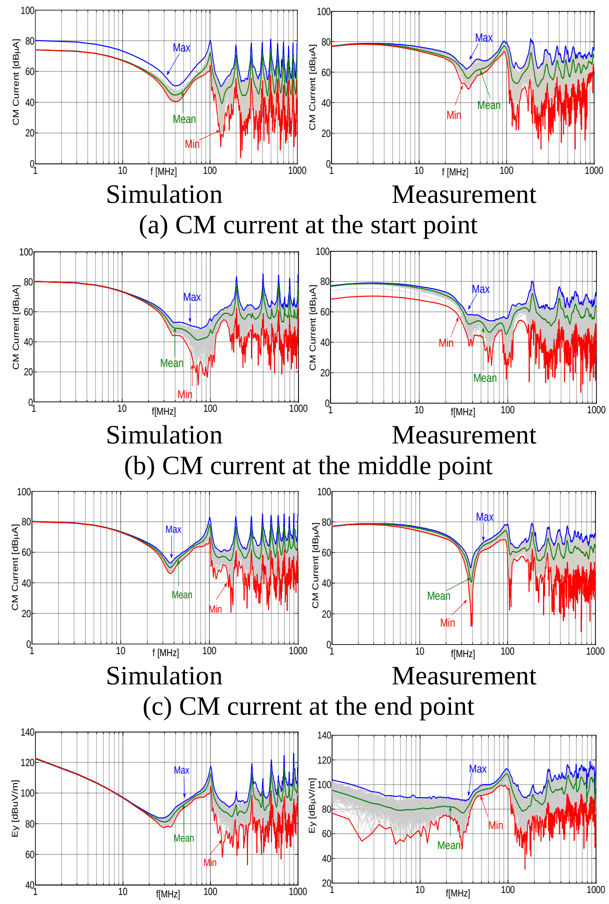


Fig. 10. Simulation and measurement configuration for a cable bundle with random wires.

The wires are terminated randomly with different resistors. In source box, one wire is driven by port 1 (P1) of a vector network analyzer (VNA); Port 2 (P2) of VNA is connected to a short rod antenna to measure electric field or a current probe to measure CM current. In the simulation and experiment, 140 different scenarios with random wire positions are implemented stochastically. Considering simulation efficiency, non-uniform transmission line in each scenario is approximated by the cascaded method. And the bundle is divided into 100 small segments. The random packed wires in cable bundles lead to non-deterministic wire position distributions in the cross sections. Stochastic non-uniform wire is difficult to be modeled via MoM and other numeric methods. Therefore with a certain stochastic function to describe random wire position, a set of deterministic non-uniform MTL can be obtained. Each non-uniform MTL produces a solution according to proposed cascaded approximation method and per-unit-length parameter deterministic method proposed above. Here stochastic-simulation sample number is 140 and total simulation time is 3.09 hours. To verify the proposed bundle stochastic-model, 140 different measurements were implemented through changing wire positions manually. CM currents at start point (beside source box), middle point and end point (beside load box) of the bundle were simulated and measured with a RF current probe. E-field at the observation point was also calculated and measured through a short rod antenna. Figure 11 shows the simulated and the measured stochastic-data distribution from 1 MHz to 1000 MHz. Maximal, mean and minimal envelop of these data are denoted. From these curves, envelop magnitudes and the main resonance frequencies from simulations and measurements agree well, which means the stochastic characteristic of a real cable bundle is close to proposed simulation model.



Simulation Measurement
(a) CM current at the start point

Simulation Measurement
(b) CM current at the middle point

Simulation Measurement
(c) CM current at the end point

Simulation Measurement
(d) E-field at observation point

Fig. 11. Simulation *vs.* measurement of common-mode (CM) currents and E-fields from the stochastic scenario.

Further we compared the maximal, mean and minimal envelops of CM current at the bundle middle point from simulation and measurement stochastic data in Fig. 12. Basically simulated maximal and mean envelopes from 1 MHz to 1000 MHz can match well with measurements; minimal envelop except below 20 MHz can also agree well with measurements. However, the simulated peaks on the maximal and mean envelopes are higher than the measured peaks at frequencies of $N \cdot 200$ MHz (N = 1, 2, 3, 4, 5). The reason for the deviation is that MTL based simulation in this work did not consider line loss and the parasitic capacitance and inductance in source and load box.

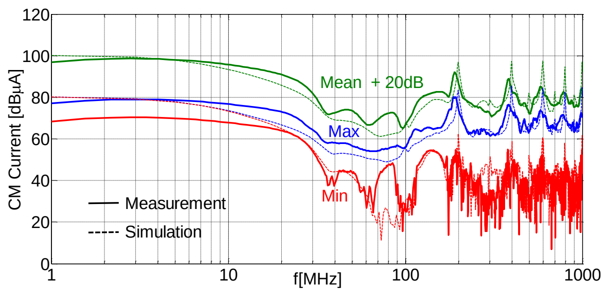


Fig. 12. Statistical analysis (Max, Mean, and Min) of CM current at the middle point on the cable bundle.

Probability density function (PDF) [15] and cumulative distribution function (CDF) [16] were also evaluated from simulated and measured CM current stochastic data. These two statistical indicators at 250 MHz were shown in Fig. 13. It can be seen that PDF from measured data express the normal distribution with mean value of 55.64 dBµA, while simulation mean value of 53.25 dBµA. CDF curves from simulated and measured data are close, which can indicate possible maximal boundary. Appearance of maximal value of common-mode current simulation data at middle point of cable bundle (250 MHz) locates at 60.35 dBµA; while appearance of maximal measured data locates at 63.22 dBµA. Similarly the maximal, mean and minimal envelopes of electric field at observation point from simulation and measurement stochastic data were compared in Fig. 14. These envelopes of electric field from 30 MHz to 1000 MHz agree well between simulated data and measured data. At low frequencies, since the used electric dipole model is sensitive to the current accuracy, the relative small error of dipole current might lead to a big difference at electric fields. Furthermore, the lower is the frequency; the bigger is the field deviation. This error can be also observed in the verification example used in [5]. Further PDF and CDF of stochastic electric field at 250 MHz from simulated and measured data are shown in Fig. 15. It can be seen that PDF from measured data expresses the normal distribution with mean value of 81.29 dBµV/m, while simulated mean value of 84.03 dBµV/m. From CDF curves from simulated and measured data, appearance of maximal value of simulation data locates at 91.03 dBµV/m; while appearance of maximal measured data locates at 94.52 dBµV/m. From simulated data, the stochastic wire position distribution (standard Gaussian rule applied in simulation) can result in CM current and field distribution obeying to lognormal rule; while from the real measured data, stochastic CM current and field due to random wire position changed manually also obey log-normal rule, but with different means and standard deviations compared with simulated data. Therefore it can be concluded that the stochastic radiation behavior of a real cable bundle due to random wire positions meets log-normal rule.

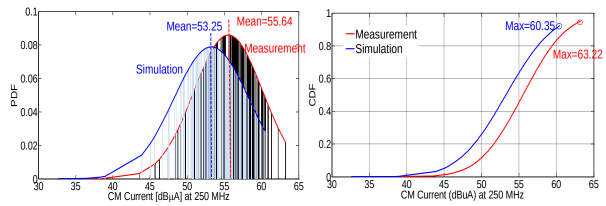


Fig. 13. Statistical analysis (PDF and CDF) of CM current at the middle point on the cable bundle (250 MHz).

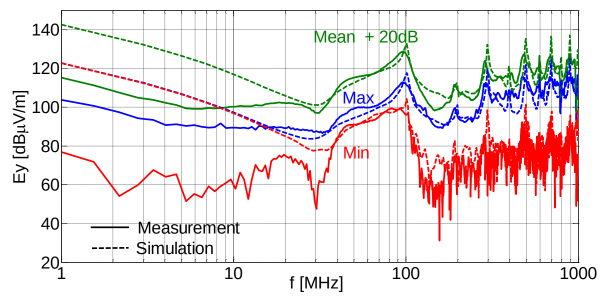


Fig. 14. Statistical analysis (Max, Mean, and Min) of E-field at the field observation point.

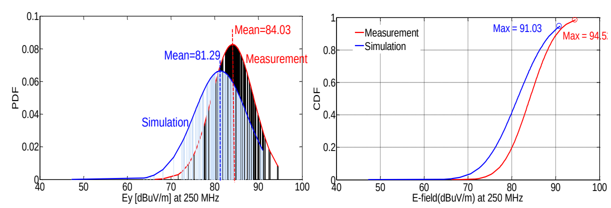


Fig. 15. Statistical analysis (PDF and CDF) of the E-field at the field observation point (250 MHz).

V. CONCLUSION

In this work, stochastic simulation models of a cable bundle were proposed based on two different approximation approaches for non-uniform transmission lines: cascaded approximation method and Chebyshev expansion method. Stochastic cable bundle model, in essence, included a set of deterministic non-uniform MTL due to random wire positions. Each non-uniform MTL can be approximated by the cascaded method through dividing them into short uniform segment; or the non-linear differential Equations (2)–(3) were directly solved by Chebyshev polynomial expansion method. This two approximation methods essentially differ from each other. One method divided the non-uniform MTL into small uniform segments physically, hence it might induce discontinuity between each two adjacent segments. The other method approximated non-linear parameters of differential equations mathematically, thereby the high order approximation smooth polynomial functions might have computation efficiency problem. Both approximation methods were verified through a parabolic MTL with analytic solutions. Subsequently, a more general cable bundle was investigated through simulation and experiment in perspective of statistic.

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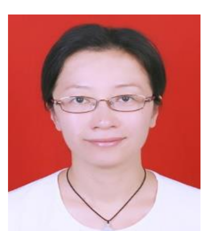

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