A Split-Step Pade Solution of 3D-PE Method for EM Scattering from PEC Targets

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Abstract **─** In this paper, the split-step Pade scheme is introduced to solve the three dimensional parabolic equation for EM scattering problems. By implementing the finite differential method, the calculation can be taken from plane to plane along the paraxial direction and a sparse-matrix equation needs to be solved in each transverse plane. In this way, the computational resources can be saved significantly when compared with the rigorous numerical methods. Numerical results demonstrate that the proposed method can obtain accurate results at wider angles up to 45°.

Index Terms **─** Electromagnetic scattering, parabolic equation method, split-step Pade.

I. INTRODUCTION

The parabolic equation (PE) method has been used as an efficient tool to analyze the EM scattering problems for a few decades [1-5]. The parabolic equation is an approximation of the wave equation and it is traditionally computed with first order Taylor expansion. By using the finite differential method (FD), the parabolic equation can be solved plane by plane along the paraxial direction. In other words, a three dimensional problem can be converted into a series of two dimensional problems to be solved by the standard parabolic equation (SPE) method. Therefore, less computational resources are needed for the SPE method than the rigorous numerical methods [6-8], such as the method of moments (MoM), the finite-difference time-domain (FDTD), and the finite element method (FEM). However, the standard PE method is a narrow-angle approximation which can only get accurate bistatic RCS results at angles within 15° of the paraxial direction.

Several kinds of high-order approximations have been introduced to the parabolic equation for wider angle bistatic computation [9-15]. These high-order approximations are based on higher order Pade approximations of a composition of the exponential or square-root functions. Both the Pade(1,1) and the Claerbout approximations were applied to the parabolic equation, which can obtain accurate results at angles within 25° of the paraxial direction for the analysis of three dimensional EM wave propagation [9]. In [10-11], the Pade(2,1) and Pade(2,2) approximations are accurate at angles even more than 40° of the paraxial direction. However, both the difference accuracy and the computation efficiency will become low with the order of Pade approximation increasing. Therefore, more efficient approximations should be developed. By using the split-step Pade scheme, a high order Pade approximation can be split as a summation of several lower order Pade approximations [12-15]. As a result, both the computational accuracy and efficiency can be guaranteed. The split-step Pade scheme was firstly introduced to solve the Helmholtz equation for propagation within optical fibers by Feit and Fleck [15]. Then more work about the split-step Pade based parabolic equation has been done by Collins and Thomson [12-14]. However, all of these works are focus on two-dimension scalar parabolic equation for EM propagating problems.

In this paper, the split-step Pade scheme is introduced to the three dimensional vector parabolic equations for the analysis of EM scattering problems. Accurate bistatic RCS results can be obtained at wider angles up to 45° of the paraxial direction. The inhomogeneous boundary conditions are added on the surface of the scattering target in each transverse plane. Moreover, the perfect matching layers (PML) are used to truncate the computational region. The rotating PE method is also used to obtain the full bistatic RCS curves.

The remainder of this paper is organized as follows. In Section 2, the theory and the formulations are given. Three numerical experiments are presented in Section 3 to show the accuracy and efficiency of the proposed

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method. Section 4 concludes this paper.

II. THEORY AND FORMULATIONS

A. Standard parabolic equation method

Suppose that a PEC object in free space is illuminated by a plane wave. The scattered field components E_x^s, E_y^s, E_z^s can be solved with the scalar wave equation:

$$\begin{aligned} &\frac{\partial^2 E_x^s}{\partial x^2}+\frac{\partial^2 E_x^s}{\partial y^2}+\frac{\partial^2 E_x^s}{\partial z^2}+k^2E_x^s=0\\ &\frac{\partial^2 E_y^s}{\partial x^2}+\frac{\partial^2 E_y^s}{\partial y^2}+\frac{\partial^2 E_y^s}{\partial z^2}+k^2E_y^s=0,\\ &\frac{\partial^2 E_z^s}{\partial x^2}+\frac{\partial^2 E_z^s}{\partial y^2}+\frac{\partial^2 E_z^s}{\partial z^2}+k^2E_z^s=0 \end{aligned} \tag{1}$$

where k is the wave number.

When the paraxial direction of the parabolic equation is chosen as the x axis, the reduced scattered fields u_x^s, u_y^s, u_z^s can be defined as:

$$\begin{aligned} u_x^s(x,y,z)&=e^{-ikx}E_x^s(x,y,z)\\ u_y^s(x,y,z)&=e^{-ikx}E_y^s(x,y,z).\\ u_z^s(x,y,z)&=e^{-ikx}E_z^s(x,y,z) \end{aligned} \tag{2}$$

Substitute Equation (2) into Equation (1), the following equations are obtained:

$$\begin{aligned} &\frac{\partial^2 u_x^s}{\partial x^2}+2ik\frac{\partial u_x^s}{\partial x}+\frac{\partial^2 u_x^s}{\partial y^2}+\frac{\partial^2 u_x^s}{\partial z^2}=0\\ &\frac{\partial^2 u_y^s}{\partial x^2}+2ik\frac{\partial u_y^s}{\partial x}+\frac{\partial^2 u_y^s}{\partial y^2}+\frac{\partial^2 u_y^s}{\partial z^2}=0.\\ &\frac{\partial^2 u_z^s}{\partial x^2}+2ik\frac{\partial u_z^s}{\partial x}+\frac{\partial^2 u_z^s}{\partial y^2}+\frac{\partial^2 u_z^s}{\partial z^2}=0 \end{aligned} \tag{3}$$

After the factorization, we can get the forward parabolic equations:

$$\begin{aligned} \frac{\partial u_x^s}{\partial x}&=-ik(1-\sqrt{Q})u_x^s\\ \frac{\partial u_y^s}{\partial x}&=-ik(1-\sqrt{Q})u_y^s,\\ \frac{\partial u_z^s}{\partial x}&=-ik(1-\sqrt{Q})u_z^s \end{aligned} \tag{4}$$

where the pseudo-differential operator Q is defined as $Q=\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+n^2.$

Then the solution of Equation (4) can be written as:

$$\begin{aligned} u_x^s(x+\Delta x,y,z)&=e^{ik\Delta x(\sqrt{Q}-1)}u_x^s(x,y,z)\\ u_y^s(x+\Delta x,y,z)&=e^{ik\Delta x(\sqrt{Q}-1)}u_y^s(x,y,z).\\ u_z^s(x+\Delta x,y,z)&=e^{ik\Delta x(\sqrt{Q}-1)}u_z^s(x,y,z) \end{aligned} \tag{5}$$

As shown in Fig. 1, the unknowns in the $(x+\Delta x)th$ transverse plane can be calculated from those at x *th* transverse plane. The calculation starts before the scattering target and ends beyond it. Moreover, the perfectly matched layer (PML) is applied to truncate the computational domain in each transverse plane. Finally, the radar cross section (RCS) results are calculated with the reduced scattered fields in the last transverse plane by near-far field conversion.

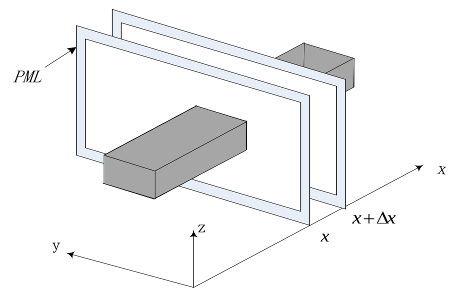


Fig. 1. The calculation process of the PE method.

B. Split-step Pade solution of parabolic equation

We substitute a rational approximation for the exponential operator:

$$e^{ik\Delta x(\sqrt{Q}-1)}=1+\sum_{l=1}^{N}\frac{a_lQ}{1+b_lQ}. \tag{6}$$

Suppose $N=2$ in Equation (6), we can rewrite the parabolic equation as:

$$\begin{aligned} u_x^s(x+\Delta x,y,z)&=u_x^s(x,y,z)+\\ &\frac{a_1Q}{1+b_1Q}u_x^s(x,y,z)+\frac{a_2Q}{1+b_2Q}u_x^s(x,y,z)\\ u_y^s(x+\Delta x,y,z)&=u_y^s(x,y,z)+\\ &\frac{a_1Q}{1+b_1Q}u_y^s(x,y,z)+\frac{a_2Q}{1+b_2Q}u_y^s(x,y,z),\\ u_z^s(x+\Delta x,y,z)&=u_z^s(x,y,z)+\\ &\frac{a_1Q}{1+b_1Q}u_z^s(x,y,z)+\frac{a_2Q}{1+b_2Q}u_z^s(x,y,z) \end{aligned} \tag{7}$$

where the coefficients are [11-12]:

$$\begin{aligned} a_1&=0.008664+0.1710i\\ b_1&=0.4929-0.1435i\\ a_2&=-0.008664+0.1431i\\ b_2&=-0.04016-0.1409i \end{aligned}. \tag{8}$$

Define

$$\begin{aligned} v_{l,x}^s(x+\Delta x,y,z)&=\frac{a_lQ}{1+b_lQ}u_x^s(x,y,z)\\ v_{l,y}^s(x+\Delta x,y,z)&=\frac{a_lQ}{1+b_lQ}u_y^s(x,y,z).\\ v_{l,z}^s(x+\Delta x,y,z)&=\frac{a_lQ}{1+b_lQ}u_z^s(x,y,z)\\ l&=1,2 \end{aligned} \tag{9}$$

It can be seen that the parabolic equation can be solved by solving $v_{1,\xi}^s(x+\Delta x,y,z)$ and $v_{2,\xi}^s(x+\Delta x,y,z)$

separately, where $\xi = x, y, z$. Then the split-step based parabolic equations can be written as follows:

$$\left[1+b_l\left(\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+1\right)\right]v_{l,x}^s(x+\Delta x,y,z) = a_l\left[\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+1\right]u_x^s(x,y,z)$$

$$\left[1+b_l\left(\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+1\right)\right]v_{l,y}^s(x+\Delta x,y,z) = a_l\left[\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+1\right]u_y^s(x,y,z)$$

$$\left[1+b_l\left(\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+1\right)\right]v_{l,z}^s(x+\Delta x,y,z) = a_l\left[\frac{1}{k^2}\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+1\right]u_z^s(x,y,z). \quad l=1,2 \tag{10}$$

When the FD scheme is used Equation (10), the forward vector parabolic equations can be written as follows:

$$\begin{aligned}&\frac{b_l}{k^2\Delta z^2}v_{l,\xi,p,q-1}^{s,m+1}+\frac{b_l}{k^2\Delta y^2}v_{l,\xi,p-1,q}^{s,m+1} +\left(1-\frac{2b_l}{k^2\Delta y^2}-\frac{2b_l}{k^2\Delta z^2}+b_l\right)v_{l,\xi,p,q}^{s,m+1} +\frac{b_l}{k^2\Delta y^2}v_{l,\xi,p+1,q}^{s,m+1}+\frac{b_l}{k^2\Delta z^2}v_{l,\xi,p,q+1}^{s,m+1}\\ &=\frac{a_l}{k^2\Delta z^2}u_{\xi,p,q-1}^{s,m}+\frac{a_l}{k^2\Delta y^2}u_{\xi,p-1,q}^{s,m} +\left(\frac{-2a_l}{k^2\Delta y^2}+\frac{-2a_l}{k^2\Delta z^2}+a_l\right)u_{\xi,p,q}^{s,m} +\frac{a_l}{k^2\Delta y^2}u_{\xi,p+1,q}^{s,m}+\frac{a_l}{k^2\Delta z^2}u_{\xi,p,q+1}^{s,m},\\ &\xi=x,y,z \quad l=1,2\end{aligned} \tag{11}$$

where $u_{p,q}^m$ is the reduced scattered field at the point of $x_m = m\Delta x, y_p = p\Delta y, z_q = q\Delta z$.

The following coordinate transformation is introduced for PML domain [16]:

$$\hat{y} = y - i\int_0^y \sigma(\xi)d\xi, \quad \hat{z} = z - i\int_0^z \sigma(\xi)d\xi \tag{12}$$

where $\sigma(\xi)=\frac{3}{2\delta}\times\frac{1}{\eta}\times\log(\frac{1}{10^{-3}})\times(\frac{\xi}{\delta})^2$, δ is the thickness of the PML and η is the wave impedance.

Similarly, the parabolic equation in the PML domain can be obtained:

$$\begin{aligned}&(1+b_l)v_{l,\xi}^s(x+\Delta x,y,z) +\frac{b_l}{k^2}e_i\frac{\partial}{\partial y}\left(e_i\frac{\partial v_{l,\xi}^s(x+\Delta x,y,z)}{\partial y}\right) +\frac{b_l}{k^2}e_j\frac{\partial}{\partial z}\left(e_j\frac{\partial v_{l,\xi}^s(x+\Delta x,y,z)}{\partial z}\right)\\ &=\frac{a_l}{k^2}e_i\frac{\partial}{\partial y}\left(e_i\frac{\partial u_\xi^s(x,y,z)}{\partial y}\right)+ \frac{a_l}{k^2}e_j\frac{\partial}{\partial z}\left(e_j\frac{\partial u_\xi^s(x,y,z)}{\partial z}\right)+a_lu_\xi^s(x,y,z),\\ &\xi=x,y,z \quad l=1,2\end{aligned} \tag{13}$$

where

$$e_i=\frac{1}{1-i\sigma(y)},\quad e_j=\frac{1}{1-i\sigma(z)},\quad \sigma(y)=\sigma_0(y/\delta)^2,$$

$$\sigma(z)=\sigma_0(z/\delta)^2,\ \sigma_0=\frac{3}{2\delta}*\frac{1}{\eta}*\log\left(\frac{1}{R_0}\right),\ \eta=120\pi,$$

$$R_0=10^{-2},10^{-3},10^{-4}.$$

The Equation (13) can be rewritten as the following equation by using the FD scheme:

$$\begin{aligned}&\left(\frac{b_le_j^2}{k^2\Delta z^2}-\frac{b_le_je_j'}{k^2\Delta z}\right)v_{l,\xi,p,q-1}^{s,m+1}+\left(\frac{b_le_i^2}{k^2\Delta y^2}-\frac{b_le_ie_i'}{k^2\Delta y}\right)v_{l,\xi,p-1,q}^{s,m+1} +\frac{b_le_i^2}{k^2\Delta y^2}v_{l,\xi,p+1,q}^{s,m+1}+\frac{b_le_j^2}{k^2\Delta z^2}v_{l,\xi,p,q+1}^{s,m+1}\\ &+\left(1+b_l-\frac{2b_le_i^2}{k^2\Delta y^2}+\frac{b_le_ie_i'}{k^2\Delta y}-\frac{2b_le_j^2}{k^2\Delta z^2}+\frac{b_le_je_j'}{k^2\Delta z}\right)v_{l,\xi,p,q}^{s,m+1}\\ &=\left(\frac{a_le_j^2}{k^2\Delta z^2}-\frac{a_le_je_j'}{k^2\Delta z}\right)u_{\xi,p,q-1}^{s,m}+\left(\frac{a_le_i^2}{k^2\Delta y^2}-\frac{a_le_ie_i'}{k^2\Delta y}\right)u_{\xi,p-1,q}^{s,m} +\frac{a_le_i^2}{k^2\Delta y^2}u_{\xi,p+1,q}^{s,m}+\frac{a_le_j^2}{k^2\Delta z^2}u_{\xi,p,q+1}^{s,m}\\ &+\left(a_l-\frac{2a_le_i^2}{k^2\Delta y^2}-\frac{2a_le_j^2}{k^2\Delta z^2}+\frac{a_le_ie_i'}{k^2\Delta y}+\frac{a_le_je_j'}{k^2\Delta z}\right)u_{\xi,p,q}^{s,m},\\ &\xi=x,y,z \qquad l=1,2\end{aligned} \tag{14}$$

where e_i' and e_j' are the first order partial derivative of e_i and e_j, respectively.

C. Boundary conditions

The above scalar parabolic equations are coupled through inhomogeneous boundary conditions on the surface of the scattering target. For the PEC objects, the tangential component of the total field equals zero on the surface:

$$\begin{cases} n_x u_y^s(p) - n_y u_x^s(p) = -e^{-ikx}\left(n_x E_y^i(p) - n_y E_x^i(p)\right) \\ n_x u_z^s(p) - n_z u_x^s(p) = -e^{-ikx}\left(n_x E_z^i(p) - n_z E_x^i(p)\right), \\ n_y u_z^s(p) - n_z u_y^s(p) = -e^{-ikx}\left(n_y E_z^i(p) - n_z E_y^i(p)\right) \end{cases} \quad (15)$$

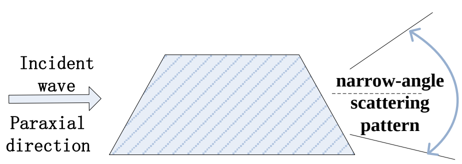
where p is a point on the surface of the scatterer and (n_x, n_y, n_z) is the outer normal to the surface at p.

To ensure the unicity of the solution, the divergence-free condition is added [1]:

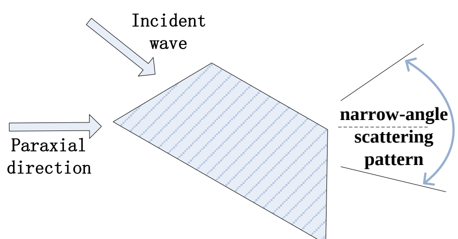
$$\frac{i}{2k}\left(\frac{\partial^2 u_x^s}{\partial y^2} + \frac{\partial^2 u_x^s}{\partial z^2}\right) + iku_x^s + \frac{\partial u_y^s}{\partial y} + \frac{\partial u_z^s}{\partial z} = 0\,. \quad (16)$$

D. Rotating PE method

As shown in Fig. 2 (a), only a narrow-angle RCS result is obtained by a single PE run. Therefore, the rotating PE method [4] is introduced to obtain the full bistatic RCS at different frequencies for the proposed method. The scattering pattern of any angle can be calculated by decoupling the paraxial direction from the direction of the incidence. As shown in Fig. 2 (b), the paraxial direction is fixed at x-axis while both the incident wave and the scattering target are rotated by a specified angle.



(a)



(b)

Fig. 2. Rotating TDPE method.

In our work, the scattering target is discretized with triangle grids. Then the cuboid grids which are needed by the FD scheme are obtained by taking advantage of the geometry information of the triangle ones. Only a narrow-angle RCS result is obtained by a single PE run. Therefore, the rotating PE method is introduced to obtain the full bistatic RCS result. After rotation, the triangle grids can be calculated directly by the coordinate transformation and then the cuboid grids should be regenerated.

III. NUMERICAL RESULTS

In this section, a series of examples are presented to demonstrate the accuracy and efficiency of the proposed method. All computations are carried out on Lenovo Intel Q9500 (2.83 GHz) with 8GB RAM.

A. The bi-static RCS for a PEC sphere

Firstly, the EM scattering from a PEC sphere is considered at the frequency of 300 MHz with the radius 4 m. The incident angle is fixed at $\theta_{inc} = 90°$ $\phi_{inc} = 0°$. The model of the PEC sphere is shown in Fig. 3. The transverse (y, z) plane of the air box is chosen to be $20\,m \times 20\,m$. There are totally 80 transverse planes to be calculated with 200×200 nodes in each transverse plane. In this simulation, all the range steps are chosen to be 0.1 m. As shown in Fig. 4, the bistatic RCS curves of the PEC sphere are compared between the traditional PE method, the proposed Split-Step Pade PE method and Mie Series. It can be seen that there is a good agreement between the Mie Series and the proposed Split-Step Pade PE method at wider angles than the standard PE method (SPE).

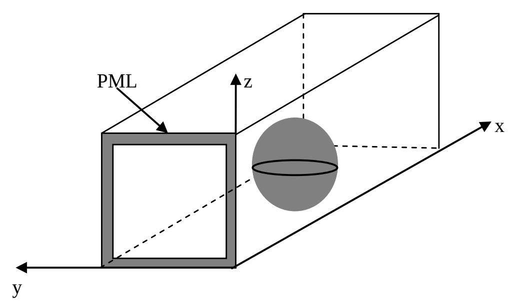


Fig. 3. Model of the PEC sphere.

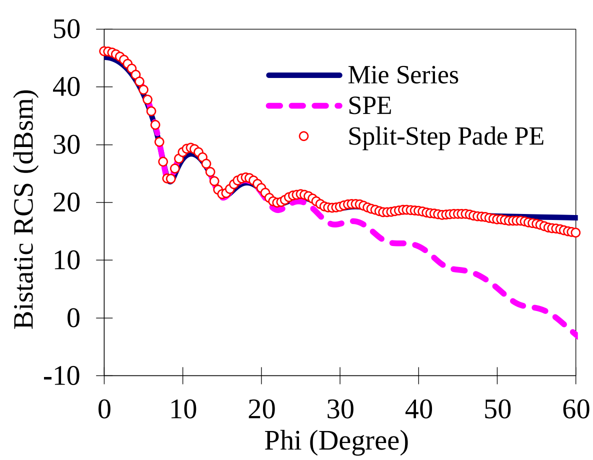


Fig. 4. Bistatic RCS result for the PEC sphere.

B. The monostatic RCS for a PEC cone

Secondly, the analysis of bisttatic RCS is taken for a PEC cone at the frequency of 300 MHz with upper radius 2 m, down radius 4 m and height 4 m. As shown in Fig. 5, the model of the cone is given. The incident angle is fixed at $\theta_{inc} = 90^\circ$ $\phi_{inc} = 0^\circ$. All the range steps are chosen to be 0.1 m and the transverse (y,z) plane of the air box is chosen to be $20m \times 20m$. As a result, there are 40 transverse planes to be calculated with 200×200 nodes in each transverse plane. As shown in Fig. 6, the bistatic RCS curves of the PEC sphere are compared between the traditional PE method, the proposed Split-Step Pade PE method and software FEKO. There is a good agreement between the FEKO and the proposed Split-Step Pade PE method at angles of 45° of the paraxial direction while 15° for the standard PE method (SPE).

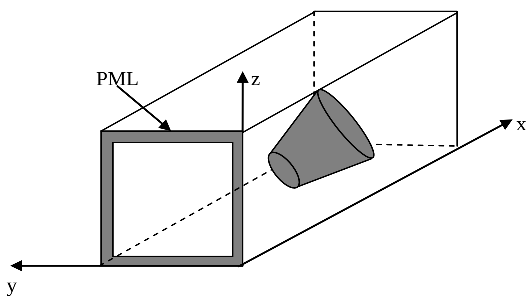


Fig. 5. Model of the PEC cone.

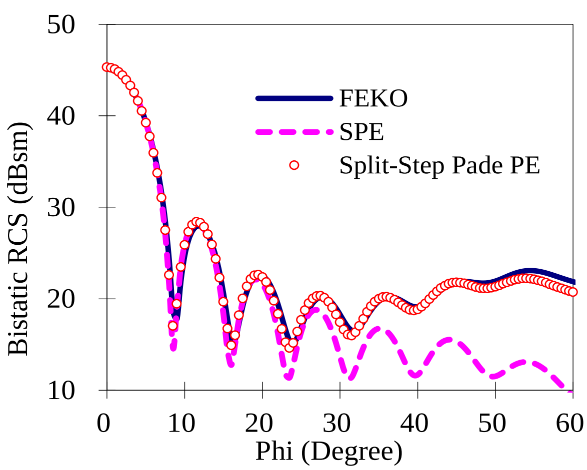


Fig. 6. Bistatic RCS result for the PEC cone.

C. Complete bistatic RCS for a PEC aircraft

At last, we consider the EM scattering from an aircraft at the frequency of 2.5 GHz and its maximum size in x, y and z directions are 10 m, 2.75 m and 8.5 m. The incident angle is fixed at $\theta_{inc} = 90^\circ$ $\phi_{inc} = 0^\circ$. In this simulation, the transverse (y,z) plane of the air box is chosen to be $12m \times 12m$. There are 167 transverse planes to be calculated with the range steps of 0.06 m and 200×200 nodes in each transverse plane. As shown in Fig. 7, the complete bistatic RCS results are compared between the proposed Split-Step Pade PE method and software FEKO. Moreover, as shown in Fig. 8, the detailed figure of the bistatic RCS result between 0° and 60° is given for better comparison. There is a good agreement between them. It should be noted that 4 rotating PE runs are used to obtain the complete bistatic RCS for the proposed method while 7 for the standard PE method. Therefore, the proposed Split-Step Pade PE method is more efficient than the standard PE method for analyzing the bistatic EM scattering problems.

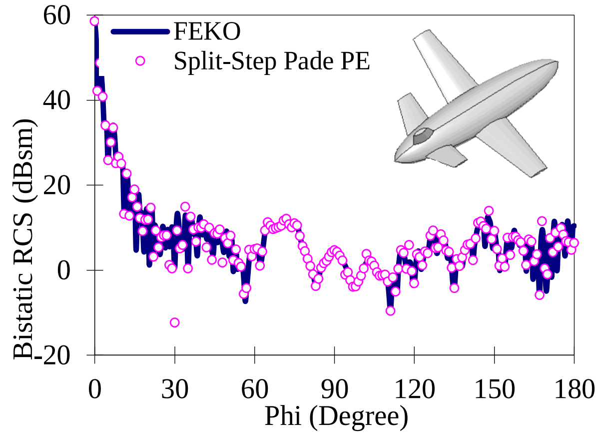


Fig. 7. Bistatic RCS result for the PEC aircraft.

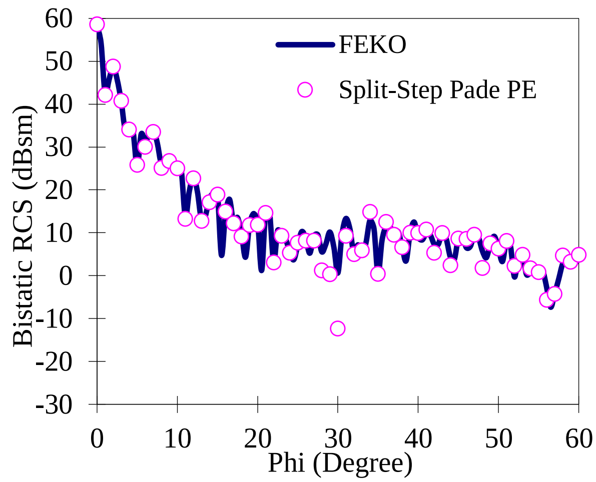


Fig. 8. Bistatic RCS result between 0° and 60° for the PEC aircraft.

IV. CONCLUSION

In this paper, the split-step Pade scheme is used to the parabolic equation for the analysis of EM scattering from electrically large PEC objects. By taking advantage of the split-step Pade scheme, a high order Pade approximation can be divided into several one-order Pade approximations. Moreover, they can be calculated separately. In this way, high computational accuracy and efficiency can be achieved by the proposed method. The numerical results demonstrate that the proposed method can obtain accurate bistatic RCS results at angles even more than 45° of the paraxial direction.

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