Efficient Representation of Multilevel QR Algorithm for Analysis of Microstrip Structures

Zhaoneng Jiang [1,3] and Ting Wan [2,3]

[1] Hefei University of Technology, Hefei 230009
jiangzhaoneng@hfut.edu.cn

[2] Nanjing University of Posts and Telecommunications, Nanjing 210003
want@njupt.edu.cn

[3] State Key Laboratory of Millimeter Waves, Nanjing 210096

Abstract ─ This paper presents a novel approach for the efficient solution of large-scale microstrip structures with the mixed potential integral equation (MPIE) in conjunction with the method of moments (MoM) based on the conventional Rao-Wilton-Glisson (RWG) basis functions. Although multilevel QR (MLQR) is efficient compared with direct method, it consumes more computation time and storage memory. A novel matrix compression technique is presented to recompress the sub-matrices of MLQR algorithm. The advantages of applying the novel recompression technique are illustrated by numerical results, the computation time as well as the memory requirements are compared to the conventional MLQR algorithm and the matrix decomposition algorithm-singular value decomposition (MDA-SVD). It is demonstrated that the use of proposed method can result in significant savings in computation time and memory requirements, with little or no compromise in the accuracy of the solution.

Index Terms ─ Compression techniques, microstrip structure, Multilevel QR Algorithm (MLQR).

I. INTRODUCTION

The method of moments (MoM) [1-5] has been widely used for the analysis of microstrip structures. The first kind is the spectral domain MoM which has been used to analyze the electromagnetic problems in [3]. The second kind is the spatial domain MoM which is proposed by Michalski and Hsu in [4] for scattering by microstrip patch antennas in a multilayered medium. However, the conventional MoM using subsectional basis functions and λ/10 discretization becomes highly inefficient for the analysis of large-scale microstrip structures. This is because the size of the associated MoM matrix grows very rapidly as the dimensions become large in terms of the wavelength, or a fine mesh is used to model a complex structure to guarantee good solution accuracy, and this in turn places an inordinately heavy burden on the CPU in terms of both memory requirement and computational complexity, which increase with $O(N^2)$ and $O(N^3)$, where N is the number of unknowns. This difficulty can be circumvented by using the Krylov iterative method, which can reduce the operation count to $O(N^2)$.

To make the iterative methods more efficient, many fast algorithms are developed to speed up the matrix-vector product operation, such as the fast multipole method (FMM) [6-7] and the matrix decomposition techniques [8-22], etc. The multilevel fast multi-pole algorithm (MLFMA) [23] has been successfully applied to the free space problem since the memory cost is reduced to the order of $N\log N$. To be noticed, though the FMM is successfully applied to the microstrip problems, the procession is always difficult because of its dependence on the Green's function. At the beginning, FMM is tried to combine with discrete complex image method (DCIM) to solve the static and two-dimensional problems [6]. Unfortunately, it will be lack of accuracy when the frequency is high. Though FMM is employed in [7] for full wave analysis, the implementation is very complicated because the surface-wave poles are extracted in DCIM. The FMM also has been applied to thin layer structures as the thin stratified medium fast multipole algorithm [24] which is adaptive to thin-stratified media. In contrast with FMM, the matrix decomposition technique is purely algebraic and, therefore, independent of the problem of Green's function. It can be easily interfaced to existing MOM codes. QR is a popular matrix decomposition technique, which has been successfully applied in [18-22] to electromagnetic problems.

The aim of this paper is to present a novel representation of MLQR algorithm for analyzing the electromagnetic scattering and radiation problems of

Submitted On: November 22, 2015
Accepted On: February 20, 2016

microstrip structures. It utilizes novel recompression technique to recompress the sub-matrices of MLQR. Simulation results show that the proposed method is computationally more efficient than the MLQR and the MDA-SVD algorithm [16-17].

II. NOVEL VERSION OF MLQR ALGORITHM

A. MLQR algorithm

The QR algorithm has been widely used to analyze the scattering/radiation [18-22], which exploits the well known fact that for well separated sub-scatterers, the corresponding sub-matrices are low rank and can be compressed [8-17]. MLQR has been successfully applied in [21-22] to electromagnetic problems. In MLQR implementation, the far-field matrix can be expressed as:

$$[Z_F]=\sum_{l=3}^{L}\sum_{i=1}^{M(l)}\sum_{j=1}^{Far(l(i))}[Q_{lij}][R_{lij}], \tag{1}$$

where $M(l)$ is the number of on empty groups at level l and, denotes the number of far interaction groups of the i-th nonempty group for each observation group $l(i)$ at level l. The largest boxes not touching each other are at level 3 in the octal tree, while the smallest boxes are at level L. The product $[Q_{lij}][R_{lij}]$ is associated with the interaction between the observer group $l(i)$ and the source group $l(j)$. For a given observation group $l(i)$, it is needed to store the matrix $[R_{lij}]$ for different source group $l(j)$, increasing the memory requirement. A novel recompression technique is utilized to recompress the sub-matrices of MLQR in this paper, which provides a sparser impedance matrix than the MLQR in solving 2-D and 3-D electromagnetic problems.

B. Novel Version of MLQR (NVMLQR)

A novel version of MLQR is proposed in this section, which utilizes matrix recompression technique to transform the far-field matrix into a sparser form. The Equation (1) can be changed into the following form:

$$[Z_F]=\sum_{l=3}^{L}\sum_{i=1}^{M(l)}[W_{li}]\sum_{j=1}^{Far(l(i))}[Y_{lij}][W_{lj}]^*, \tag{2}$$

where the $[W]$ is a unitary matrix, which is also block-diagonal matrix. The $[W]^*$ is the conjugate matrix of $[W]$. It needs to store the matrix $[W_{li}]$ only once for a given the observer group $l(i)$ and the dimensions of matrix $[Y_{lij}]$ are very small. Therefore, the form of Equation (2) is much sparser than that of Equation (1). The following process is used to form the two matrices in the Equation (2):

(i) The formation process of matrix [W]

For level l, loop over all level l source groups $l(i)$, which is not near-neighbors of the group $l(j)$. Extract the corresponding sub-matrix $[Z_{l(i),l(j)}]$, $l(i)\in Far(l(j))$ of impedance matrix $[Z]$, which is approximated by Equation (1). Then concatenate all matrix $[R_{l(i),l(j)}]$ in a row and obtain the matrix A. Maintaining the admissible error ε, use adaptive cross approximation-singular value decomposition (ACA-SVD) [25] (which does need to form the whole information of matrix $[A]$) to decompose the matrix $[A]$:

$$[A_{pn}]=\left\{\begin{matrix}\vdots\\ [R_{l(i),l(j)}]\\ \vdots\end{matrix}\right\}\xrightarrow{ACA-SVD}[U'_{pk}][V'_{nk}]^*,\ k<\min(p,m), \tag{3}$$

where n indicates the number of the basis functions in the box $l(j)$. p denotes the sum of the rank r of sub-matrix $[Z_{l(i),l(j)}]$, $l(i)\in Far(l(j))$ in Equation (1), while k indicates the rank of the matrix $[A]$. $[V'_{nk}]^*$ is the j-th diagonal block of $[W_l]$. Implement the procedure as the above, we can build $[W_l]$. Using the procedures at all level of octal tree structure, we can obtain the whole matrix of $[W]$.

(ii) The formation process of matrix [Y]

The matrix $[Y]$ is formed by using the matrix $[W]$ and Equation (1), which utilizes the matrix $[W]$ multiply both sides of the Equation (1). For level l, the detail process is shown in the following:

$$[Y_l]=\sum_{i=1}^{M(l)}[W_{li}]^*\sum_{j=1}^{Far(l(i))}[Q_{lij}][R_{lij}][W_{lj}]. \tag{4}$$

The dimensions of matrix $[Y_l]$ are very small. Using the procedures at all level of octal tree structure, we can obtain the whole matrix of $[Y]$.

The essential of above procedure is to recompress the far-field matrix of MLQR. The dominant memory usage of the novel version of MLQR is the storage of the matrices $[W]$ and $[Y]$. The forms of the matrices $[W]$ and $[Y]$ are very sparse. Because of recompression of far interactions, the matrix-vector multiplication of proposed method is more efficient than that of MLQR.

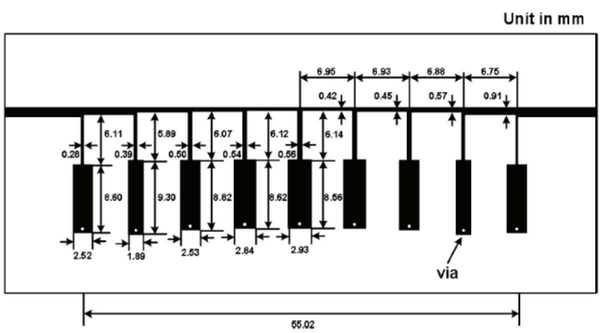
III. NUMBERICAL RESULTS

To validate and demonstrate the accuracy and efficiency of the proposed novel version of MLQR (NVMLQR), some numerical results are presented in this section. In the implementation of the NVMLQR, the restarted version of generalized minimal residual (GMRES) algorithm [26-27] is used as the iterative method. The restart number of GMRES is set to be 30 and the stop precision for restarted GMRES is denoted to be 10^{-3}. The truncating tolerance of MDA-SVD is 10^{-3} relative to the largest singular value. All experiments are performed on a Core-i5 3350P with 3.1 GHz CPU and

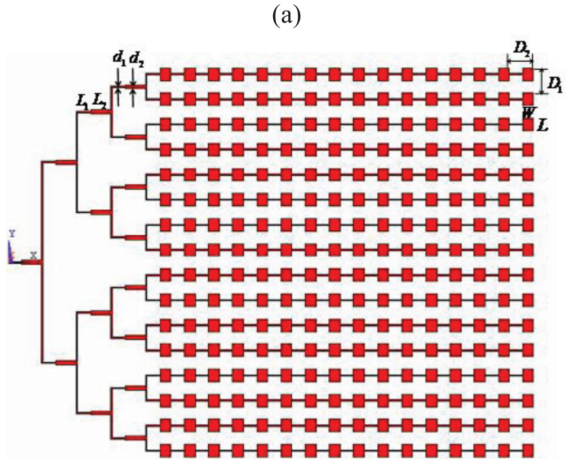
4 GB RAM in double precision.

A. The accuracy of the NVMLQR

The first example is a dual wideband filter shown in Fig. 1 (a). The dimensions of the structure are the same as that presented in [28]. The second example concerns the radiation from microstrip corporate fed planar arrays. The geometry of an 16×16 antenna array is depicted in Fig. 1 (b), where $\varepsilon_r = 2.2$ and the substrate thickness is $h = 1.59\,mm$. The width of rectangular patch is $W = 10.08\,mm$ with $L = 11.79\,mm$. The fed line is of the width $d_1 = 1.3\,mm$, $d_2 = 3.93\,mm$ with the length $L_1 = 12.32\,mm$, $L_2 = 18.48\,mm$. The interval between patches are $D_1 = 23.58\,mm$ and $D_2 = 22.40\,mm$, respectively. The frequency is 9.42 GHz. The *S*-parameter of the dual wideband filter is analyzed in Fig. 2, while the radiation pattern which is observed at $\phi = 90^0$ shown in Fig. 3. It can be found that the results of the proposed method agree well with the HFSS and the Ansoft Designer simulation results. The results validate the accuracy of the proposed method.



(a)



(b)

Fig. 1. (a) The topology and dimensions of the dual wideband filter, and (b) the 16×16 series-fed microstrip antenna array.

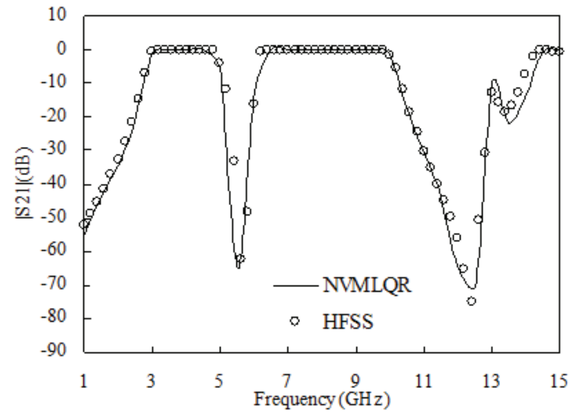


Fig. 2. The *S*-parameter of the dual wideband filter.

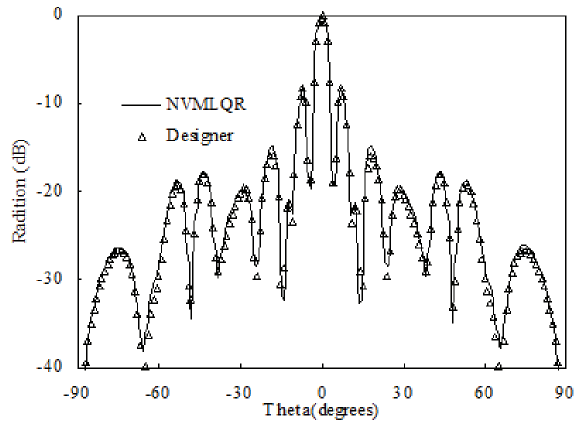


Fig. 3. The radiation pattern of the series-fed microstrip antenna.

The relative errors of the NVMLQR and MDA-SVD and MLQR for the two structures as mentioned above are shown in Table 1. The formulation of the relative error is given by:

$$\text{Relative error} = \|F - E\| / \|E\|, \tag{5}$$

where E denotes the induced current computed using the MoM iterative solution, and F is the induced current computed using the NVMLQR and MDA-SVD and MLQR.

Table 1: The relative errors for the two structures

Structures	NVMLQR	MDA-SVD	MLQR
Dual wideband filter	0.49%	0.44%	0.24%
Series-fed microstrip antenna	0.58%	0.53%	0.31%

B. The efficiency analysis of the NVMLQR

In order to check the efficiency of the algorithm, Fig. 4 and Fig. 5 shows the memory behavior and the CPU time per iteration of the NVMLQR observed on the series-fed microstrip antenna when the number of unknowns is increased, respectively. MVP indicates one matrix-vector operation in Fig. 5. According to the plots, the memory requirement of the NVMLQR is much less than that of MLQR, while it is much less than sixth of the MDA-SVD's memory and a seventh of the MLQR's memory. The MVP time of the NVMLQR is much less than a quarter of that of MDA-SVD and a fifth of that of MLQR.

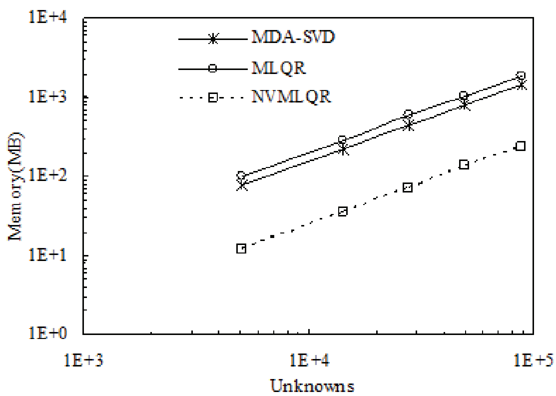


Fig. 4. Plot of the far-field memory consumption for the series-fed microstrip antenna.

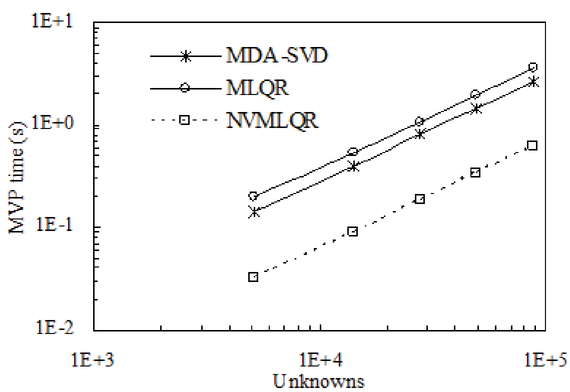


Fig. 5. Plot of the MVP time for the series-fed microstrip antenna.

IV. CONCLUSION

In this paper, a novel version of MLQR (NVMLQR) is applied to analyze the properties of microstrip structures efficiently. It is more convenient than MLFMA for solving the complex Green's functions' problems. Since recompression of far interactions of MLQR, the matrix-vector multiplication of the NVMLQR is much more efficient than MLQR. The numerical results demonstrate that the memory requirement and matrix-vector multiplication time of NVMLQR are much less than that of MLQR and MDA-SVD, while the accuracy of the NVMLQR is controllable. It can be used to compute monostatic RCS's of complex objects efficiently.

ACKNOWLEDGEMENT

This work was supported in part by HeFei University of Technology Teacher Foundation of JZ2014HGBZ0372, National Natural Science Foundation of China under Contract Number 61501159, Natural Science Foundation of Anhui Province (1608085MF124), the Open Research Program of State Key Laboratory of Millimeter Waves of Southeast University under Contract Number K201602 and K201411.

REFERENCES

[1] R. F. Harrington, *Field Computation by Moment Methods*. Malabar, Fla.: R. E. Krieger, 1968.

[2] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 409-418, May 1982.

[3] E. H. Newman and D. Forrai, "Scattering from a microstrip patch," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 245~251, Mar. 1987.

[4] K. A. Michalski and C. G. Hsu, "RCS computation of coax-loaded microstrip patch antennas of arbitrary shape," *Electromagn.*, vol. 14, pp. 33~62, Jan.~Mar. 1994.

[5] K. A. Michalski and D. L. Zheng, "Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media, part I: theory," *IEEE Trans. Antennas Propag.*, vol. 38, no. 3, pp. 335-344, 1990.

[6] L. Gurel and M. I. Aksun, "Electromagnetic scattering solution of conducting strips in layered media using the fast multipole method," *IEEE Microwave Guided Wave Lett.*, vol. 6, pp. 277~279, Aug. 1996.

[7] P. A. Macdonald and T. Itoh, "Fast simulation of microstrip structures using the fast multipole method," *Int. J. Numer. Modeling: Electron. Networks, Devices, Fields*, vol. 9, pp. 345~357, 1996.

[8] E. Michielssen and A. Boag, "A multilevel matrix decomposition algorithm for analyzing scattering from large structures," *IEEE Trans. Antennas Propag.*, vol. 44, no. 8, pp. 1086-1093, Aug. 1996

[9] A. Heldring, J. M. Tamayo, J. M. Rius, and J. Parron, "Multilevel MDA-CBI for fast direct solution of large scattering and radiation problems," *IEEE AP-S International Symposium 2007*, Honolulu, Hawaii, USA, 10-15 June 2007.

[10] J. M. Rius, J. Parron, E. Ubeda, and J. Mosig, "Multilevel matrix decomposition algorithm for

analysis of electrically large electromagnetic problems in 3-D," *Microw. Opt. Technol. Lett.*, vol. 22, no. 3, pp. 177-182, Aug. 1999.

[11] J. Parron, G. Junkin, and J. M. Rius, "Improving the performance of the multilevel matrix decomposition algorithm for 2.5-d structures application to metamaterials," *in Proc. Antennas Propag. Soc. Int. Symp.*, pp. 2941-2944, July 2006.

[12] J. Parron, J. M. Rius, and J. Mosig, "Application of the multilevel decomposition algorithm to the frequency analysis of large microstrip antenna arrays," *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 721-724, Mar. 2002.

[13] J. Parron, J. M. Rius, A. Heldring, E. Ubeda, and J. R. Mosig, "Analysis of microstrip antennas by multilevel matrix decomposition algorithm," in *Proc. Euro. Congress Computational Methods Applied Sciences in Engineering*, Barcelona, Spain, Sept. 2000.

[14] J. Parron, J. M. Rius, and J. Mosig, "Method of moments enhancement technique for the analysis of sierpinski pre-fractal antennas," *IEEE Trans. Antennas Propag.*, vol. 51, no. 8, pp. 1872-1876, Aug. 2003.

[15] J. M. Rius, A. Heldring, J. M. Tamayo, and J. Parron, "New and more efficient formulation of MLMDA for arbitrary 3D antennas and scatterers," *Antennas Propag., EuCAP The First European Conference on*, pp. 1-6, 2006.

[16] J. M. Rius, J. Parron, A. Heldring, J. M. Tamayo, and E. Ubeda, "Fast iterative solution of integral equations with method of moments and matrix decomposition algorithm - singular value decomposition," *IEEE Trans. Antennas Propag.*, vol. 56, no. 8, pp. 2314-2324, Aug. 2008.

[17] J. M. Rius, A. Heldring, J. M. Tamayo, and J. Parron, "The MDA-SVD algorithm for fast direct or iterative solution of discrete integral equations," *Antennas Propag., EuCAP The Second European Conference on*, pp. 1-8, 2007.

[18] N. A. Ozdemir and J. F. Lee, "A low-rank IE-QR algorithm for matrix compression in volume integral equations," *IEEE Trans. On Magnetics*, vol. 40, no. 2, pp. 1017-1020, Mar. 2004.

[19] S. M. Seo and J. F. Lee, "A single-level low rank IE-QR algorithm for PEC scattering problems using EFIE formulation," *IEEE Trans. Antennas Propag.*, vol. 52, no. 8, pp. 2141-2146, Aug. 2004.

[20] K. Z. Zhao and J. F. Lee, "A single-level dual rank IE-QR algorithm to model large microstrip antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 52, no. 10, pp. 2580-2585, Oct. 2004.

[21] D. Gope and V. Jandhyala, "Oct-tree-based multilevel low-rank decomposition algorithm for rapid 3-D parasitic extraction," *IEEE Trans. Computer-Aided Design*, vol. 23, no. 4, pp. 1575-1580, Nov. 2004.

[22] D. Gope and V. Jandhyala, "Efficient solution of EFIE via low-rank compression of multilevel predetermined interactions," *IEEE Trans. Antennas Propagat.*, vol. 53, no. 10, pp. 3324-3333, Oct. 2005.

[23] W. C. Chew, J. M. Jin, E. Michielssen, and J. Song, *Fast Efficient Algorithms in Computational Electromagnetics*. Boston, MA: Artech House, 2001.

[24] J. S. Zhao, W. C. Chew, C. C. Lu, E. Michielssen, and J. M. Song, "Thin-stratified medium fast-multipole algorithm for solving microstrip structures," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 395~403, Apr. 1998.

[25] M. Bebendorf and S. Kunis, "Recompression techniques for adaptive cross approximation," *J. Integral Equations Appl.*, vol. 21, no. 3, pp. 331-357, 2009.

[26] Y. Saad and M. H. Schultz, "GMRES: A generalized minimal residual algorithm for solving nonsymmetric linear systems," *SIAM J. Sci. Statist. Comput.*, vol. 7, pp. 856-869, July 1986.

[27] Y. Saad, *Iterative Methods for Sparse Linear System*. PWS Publishing Company, 1996.

[28] A.-S. Liu, T.-Y. Huang, and R.-B. Wu, "A dual wideband filter design using frequency mapping and stepped-impedance resonators," *IEEE Transaction on Microwave and Theory Technology*, vol. 12, pp. 2921-2929, 2008.