New Equivalent Circuit Analysis and Synthesis for Broadband Composite Right/Left-Handed Transmission Line Metamaterials

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Abstract **─** This paper proposes a new equivalent circuit analysis and synthesis method for the broadband composite right/left-handed transmission line (CRLH TL) and Dual-CRLH (DCRLH) TL metamaterials. In order to consider magnitude and phase of S-parameter of the CRLH/DCRLH TL element simultaneously, a pseudo-inverse technique is first implemented to extract the primary equivalent circuit. By using the filter synthesis technique, the retrieval primary equivalent circuit is further transformed to a typical band-pass/band-stop filter circuit as the secondary equivalent circuit. With two-step retrieval procedure, the response of the secondary equivalent circuit can more accurately agree with that of CRLH/DCRLH TL unit cell, compared with the sole use of the pseudo-inverse technique or the filter synthesis technique. Based on the equivalent circuit models, the topological structure and parameters of CRLH/DCRLH TL can be effectively synthesized for a given center frequency and bandwidth. Numerical results are given to demonstrate good performance of the proposed analysis and synthesis methods.

Index Terms **─** Composite right/left handed transmission line (CRLH TL), Dual-CRLH (DCRLH) TL, equivalent circuit, filter synthesis, pseudo-inverse, two-step retrieval procedure.

I. INTRODUCTION

Electromagneitc metamaterials are artificial effectively homogeneous electromagnetic structures, which exhibit unusual electromagnetic properties not found in natural media. One of physical realization of the metamaterials is based on composite right/left-handed transmission line (CRLH TL) [1]-[3]. Compared with resonant left-handed (LH) metamaterials, the CRLH TL is broad-band and low-loss. Up to now, many different types of the CRLH TLs including Dual CRLH (DCRLH) TL [4], [5], simplified CRLH TL [6] and extended CRLH TL [7] were reported. Due to unprecedented properties and performances, various CRLH-based components, antennas and system applications have been developed [8-13].

When designing the CRLH TL structure, a large number of periodic cells are usually cascaded to achieve special performance. However, analyzing the multi-cell of CRLH TL often needs to use full-wave electromagnetic simulations, and thus plenty of computational resource and time are consumed. By contrast, the equivalent circuit (EC) analysis of the CRLH TL structure is more efficient and costs less computational resource. As a result, we need to extract the relatively precise equivalent circuit of the CRLH TL unit cell firstly, and then the equivalent circuit of multi-cell CRLH TL structure can be obtained to flexibly synthesize new CRLH metamaterial structures. The CRLH TL unit cell is often categorized as a series inductor (L_R), series capacitor (C_L), shunt inductor (L_L), and shunt capacitor (C_R), and exhibits a band-pass filter, in the sense that it has left-handed (LH) high-pass with low-frequency stop-band and right-handed (RH) low-pass with high-frequency stop-band. Most of methods, such as an unwrapping method [1], a calibration method based on transmission matrix [14], etc., have been developed to extract the propagation constant of the CRLH TL structure. However, those methods cannot be implemented to solve the equivalent circuit of the CRLH TL unit cell. The filter technique [15] can be used to extract the equivalent circuit of the CRLH TL structure according to its band-pass response. But the specific phase response of the CRLH TL structure distinguishes itself from the conventional filters which are generally designed to meet the magnitude specifications. In fact, the phase response is a key parameter in the filter design. For example, the phase of S_{21} can be used to calculate the group delay of the filter [15]. A pseudo-inverse method [16] has been proposed to extract the equivalent circuit of a CRLH TL unit cell to simultaneously meet magnitude and phase responses. By transforming *S*-parameters of a CRLH TL unit cell obtained by full-wave simulation or measurement to *ABCD* parameters of the corresponding lumped LC circuit, the capacitance and inductance in the lumped LC circuit can be solved. But by use of the pseudo-inverse matrix, the retrieval lumped LC circuit can only

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roughly feature electromagnetic responses of the CRLH TL structures.

This paper presents a new method to extract the equivalent circuits of the CRLH TL and DCRLH TL unit cells which combine the aforementioned pseudo-inverse method and filter synthesis technique. The new method combines the advantages of the two traditional extraction methods which owns good performance to meet both the magnitude and phase responses. The pseudo-inverse method is first implemented to obtain the primary equivalent circuit according to both magnitude and phase of the scattering parameters of the CRLH TL and DCRLH TL unit cells. In order to further improve retrieval accuracy, the primary equivalent circuit is transformed to typical band-pass/band-stop filter circuit as the secondary equivalent circuit according to the filter synthesis technique. Numerical results show that the scattering parameter responses solved by the extracted circuit and the original unit cell agree well with each other. Furthermore, three CRLH TL cells are cascaded to verify the accuracy of the equivalent circuit, results shows the *S*-parameters of the cascaded equivalent circuit agrees well with that of the three-cell CRLH TL structure. Finally, a typical CRLH TL structure with specific feature is synthesized by use of the equivalent circuit. Numerical results also show a good agreement between the equivalent circuit and the CRLH TL structure.

II. PSEUDO-INVERSE TECHNIQUE TO EXTRACT PRIMARY EQUIVALENT CIRCUIT OF CRLH AND DCRLH TL UNIT CELL FROM S PARAMETERS

The equivalent circuit model of a CRLH unit cell can be featured by the impedance and admittance from the lumped elements L_R, C_L, L_L, and C_R, as shown in Fig. 1. Complex *S*-parameters of the CRLH unit cell at *N* distinct frequency points can be obtained by full-wave simulation or measurement. In order to approximate the *S* parameters of the CRLH TL unit cell by using its equivalent circuit model as accurately as possible, the two-step retrieval procedure is implemented. The parameter extraction of the lumped elements in the primary equivalent circuit is achieved by implementing the pseudo-inverse method. First we convert the *S* parameters of a CRLH TL unit cell to *ABCD* parameters. Specifically, the *ABCD* parameters of the unit cell are related to its *S* parameters as follows [17], [18]:

$$A=\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}, \tag{1}$$

$$B=Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}, \tag{2}$$

$$C=\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2Z_0S_{21}}, \tag{3}$$

$$D=\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}. \tag{4}$$

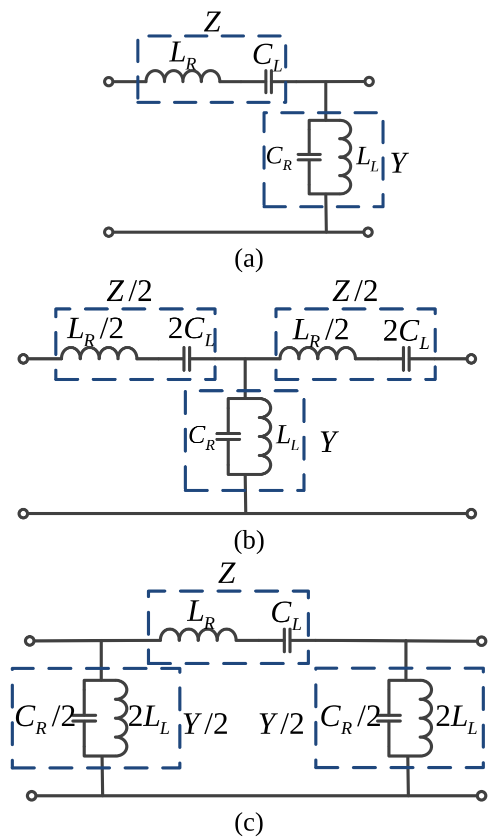


Fig. 1. The equivalent lumped LC circuits of the CRLH TL unit cell: (a) asymmetrical network, (b) symmetrical T-network, and (c) symmetrical π-network.

Second, we solve the *ABCD* parameters according to the equivalent circuit model of the CRLH unit cell. The equivalent circuit of a classical CRLH TL unit cell includes three types such as asymmetrical network, symmetrical T-network and symmetrical π-network [1], as shown in Fig. 1. Corresponding to three kinds of the equivalent circuits, the *ABCD* parameters can be expressed as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix}, \tag{5}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\begin{bmatrix} 1+\frac{ZY}{2} & Z(1+\frac{ZY}{4}) \\ Y & 1+\frac{ZY}{2} \end{bmatrix}, \tag{6}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\begin{bmatrix} 1+\frac{ZY}{2} & Z \\ Y(1+\frac{ZY}{4}) & 1+\frac{ZY}{2} \end{bmatrix}, \tag{7}$$

where the impedance parameter Z and admittance parameter Y are:

$$Z = j(\omega L_R' - \frac{1}{\omega C_L'}), \tag{8}$$

$$Y = j(\omega C_R' - \frac{1}{\omega L_L'}). \tag{9}$$

Here we have $L_R' = L_R$, $C_R' = C_R$, $L_L' = L_L$, and $C_L' = C_L$ for three types of the equivalent lumped circuits. It can be seen from (5)-(7) that $Y = C$ and $Z = B$ for the asymmetrical network, $Y = C$ and $Z = 2B/(A+1)$ for the symmetrical T-network, and $Z = B$ and $Y = 2C/(A+1)$ for the symmetrical π-network, respectively.

The third step is to extract the parameters of L_R, C_L, L_L, and C_R from the parameters Z and Y. Substituting the S parameters at N distinct frequency points into (1)-(4) and inserting the resultant $ABCD$ parameters into (5)-(9), the parameters Z and Y can be obtained as follows:

$$\begin{bmatrix} Z(\omega_1) \\ Z(\omega_2) \\ \vdots \\ Z(\omega_N) \end{bmatrix} = \begin{bmatrix} j\omega_1 & -\frac{j}{\omega_1} \\ j\omega_2 & -\frac{j}{\omega_2} \\ \vdots & \vdots \\ j\omega_N & -\frac{j}{\omega_N} \end{bmatrix} \begin{bmatrix} L_R' \\ \frac{1}{C_L'} \end{bmatrix}, \tag{10}$$

$$\begin{bmatrix} Y(\omega_1) \\ Y(\omega_2) \\ \vdots \\ Y(\omega_N) \end{bmatrix} = \begin{bmatrix} j\omega_1 & -\frac{j}{\omega_1} \\ j\omega_2 & -\frac{j}{\omega_2} \\ \vdots & \vdots \\ j\omega_N & -\frac{j}{\omega_N} \end{bmatrix} \begin{bmatrix} C_R' \\ \frac{1}{L_L'} \end{bmatrix}. \tag{11}$$

Introducing the matrix,

$$W = \begin{bmatrix} j\omega_1 & -\frac{j}{\omega_1} \\ j\omega_2 & -\frac{j}{\omega_2} \\ \vdots & \vdots \\ j\omega_N & -\frac{j}{\omega_N} \end{bmatrix}, \tag{12}$$

(10) and (11) can be rewritten as:

$$[Z] = [W] \begin{bmatrix} L_R' \\ \frac{1}{C_L'} \end{bmatrix}, \tag{13}$$

$$[Y] = [W] \begin{bmatrix} C_R' \\ \frac{1}{L_L'} \end{bmatrix}. \tag{14}$$

Solving (13) and (14) by using the pseudo-inverse technique, the lumped parameters of the equivalent circuit can be obtained as follows:

$$\begin{bmatrix} L_R' \\ \frac{1}{C_L'} \end{bmatrix} = [W]^+ [Z], \tag{15}$$

$$\begin{bmatrix} C_R' \\ \frac{1}{L_L'} \end{bmatrix} = [W]^+ [Y], \tag{16}$$

in which $[\ ]^+$ denotes the matrix pseudo-inverse. Similar procedure can be implemented for a DCRLH TL unit cell. Three types of the equivalent circuits of the DCRLHTL unit cell are shown in Fig. 2.

The impedance Z and admittance Y for the DCRLH TL unit cell can be written in terms of the lumped elements as follows:

$$\frac{1}{Z} = j(\omega C_L' - \frac{1}{\omega L_R'}), \tag{17}$$

$$\frac{1}{Y} = j(\omega L_L' - \frac{1}{\omega C_R'}), \tag{18}$$

where $L_R'=L_R$, $C_R'=C_R$, $L_L'=L_L$, and $C_L'=C_L$ for three types of the equivalent lumped circuits.

Similarly inserting the S parameters at N distinct frequency points into (1)-(7), (17) and (18), we have:

$$\left[\frac{1}{Z}\right] = [W] \begin{bmatrix} C_L' \\ \frac{1}{L_R'} \end{bmatrix}, \tag{19}$$

$$\left[\frac{1}{Y}\right] = [W] \begin{bmatrix} L_L' \\ \frac{1}{C_R'} \end{bmatrix}. \tag{20}$$

With the pseudo-inverse technique, (19) and (20) can be solved as:

$$\begin{bmatrix} C_L' \\ \frac{1}{L_R'} \end{bmatrix} = [W]^+ \left[\frac{1}{Z}\right], \tag{21}$$

$$\begin{bmatrix} L_L' \\ \frac{1}{C_R'} \end{bmatrix} = [W]^+ \left[\frac{1}{Y}\right]. \tag{22}$$

According to (15), (16), (21) and (22), the lumped capacitance and inductance of the primary equivalent circuits of the CRLH TL and DCRLH TL unit cells can be obtained.

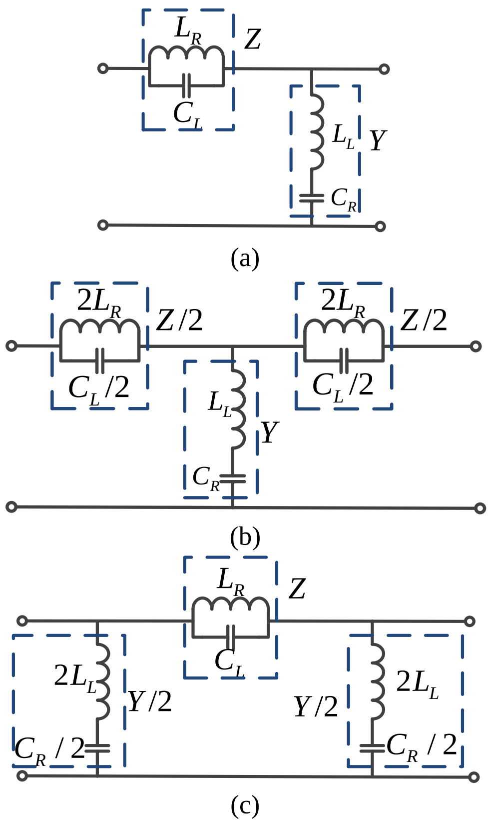


Fig. 2. The equivalent lumped LC circuits of the DCRLH TL unit cell: (a) asymmetrical network, (b) symmetrical T-network, and (c) symmetrical π-network.

III. FILTER SYNTHESIS TECHNIQUE TO EXTRACT SECONDARY EQUIVALENT CIRCUIT OF CRLH AND DCRLH TL UNIT CELL

The pseudo-inverse method can simultaneously approximate the magnitude and phase of *S* parameters of the CRLH and DCRLH TL unit cells. However, the retrieval equivalent circuit model roughly features the electromagnetic responses of the CRLH and DCRLH TL unit cells due to the use of the matrix pseudo-inverse. On the other hand, according to viewpoint of the filter design, the *S*-parameter responses of the CRLH TL unit cell demonstrate the band-pass characteristics, while those of DCRLH TL unit cell show the band-stop characteristics. Hence, a filter synthesis procedure can be introduced into the primary equivalent circuit to improve its approximation accuracy. First, we determine low-pass prototype of the primary equivalent circuit. Second, the resultant low-pass prototype is employed to re-synthesize the desirable band-pass or band-stop responses.

Considering the band-pass characteristics of the CRLH TL unit cell, its primary equivalent circuit is chosen as the symmetrical T network, as shown in Fig. 1 (b), and the corresponding low-pass prototype is shown in Fig. 3.

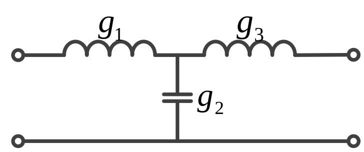


Fig. 3. The low-pass prototype.

According to the band-pass-to-low-pass transformation, we can obtain:

$$g_1 = g_3 = \frac{L_R}{2} \cdot \frac{2\pi f_0 \cdot FBW}{Z_0}, \tag{23}$$

$$g_2 = \frac{1}{L_L} \cdot \frac{Z_0 \cdot FBW}{2\pi f_0}, \tag{24}$$

where f_0 is the central frequency of the band-pass response, *FBW* is the corresponding fractional bandwidth, and Z_0 is the source impedance of the circuit. Following the band-pass synthesis procedure in terms of the coupled resonator cavity filter, as shown in Fig. 4 (a), all parameters can be determined as follows [15]:

$$L = \frac{1}{2\pi f_0}, \tag{25}$$

$$C = \frac{1}{2\pi f_0}, \tag{26}$$

$$K_{01} = K_{34} = \sqrt{\frac{FBW \cdot Z_0}{g_1}}, \tag{27}$$

$$K_{12} = K_{23} = \frac{FBW}{\sqrt{g_1 \cdot g_2}}. \tag{28}$$

Similarly for the DCRLH TL unit cell, the band-stop-to-low-pass transformation is employed, and the parameters in the low-pass prototype shown in Fig. 3 are obtained as:

$$g_1 = g_3 = \frac{L_R}{2} \cdot \frac{2\pi f_0}{FBW \cdot Z_0}, \tag{29}$$

$$g_2 = \frac{1}{L_L} \cdot \frac{Z_0}{2\pi f_0 \cdot FBW}. \tag{30}$$

With the band-stop synthesis procedure in terms of the coupled resonator cavity filter, as shown in Fig. 4 (b), we can get all parameters as follows:

$$L = \frac{1}{2\pi f_0}, \tag{31}$$

$$C = \frac{1}{2\pi f_0}, \tag{32}$$

$$\frac{1}{J_{01}} = \frac{1}{J_{34}} = \sqrt{g_1 \cdot FBW \cdot Z_0}, \tag{33}$$

$$\frac{1}{J_{12}} = \frac{1}{J_{23}} = \frac{FBW}{\sqrt{g_1 \cdot g_2}}. \tag{34}$$

It is worthwhile pointing out that the use of the filter synthesis procedure can better meet the magnitude characteristics of the *S* parameters of the CRLH and DCRLH TL unit cells, irrespective of their phase information.

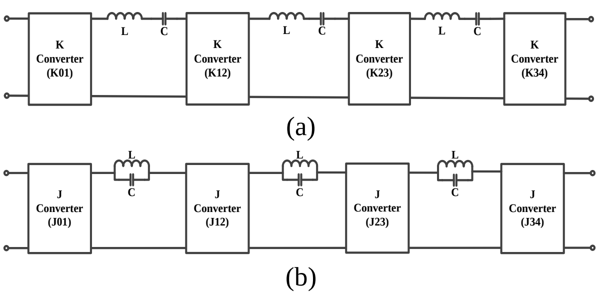


Fig. 4. The coupling resonator cavity filter circuit: (a) band-pass filter and (b) band-stop filter.

With the developed two-step retrieval procedure, the secondary equivalent circuit in terms of the coupled resonator cavity filter can accurately feature the electromagnetic responses of the CRLH/DCRLH TL unit cell. Figure 5 shows the flow chart of whole retrieval procedure.

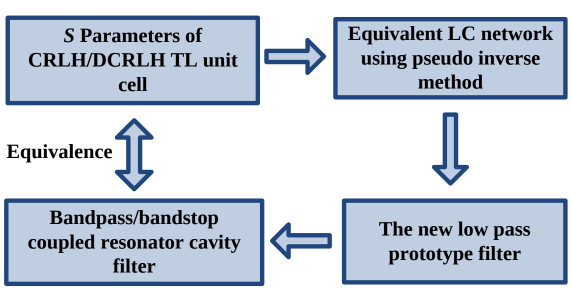


Fig. 5. The flow chart of the proposed two-step retrieval method.

IV. SYNTHESIS OF THE CRLH AND DCRLH TL UNIT CELLS

According to above discussion, we can know that the coupled resonator cavity filter obtained by the two-step retrieval method has nearly the same *S*-parameter responses in magnitude and phase as the CRLH and DCRLH TL unit cells. Hence, given the desirable center frequency and fractional bandwidth, we can synthesize the CRLH and DCRLH TL unit cell structures using the inverse two-step retrieval procedure. The flow chart of the synthesize procedure is shown in Fig. 6.

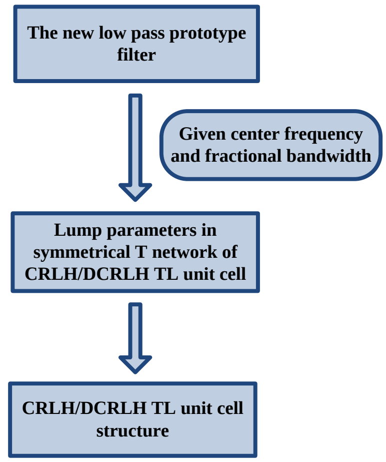


Fig. 6. The flow chart of the synthesis procedure of the CRLH/DCRLH TL unit cell.

According to the low-pass prototype of the CRLH and DCRLH TL unit cells shown in Fig. 3, the lumped elements L_R, C_L, L_L, and C_R in the primary equivalent circuit can be determined, when the operating center frequency and the fractional bandwidth are given. Specifically, according to (23) and (24), the lumped elements for the CRLH TL unit cell are:

$$L_R = \frac{2Z_0 \cdot g_1}{2\pi f_0 \cdot FBW}, \tag{35}$$

$$L_L = \frac{Z_0 \cdot FBW}{2\pi f_0 \cdot g_2}, \tag{36}$$

$$C_L = \frac{1}{(2\pi f_0)^2 \cdot L_R}, \tag{37}$$

$$C_R = \frac{1}{(2\pi f_0)^2 \cdot L_L}. \tag{38}$$

Similarly according to (29) and (30), the lumped elements for the DCRLH TL unit cell are:

$$L_R = \frac{FBW \cdot 2Z_0 \cdot g_1}{2\pi f_0}, \tag{39}$$

$$L_L = \frac{1}{g_2} \cdot \frac{Z_0}{2\pi f_0 \cdot FBW}, \tag{40}$$

$$C_L = \frac{1}{(2\pi f_0)^2 \cdot L_R}, \tag{41}$$

$$C_R = \frac{1}{(2\pi f_0)^2 \cdot L_L}. \tag{42}$$

Once these lumped elements are solved, we can design the geometries of the CRLH and DCRLH TL unit cells by the computer-aided design method. A general design optimization and modeling of microwave circuits is space-mapping optimization algorithm [19], which can automatically mate the equivalent circuits of the CRLH and DCRLH TL unit cells with their corresponding structures. Besides, for some classical structures of the CRLH and DCRLH TL unit cells, for example the CRLH TL unit cell consisting of the interdigital capacitors and stub inductors shorted to the ground plane by a via [20], [21], some approximately empirical formula can be employed to fast achieve the design.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, some CRLH and DCRLH TL structures are presented to show good performance of the proposed two-step retrieval method.

A. Example of the CRLH TL Structure

As the first example, consider a typical CRLH TL structure composed of interdigital capacitors and stub inductors shorted to the ground plane by a via [19], as shown in Fig. 7. The geometry parameters of the CRLH TL structure are given in Table 1. The pseudo-inverse method, the conventional filter synthesis method and the proposed two-step retrieval method have been implemented to extract the equivalent circuits of the CRLH unit cell, as shown in Fig. 8. Here, according to the band-pass response of the CRLH TL structure, we choose f_0=4 GHz and *FBW*=1.125, and the parameters in the corresponding low-pass prototype circuit can be obtained as $g_1 = g_3 = 0.5055$, $g_2 = 1.9355$. Figure 9 shows the *S*-parameter comparison between the pseudo-inverse method, the filter synthesis method and two-step retrieval method. It can be seen from Fig. 9 that among three retrieval methods, two-step retrieval method can obtain the best equivalent circuit to match the magnitude and phase of S_{21}.

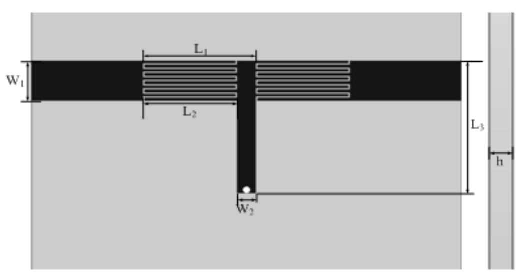


Fig. 7. Microstrip implementation of the CRLH TL unit cell.

Table 1: Parameters of the CRLH TL Unit Cell

Parameters	mm
L_1	6.10
L_2	5.00
L_3	8.00
W_1	2.40
W_2	1.00
H	1.00

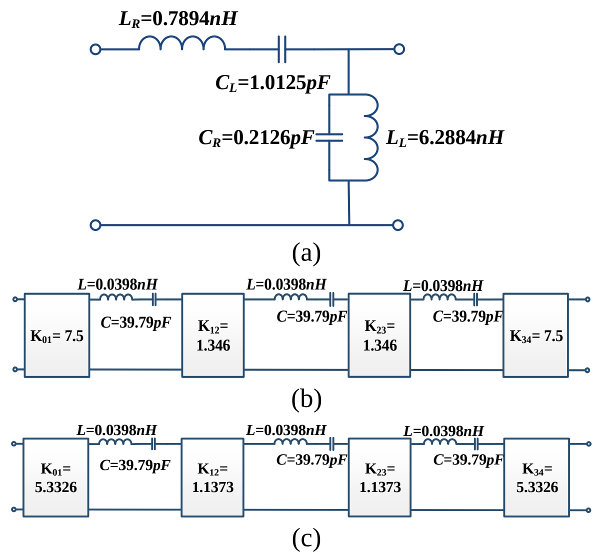


Fig. 8. Equivalent circuits of the CRLH TL unit cell: (a) the pseudo-inverse method, (b) filter synthesis method, and (c) two-step retrieval method.

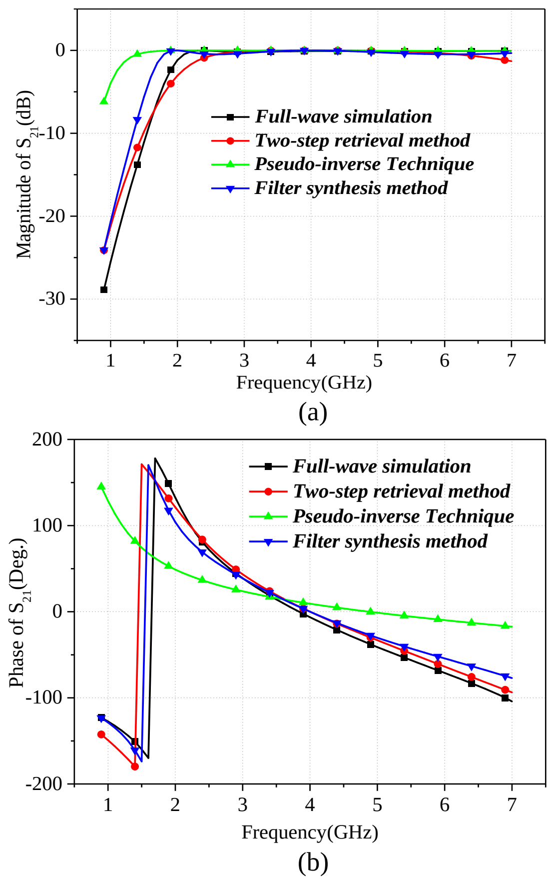


Fig. 9. *S*-parameter comparison of the equivalent circuits of the CRLH TL between the pseudo-inverse method, filter synthesis method and two-step retrieval method: (a) magnitude of S_{21} and (b) phase of S_{21}.

Furthermore, three CRLH TL unit cells is cascaded for a wider band-pass response, as shown in Fig. 10. Three kinds of the equivalent circuits obtained by three retrieval methods are cascaded to realize the equivalent circuits of the cascading CRLH TL structure, respectively. Figure 11 shows *S*-parameter comparison between three retrieval methods. It can be seen that the cascading equivalent circuit obtained by the two-step retrieval method can best match the cascading CRLH TL structure according to *S*-parameter response in magnitude and phase.

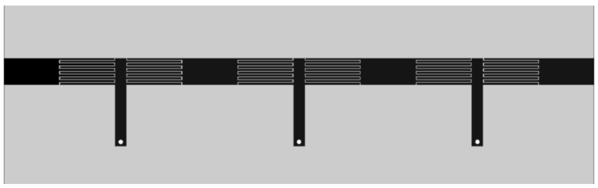


Fig. 10. Topology structure of the 3-cell CRLH TL structure.

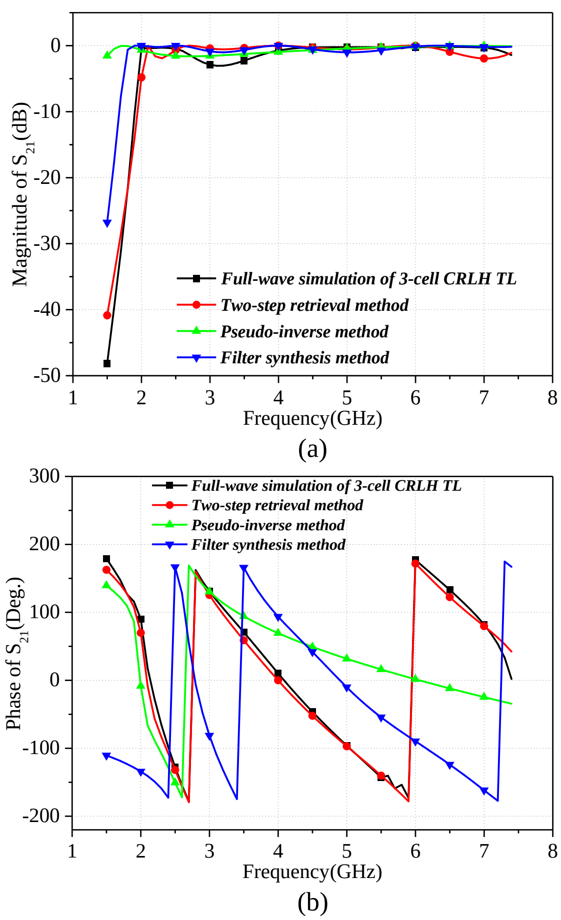


Fig. 11. *S*-parameter comparison of the 3-cell CRLH TL structure and couple resonator cavity filter: (a) magnitude of S_{21} and (b) phase of S_{21}.

B. Example of the DCRLH TL structure

Next, a microstrip implementation of the DCRLH TL structure [22] is considered. The unit cell consists of one interdigital capacitor paralleled by two high impedance stubs and four series branches of the plane capacitors and stubs, as shown in Fig. 12. The dimension of the DCRLH TL unit cell is given in Table 2. We use the pseudo-inverse method, the filter synthesis method, and the two-step retrieval method to extract the equivalent circuits of the DCRLH TL unit cell, respectively, as shown in Fig. 13. Considering the band-stop response of the DCRLH TL unit cell, we choose f_0=4.7 GHz and *FBW*=0.87 in this example. The obtained parameters in the low-pass prototype circuit are $g_1 = g_3$=0.5676 and g_2 =1.5986.

Table 2: Parameters of the DCRLH TL Unit Cell

Parameters	mm
L_1	10.0
L_2	6.00
L_3	2.00
L_4	2.80
W_1	2.80
W_2	0.75
W_3	3.50
H	1.00

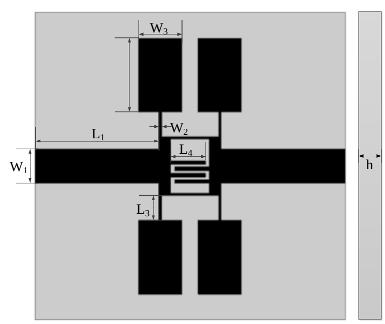
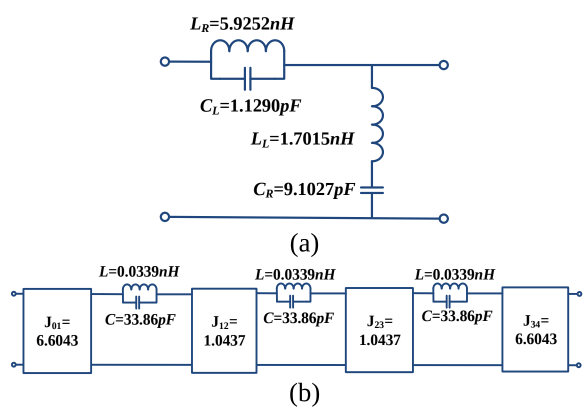
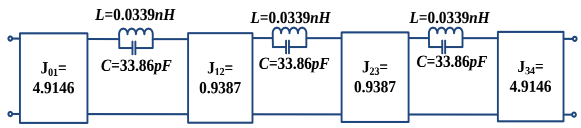


Fig. 12. Microstrip implementation of the DCRLH TL unit cell.

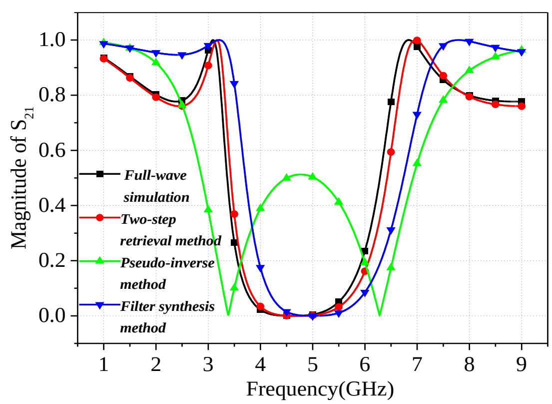




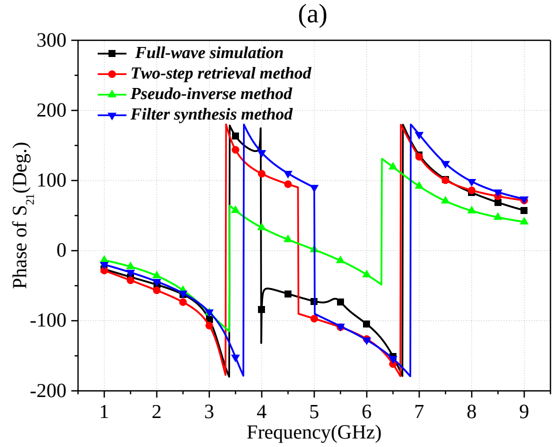
(c)

Fig. 13. Equivalent circuits of the DCRLH TL unit cell: (a) the pseudo-inverse method, (b) filter synthesis method, and (c) two-step retrieval method.

Due to the simultaneous use of the pseudo-inverse and the filter synthesis techniques, the *S*-parameter responses of the equivalent circuit extracted by the two-step retrieval method agree well with that of the original DCRLH TL unit cell, as shown in Fig. 14.



(a)



(b)

Fig. 14. *S*-parameter comparison of the equivalent circuits of the CRLH TL unit cell between the pseudo-inverse method, filter synthesis method and two-step retrieval method: (a) magnitude of S_{21} and (b) phase of S_{21}.

C. Synthesis of the CRLH TL structure

Finally, we design a CRLH TL unit cell with the operating center frequency of 10 GHz, and the fractional bandwidth of 0.6. For simplicity, the CRLH TL structure same as the first example is chosen, as shown in Fig. 15. Hence, the parameters in the low-pass prototype circuit are same as those in the first example, e.g., $g_1 = g_3 = 0.5055$ and $g_2 = 1.9355$. According to (35)-(38), the lumped elements L_R, C_L, L_L, and C_R can be solved as 0.6704 *nH*, 0.1305 *pF*, 0.2467 *nH*, and 1.0 *pF*, respectively. Considering approximation empirical formula of the CRLH TL structure [1], i.e.,

$$L_L = \frac{Z_0}{\omega_0} \tan(\beta L_3), \tag{43}$$

$$C_L = (\varepsilon_r + 1) L_2 [-2A_1 + A_2] \quad (pF), \tag{44}$$

$$A_1 = 4.409 \tanh[0.55(\frac{h}{W_1})^{0.45}] \cdot 10^{-6} (pF / \mu m), \tag{45}$$

$$A_2 = 9.92 \tanh[0.52(\frac{h}{W_1})^{0.5}] \cdot 10^{-6} (pF / \mu m), \tag{46}$$

where Z_0 and β represent the characteristic impedance and the propagation constant of the microstrip line, respectively, the geometries of the CRLH unit cell can be obtained, as shown in Table 3. For comparison, the two-step retrieval method is implemented to obtain the equivalent circuit of the CRLH TL unit cell, as shown in Fig. 16. Figure 17 shows the comparison between *S* parameter of the designed CRLH unit cell obtained by the full-wave simulations and that of the equivalent circuit extracted by the two-step retrieval method, in a good agreement with each other.

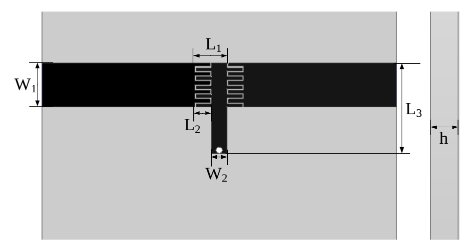


Fig. 15. Topology structure of the designed CRLH TL unit cell.

Table 3: Parameters of the DCRLH TL Unit Cell

Parameters	mm
L_1	1.05
L_2	0.55
L_3	3.00
W_1	1.45
W_2	0.50
H	1.00

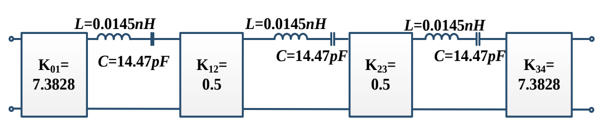


Fig. 16. Couple resonator cavity filter circuit of the designed CRLH TL unit cell.

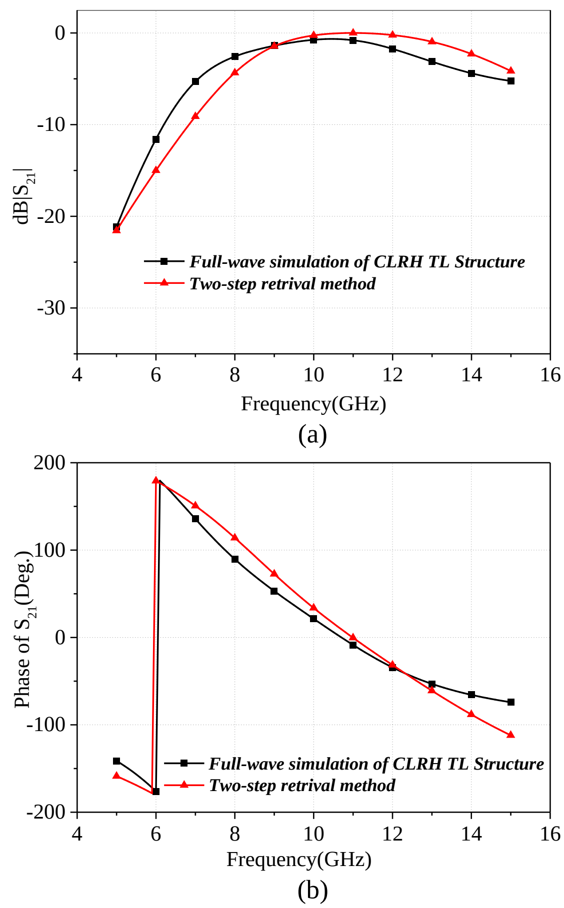


Fig. 17. *S*-parameter comparison of the designed CRLH TL unit cell and the coupled resonator cavity filter: (a) magnitude of S_{21} and (b) phase of S_{21}.

VI. CONCLUSION

The two-step retrieval method to extract the equivalent circuits of the CRLH TL and DCRLH TL unit cells is proposed. The new extraction procedure combines the pseudo-inverse technique and the filter synthesis method based on the coupled resonator cavity filter, of which the electromagnetic responses can be equivalent to the bandpass of the CRLH TL unit cell and bandstop of the DCRLH TL unit cell. The equivalent circuit obtained by the two-step retrieval method can better match the magnitude and phase of *S* parameters from the original CRLH and DCRLH TL unit cells. Based on the equivalent circuit models, the topological structure and parameters of CRLH/DCRLH TL can be effectively synthesized for a given center frequency and bandwidth. The proposed method makes the analysis and synthesis of the metamaterial-based CRLH TL structures more accurate and efficient.

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