Formulations for Modeling Voltage Sources with RLC Impedances in the FDTD Method

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Abstract — Voltage sources are modeled with constant resistive internal impedances when simulated in the finite-difference time-domain (FDTD) method in most of the applications. However, some applications, such as an RFID tag antenna in radiation mode, require modeling of voltage sources with frequency dependent complex impedances. If the frequency dependent behavior of the complex impedance of a voltage source follows that of an RLC circuit, the voltage source can be modeled with an RLC internal impedance in the FDTD method for wide-band simulations. This paper presents FDTD formulations to model a voltage source with an RLC impedance for the cases where the RLC circuit is a series or a parallel connected RL, RC, or an RLC configuration.

Index Terms — Finite-difference time-domain method, impedance modeling, source modeling.

I. INTRODUCTION

Integration of complex behavior of lumped components and sources into the finite-difference time-domain (FDTD) [1, 2] simulations has been an active research area during the past decades. Though there are various types of local sources that can be used as the excitation of an FDTD simulation, voltage source is the most common one and several formulations are available in the literature to integrate a resistive voltage source into the FDTD method [3-8]. However, in some applications the input impedance of a voltage source needs to be modeled as a reactive impedance. For instance, a radio-frequency identification (RFID) chip can be modeled as an equivalent of a voltage source together with an RLC circuit in the radiation mode [9]. For such applications where reactive impedances need to be considered, the reactive nature of the voltage source impedance needs to be modeled by appropriate formulations in the FDTD method to achieve wide-band time domain analyses. This paper presents formulations for the cases where the impedance of a voltage source is an equivalent of an RLC circuit; updating equations for series or parallel connected RL, RC, and RLC configurations are presented here.

When a lumped component is modeled in the FDTD method, the updating equation is developed assuming that the lumped component lies along an edge of a Yee cell. A voltage source with a complex impedance can be simulated by placing the source component of the voltage source along an edge of a Yee cell while placing the impedance component along the edge of the neighboring Yee cell to form a series connection, thus occupying two cells length of space in the FDTD grid. The goal here is to formulate the updating equation such that both the source and the impedance components are placed along a single edge of a Yee cell.

Various methods have been presented in the literature to incorporate arbitrary lumped devices and networks into the FDTD method [10-16]. It should be mentioned that modeling a voltage source can be considered as an extension to modeling a load, as will be discussed in the following section, therefore, some of these general load modeling methods can be extended to model voltage sources as well where the load is considered as the impedance of the voltage source.

Here, we present development of FDTD updating equations for each of the RLC configurations based on the auxiliary differential equation (ADE) method [17-21], one of the well-established methods to model dispersive materials in the FDTD method. Next section presents development of updating equations for configurations in consideration. The following section discusses the validity and accuracy of the presented formulations.

II. UPDATING EQUATIONS FOR VOLTAGE SOURCE MODELING

Following the procedure in [3], we utilize the impressed current density $\bar{J}$ in Maxwell's magnetic field curl equation:

$$\nabla \times \bar{H} = j\omega\varepsilon\bar{E} + \bar{J} , \qquad (1)$$

to model a voltage source in FDTD. Figure 1 illustrates model of a voltage source and its impedance Z_S along an edge of a Yee cell oriented in the z direction. For the voltage source, we can express the impressed current

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density as:

$$J_z = \frac{I_S}{\Delta x \Delta y}, \quad (2)$$

where the current is,

$$I_S = \frac{\Delta V + V_S}{Z_S}. \quad (3)$$

Here, V_S is the source voltage and ΔV is the voltage induced along the cell edge, which can be expressed in terms of the electric field as:

$$\Delta V = E_z \Delta z. \quad (4)$$

We can therefore express the current density in terms of the source voltage and the electric field as:

$$J_z = \frac{\Delta z}{\Delta x \Delta y}\frac{1}{Z_S}E_z + \frac{1}{\Delta x \Delta y}\frac{1}{Z_S}V_S. \quad (5)$$

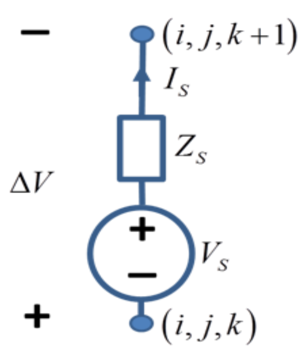

Fig. 1. A voltage source with an internal impedance of Z_S.

A. Modeling a voltage source with a series RLC impedance

For a series RLC circuit, the impedance is:

$$Z_S = R + j\omega L + \frac{1}{j\omega C} = \frac{j\omega CR - \omega^2 CL + 1}{j\omega C}, \quad (6)$$

which leads (5) to:

$$J_z = \frac{\Delta z}{\Delta x \Delta y}\frac{j\omega C}{j\omega CR - \omega^2 CL + 1}E_z + \frac{1}{\Delta x \Delta y}\frac{j\omega C}{j\omega CR - \omega^2 CL + 1}V_S. \quad (7)$$

We can write (7) briefly as:

$$J_z = j\omega Q + \frac{1}{\Delta x \Delta y}F. \quad (8)$$

The term Q is in the form of a single term Lorenzian as:

$$Q = \frac{\Delta z C}{\Delta x \Delta y}\frac{1}{j\omega CR - \omega^2 CL + 1}E_z = \frac{\psi}{\omega_0^2 + 2j\omega\delta - \omega^2}E_z, \quad (9)$$

where $\psi = \frac{\Delta z}{\Delta x \Delta y L}$, $\delta = \frac{R}{2L}$, and $\omega_0 = \frac{1}{\sqrt{LC}}$. Then construction of updating equation including the Q term is similar to the one for modeling Lorentzian dispersive medium as presented in [21]. Following the procedure in [21], we can construct an ADE from (9) as:

$$\left(\frac{\partial^2}{\partial t^2} + 2\delta\frac{\partial}{\partial t} + \omega_0^2\right)Q = \psi E_z, \quad (10)$$

which can be expressed in discrete time using central difference approximation at time step n as:

$$\frac{Q^{n+1} - 2Q^n + Q^{n-1}}{(\Delta t)^2} + \delta\frac{Q^{n+1} - Q^{n-1}}{(\Delta t)^2} + \omega_0^2 Q^n = \psi E_z^n. \quad (11)$$

Equation (11) can be arranged to calculate the future value of Q, such that,

$$Q^{n+1} = \frac{2 - (\Delta t)^2\omega_0^2}{\delta\Delta t + 1}Q^n + \frac{\delta\Delta t - 1}{\delta\Delta t + 1}Q^{n-1} + \frac{(\Delta t)^2\psi}{\delta\Delta t + 1}E_z^n. \quad (12)$$

The additional source term in (8) is denoted as F. From (7) and (8), we can write:

$$\left(j\omega CR - \omega^2 CL + 1\right)F = j\omega C V_S. \quad (13)$$

This equation can be expressed in time domain as:

$$CL\frac{\partial^2}{\partial t^2}F + CR\frac{\partial}{\partial t}F + F = C\frac{\partial}{\partial t}V_S. \quad (14)$$

The term F can be calculated analytically or numerically for a given time waveform V_S before the FDTD iterations start. For instance, if we use second order accurate central difference for (14) we obtain an iterative equation to calculate F for all time steps as:

$$F^{n+0.5} = \left(\frac{4L - 2(\Delta t)^2 / C}{2L + R\Delta t}\right)F^{n-0.5} + \left(\frac{R\Delta t - 2L}{2L + R\Delta t}\right)F^{n-1.5} + \frac{\Delta t}{2L + R\Delta t}\left({V_S}^{n+0.5} - {V_S}^{n-1.5}\right). \quad (15)$$

Using (8) in (1) and transforming into time domain, we obtain:

$$\left(\nabla \times \bar{H}\right)_z = \varepsilon\frac{\partial}{\partial t}E_z + \frac{\partial}{\partial t}Q + \frac{1}{\Delta x \Delta y}F, \quad (16)$$

which leads to an FDTD updating equation for a voltage source with a series RLC impedance as:

$$E_z^{n+1} = E_z^n + \frac{\Delta t}{\varepsilon}\left(\nabla \times \bar{H}\right)_z^{n+0.5}$$
$$-\frac{1}{\varepsilon}\left(Q^{n+1} - Q^n\right) - \frac{1}{\varepsilon}\frac{\Delta t}{\Delta x \Delta y}F^{n+0.5}. \quad (17)$$

The field update procedure can be summarized as follows: At every time step, magnetic field components are updated as usual. Next, the new value of Q is calculated using its past values and the past values of E_z using (12). Then, E_z is updated using the past value of E_z and magnetic field components, current value of F, as well as the current and past values of Q as in (17).

B. Modeling a voltage source with a series RL impedance

We can develop the equations for a series RL configuration starting from series RLC equations; we assume that capacitance is infinite. Then (9) becomes:

$$Q = \frac{\Delta z}{\Delta x \Delta y L}\frac{1}{j\omega R / L - \omega^2}E_z. \quad (18)$$

Then we can use $\psi = \frac{\Delta z}{\Delta x \Delta y L}, \delta = R / 2L, \omega_0 = 0$, which leads (12) to:

$$Q^{n+1} = \frac{2}{\left(\delta\Delta t + 1\right)}Q^n + \frac{\left(\delta\Delta t - 1\right)}{\left(\delta\Delta t + 1\right)}Q^{n-1} + \frac{\left(\Delta t\right)^2\psi}{\left(\delta\Delta t + 1\right)}E_z^n. \quad (19)$$

Using infinite capacitance value, (15) becomes:

$$F^{n+0.5} = \left(\frac{4L}{2L + R\Delta t}\right)F^{n-0.5} + \left(\frac{R\Delta t - 2L}{2L + R\Delta t}\right)F^{n-1.5}$$
$$+\frac{\Delta t}{2L + R\Delta t}\left(V_S^{n+0.5} - V_S^{n-1.5}\right). \quad (20)$$

FDTD updating equation for a voltage source with a series RL impedance is (17), where Q and F are calculated using (19) and (20), respectively.

C. Modeling a voltage source with a series RC impedance

If we use the equations for the series RLC case in Section A as is and just let $L = 0$, we face a few problems to formulate the series RC case. First of all the terms ψ, δ, and ω_0 tend to infinity as L tends to zero. Thus, (12) cannot be used.

It is possible to go back to (9) and let $L = 0$ there and derive an equation to calculate Q. However, notice that, the relationship between Q and E is a second order derivative when L is non-zero and it reduces to a first order derivative for $L = 0$ when moving from (9) to (10). When the equation is a second order derivative, then it is more convenient and accurate to calculate Q at integer time steps (i.e., as Q^{n+1} or Q^n) using E^n and using the derivative of Q in (16) (or difference of Q in (17)) rather than Q itself in (16) (or $Q^{n+0.5}$ in (17)). When L is zero and (9) leads to a first order derivative, then it is more convenient and accurate to calculate Q at half integer time steps as $Q^{n+0.5}$ and reformulate (16) to use Q directly rather than its derivative. Hence, a new formulation that uses Q directly is needed instead of (16) and such a formulation can be derived by going back to (1). The details of this procedure are presented below.

The admittance of a series RC circuit is:

$$Y_S = \frac{j\omega C}{j\omega CR + 1} = \frac{1}{R}\left(\frac{j\omega CR}{j\omega CR + 1}\right)$$
$$= \left(\frac{1}{R} - \frac{1}{j\omega CR^2 + R}\right). \quad (21)$$

Using (21) in (5) yields:

$$J_z = \frac{\Delta z}{\Delta x \Delta y}\left(\frac{1}{R} - \frac{1}{j\omega CR^2 + R}\right)E_z$$
$$+\frac{1}{\Delta x \Delta y}\left(\frac{1}{R} - \frac{1}{j\omega CR^2 + R}\right)V_S. \quad (22)$$

Using J_z in (1) leads to:

$$\left(\nabla \times \bar{H}\right)_z = j\omega\varepsilon E_z + \frac{\Delta z}{\Delta x \Delta y R}E_z + \frac{1}{\Delta x \Delta y R}V_S$$
$$-\frac{\Delta z}{\Delta x \Delta y}\frac{1}{j\omega CR^2 + R}E_z - \frac{1}{\Delta x \Delta y}\frac{1}{j\omega CR^2 + R}V_S. \quad (23)$$

First three terms on the RHS are equivalent to the equation for a resistive voltage source. The fourth term can be expressed as:

$$Q = \frac{\Delta z}{\Delta x \Delta y}\frac{1}{j\omega CR^2 + R}E_z, \quad (24)$$

which leads in discrete time domain to:

$$Q^{n+0.5} = \frac{\left(2CR - \Delta t\right)}{\left(2CR + \Delta t\right)}Q^{n-0.5}$$
$$+\frac{2\Delta t \Delta z}{\left(2CR + \Delta t\right)R\Delta x \Delta y}E_z^n. \quad (25)$$

We can use the source terms (the third and fifth terms) in (23) to write:

$$F = V_S - \frac{1}{j\omega CR + 1}V_S = \frac{j\omega CR}{j\omega CR + 1}V_S, \quad (26)$$

which leads to:

$$\frac{\partial}{\partial t}F + \frac{1}{CR} = \frac{\partial}{\partial t}V_S. \quad (27)$$

In discrete time, (27) yields:

$$F^{n+0.5} = \frac{2CR - \Delta t}{2CR + \Delta t} F^{n-0.5} + \frac{2CR}{2CR + \Delta t}\left(V_S{}^{n+0.5} - V_S{}^{n-0.5}\right). \tag{28}$$

We can obtain an updating equation from (23) as:

$$E_z^{n+1} = K_1 K_2 E_z^n + K_2 \left(\nabla \times \bar{H}\right)_z^{n+0.5} + K_2 Q^{n+0.5} - K_2 \frac{1}{\Delta x \Delta y R} F^{n+0.5}, \tag{29}$$

where

$$K_1 = \frac{\varepsilon}{\Delta t} - \frac{\Delta z}{2\Delta x \Delta y R}, \tag{30}$$

and

$$K_2 = \left(\frac{\varepsilon}{\Delta t} + \frac{\Delta z}{2\Delta x \Delta y R}\right)^{-1}. \tag{31}$$

FDTD updating equation for a voltage source with a series RC impedance is (29), where Q and F are calculated using (25) and (28), respectively.

D. Modeling a voltage source with a parallel RLC impedance

For a parallel RLC circuit, the source admittance becomes:

$$Y_S = \frac{1}{Z_S} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C, \tag{32}$$

which leads (5) to:

$$J_z = \frac{\Delta z}{\Delta x \Delta y} Y_S E_z + \frac{1}{\Delta x \Delta y} Y_S V_S. \tag{33}$$

Using (33), we can write (1) in open form as:

$$\left(\nabla \times \bar{H}\right)_z = j\omega\left(\varepsilon + \frac{\Delta z}{\Delta x \Delta y} C\right) E_z + \left(\frac{\Delta z}{\Delta x \Delta y}\frac{1}{R}\right) E_z + \frac{\Delta z}{\Delta x \Delta y L} Q + \frac{1}{\Delta x \Delta y} F, \tag{34}$$

where

$$Q = \frac{1}{j\omega} E_z, \tag{35}$$

and

$$F = Y_S V_S = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right) V_S. \tag{36}$$

We can derive an updating equation for Q using (35) as:

$$Q^{n+0.5} = Q^{n-0.5} + \Delta t E_z^n. \tag{37}$$

This $Q^{n+0.5}$ term can be calculated at every time step using E_z^n and can be used to update E_z^{n+1}. Equation (36) leads to:

$$j\omega L R F = \left(-\omega^2 C L R + j\omega L + R\right) V_S, \tag{38}$$

which can be transformed into discrete time domain using the central difference approximation to obtain an iterative equation to calculate F as:

$$F^{n+0.5} = F^{n-1.5} + \frac{\left(2CR + \Delta t\right)}{R\Delta t} V_S^{n+0.5} + \frac{\left(2\Delta t^2 R / L - 4CR\right)}{R\Delta t} V_S^{n-0.5} + \frac{\left(2CR - \Delta t\right)}{R\Delta t} V_S^{n-1.5}. \tag{39}$$

Finally, (34) can be transformed into discrete time domain using the central difference approximation to obtain:

$$E_z^{n+1} = K_1 K_2 E_z^n + 2R\Delta x \Delta y \Delta t K_2 \left(\nabla \times \bar{H}\right)_z^{n+0.5} - 2\left(R / L\right)\Delta z \Delta t K_2 Q^{n+0.5} - 2R\Delta t K_2 F^{n+0.5}, \tag{40}$$

where

$$K_1 = \varepsilon \Delta x \Delta y 2R + \Delta z C 2R - \Delta z \Delta t, \tag{41}$$

and

$$K_2 = \left(\varepsilon \Delta x \Delta y 2R + \Delta z C 2R + \Delta z \Delta t\right)^{-1}. \tag{42}$$

FDTD updating equation for a voltage source with a parallel RLC impedance is (40), where Q and F are calculated using (37) and (39), respectively.

E. Modeling a voltage source with a parallel RL impedance

We can use the equations developed for the parallel RLC configuration to model the parallel RL case: we just set the capacitance as zero in all respective equations in Section II.D such that:

$$F^{n+0.5} = F^{n-1.5} + \frac{1}{R} V_S^{n+0.5} + \frac{2\Delta t}{L} V_S^{n-0.5} - \frac{1}{R} V_S^{n-1.5}, \tag{43}$$

$$K_1 = \varepsilon \Delta x \Delta y 2R - \Delta z \Delta t, \tag{44}$$

and

$$K_2 = \left(\varepsilon \Delta x \Delta y 2R + \Delta z \Delta t\right)^{-1}. \tag{45}$$

F. Modeling a voltage source with a parallel RC impedance

We can use the equations developed for the parallel RLC configuration to model the case of parallel RC circuit: we just set the inductance as infinite in all respective equations in Section II.D. Equation (39) becomes:

$$F^{n+0.5} = F^{n-0.5} + \frac{\left(2CR + \Delta t\right)}{R\Delta t} V_S{}^{n+0.5} + \frac{\left(-4C\right)}{\Delta t} V_S^{n-0.5} + \frac{\left(2CR - \Delta t\right)}{R\Delta t} V_S{}^{n-1.5}. \tag{46}$$

The updating equation for electric field can be obtained as:

$$E_z^{n+1} = K_1 K_2 E_z^n + 2R\Delta x\Delta y\Delta t K_2 \left(\nabla\times\bar{H}\right)_z^{n+0.5}$$
$$-2R\Delta t K_2 F^{n+0.5}, \quad (47)$$

where K_1 and K_2 are the same as (41) and (42), respectively.

G. Modeling a voltage source across multiple FDTD cells

The updating equations presented above are developed for a voltage source connected across an edge of a single cell. Another case that needs consideration is where the voltage source is defined across multiple three dimensional FDTD cells. Such a case is discussed in [22] as an extension to the source modeling formulation presented in [4].

Here, as a simple procedure, we can assign an equivalent voltage, resistance, inductance, and capacitance values to each cell edge contained in the volume of the voltage source region and still use the above equations to update the associated electric fields.

For instance, Fig. 2 illustrates a voltage source across multiple cells. Here each edge is assigned an equivalent voltage of $V_{S,eq}$ and impedance of $Z_{S,eq}$, where $Z_{S,eq}$ can be a parallel or series connection of R_{eq}, L_{eq}, and/or C_{eq}. The indices *is*, *js*, and *ks*, indicate the start node of the region where the voltage source is located, whereas the indices *ie*, *je*, and *ke*, indicate the end node. Number of cells in the voltage source region is $nx = ie - is$, $ny = je - js$, $nz = ke - ks$, in the x, y, and z directions, respectively. The equivalent voltage for an edge can be obtained by:

$$V_{S,eq} = \frac{1}{nz} V_S. \quad (48)$$

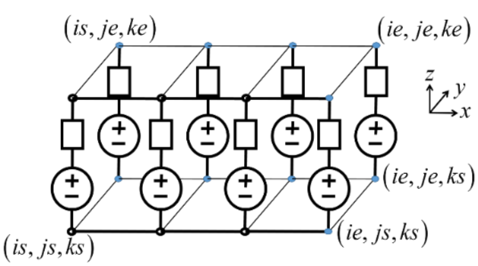

Fig. 2. A voltage source with an internal impedance across multiple cells.

The equivalent impedance is:

$$Z_{eq} = \frac{(nx+1)(ny+1)}{nz} Z_S, \quad (49)$$

which leads to:

$$R_{eq} = \frac{(nx+1)(ny+1)}{nz} R, \quad (50)$$

$$L_{eq} = \frac{(nx+1)(ny+1)}{nz} L, \quad (51)$$

and

$$C_{eq} = \frac{nz}{(nx+1)(ny+1)}. \quad (52)$$

III. VALIDATION OF FORMULATIONS

In this section we present results of some example cases to demonstrate the validity and accuracy of the presented formulations. Figure 3 shows two parallel PEC plates terminated by a voltage source and a load. Each plate is 1 mm by 2 mm in size. The separation between the plates is 1 mm. This geometry is simulated using $\Delta x = 1mm$, $\Delta y = 1mm$, and $\Delta z = 1mm$ cell size. In the simulations the voltage source V_S is excited by a derivative of Gaussian waveform. In the following tests each time a different source impedance configuration is modeled using the values of $R = 50\Omega$, $L = 5nH$, and $C = 10\,pF$. The load is modeled as a 50Ω resistor.

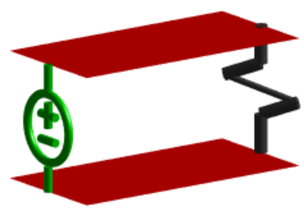

Fig. 3. Two parallel PEC plates terminated by a voltage source and a load.

During impedance calculations after the simulations were performed, it was observed that a capacitance in parallel with the load and source exists at the position of the load and the source. This numerical parasitic reactance in the FDTD grid has been discussed in the literature and correction procedures were suggested [6], [23-25].

The parasitic capacitance along the location of the voltage source in Fig. 3 is determined as $C_P = 0.0175\,pF$ by initial FDTD analyses for the cell size being used. To extract the simulated impedance of the voltage source, the effect of this capacitance is removed from the calculated impedances in the following example test cases.

It should be noted that a value for the numerical parasitic capacitance can be obtained by:

$$C_z = \varepsilon \frac{\Delta x \Delta y}{\Delta z}, \quad (53)$$

where C_z is the capacitance related to the edge of a cell in the z direction as proposed by [24]. In Fig. 3, two parasitic capacitors can be imagined along the two edges of the voltage source. These capacitors are connected in parallel and their equivalent capacitance can be calculated as $C_P = 0.0177\,pF$ using (53), which agrees well with the above determined value out of the simulations.

Figure 4 shows the model of the voltage source together with the parasitic capacitor C_P. In each test case the sampled voltage (V_1) and current (I_1) are captured on the voltage source in Fig. 4 and then the source impedance is calculated as:

$$Z_S = \frac{V_{S1} - V_1}{I_S} = \frac{V_{S1} - V_1}{I_1 + I_C}, \tag{54}$$

where $I_C = j\omega C_P V_1$.

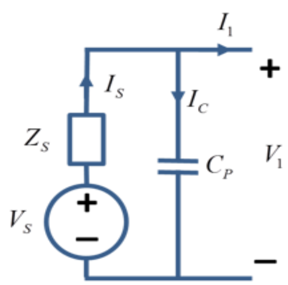

Fig. 4. Equivalent model of a voltage source including the parasitic capacitance.

Figures 5 to 10 show the results of different test cases, where the expected impedances of the voltage sources are compared with the ones obtained from FDTD simulations using (54). The results show very good agreement on a wide frequency range.

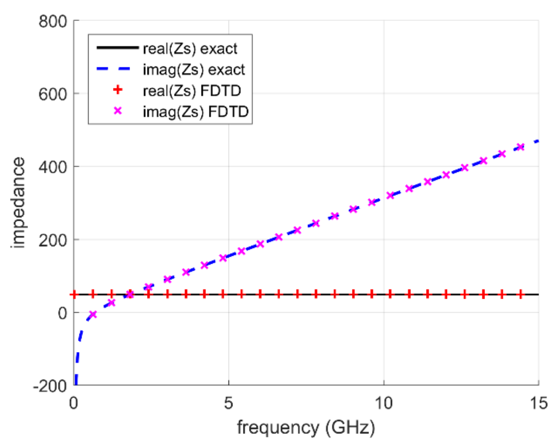

Fig. 5. Impedance of a voltage source as a series RLC.

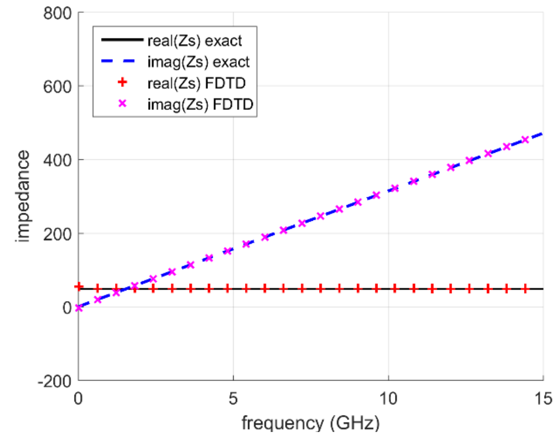

Fig. 6. Impedance of a voltage source as a series RL.

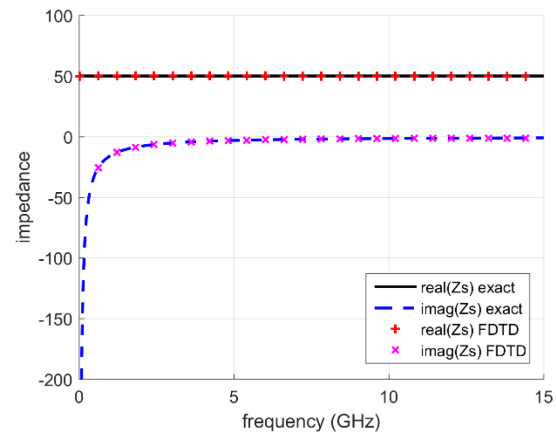

Fig. 7. Impedance of a voltage source as a series RC.

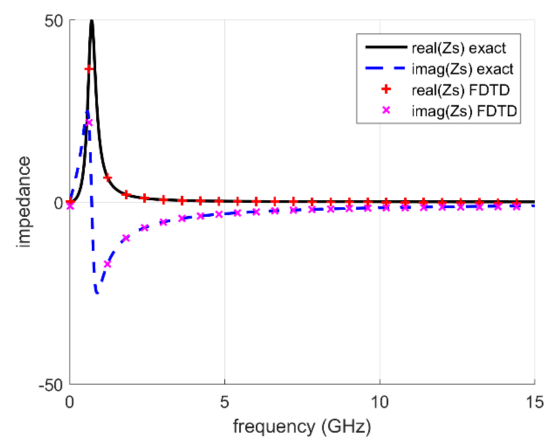

Fig. 8. Impedance of a voltage source as a parallel RLC.

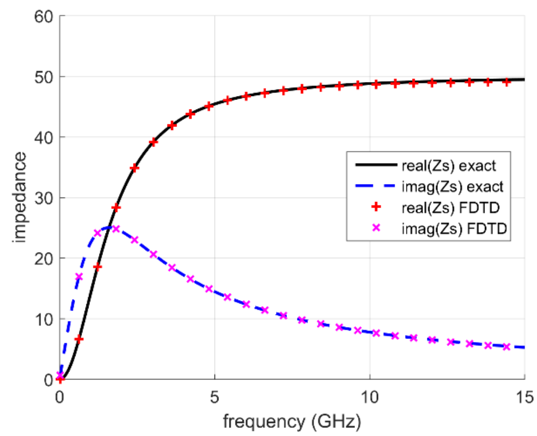

Fig. 9. Impedance of a voltage source as a parallel RL.

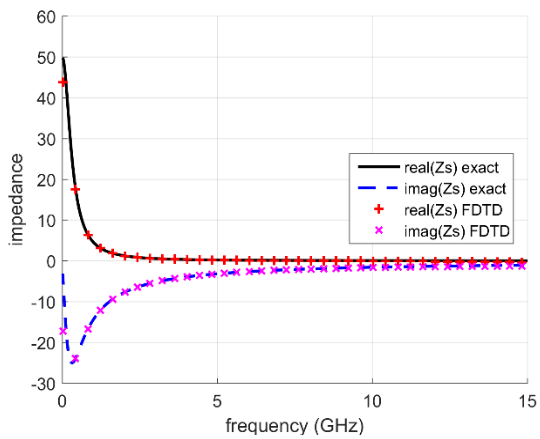

Fig. 10. Impedance of a voltage source as a parallel RC.

IV. CONCLUSION

Formulations are presented to model a voltage source with an RLC internal impedance in the FDTD method. It has been shown that these formulations can be used to model the RLC impedance of a voltage source on a wide frequency band. It should be noted that while these formulations are developed to model voltage sources with RLC impedances, they can be equivalently used to model passive RLC impedances as well by simply removing the source terms from the equations.

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