The Analogy between Offset Configurations of Parabolic Reflectors and Reflectarrays

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Abstract ─ The analogous relationship between the design parameters of offset parabolic reflectors and offset reflectarrays is delineated. Each antenna is specified with four design parameters that best describe the system setup, and the mathematical relationship between the design parameters of these two antenna systems are determined. The presented study allows one to determine the analogous reflectarray system based on the design parameters of the offset reflector and vice versa. This makes it possible to simplify the configuration setup in offset reflectarrays using a coordinate system and a set of parameters that is very suitable for reflectarray design and analysis.

Index Terms ─ Antenna array, focal depth, offset parabolic reflector, reflectarray antenna.

I. INTRODUCTION

Parabolic reflector antennas have long served as the most suitable antenna for high-gain operation in both terrestrial and spaceborne applications [1, 2]. Since the breakthrough of printed antenna technology however, a new generation of high-gain antennas has been introduced, which not only possesses many of the favorable features of parabolic reflectors, but also integrates them with printed array technology. This hybrid antenna, known as reflectarray, offers many promising features such as low-weight and low-profile, and low-cost, and as such has received a great deal of attention over the years [3, 4].

Reflectarray antennas are essentially built based on the concept of reflector antennas, and given the fact that reflector antenna technology is mature and well developed, it certainly is beneficial to take advantage of the developments that have been made in reflector design over generations. To this end, one should be able to describe a reflectarray system that exactly mimics the parabolic reflector system. However, in many cases, the design parameters for the reflectarray don't correlate directly with the reflector design parameters. This is due to the fact that for each antenna, the design parameters are given in manner that best describes its own system. While this is less of an issue for axisymmetric designs [5], the offset configuration is more complex. In particular, many important reflector design parameters such as focal depth are typically misinterpreted, while other parameters such as feed tilt angle, and offset height are essentially nonexistent in offset reflectarrays.

In this paper, we first review the offset parabolic reflector design characteristics, and then define a unique set of parameters that describes the offset reflectarray system. We then present the mathematical formulation that converts the design parameters of the parabolic reflector system to the reflectarray design parameters and vice versa. Comparison between the radiation patterns of an analogous reflector and reflectarray antenna is also presented which shows very similar performance. The work presented here simplifies the configuration setup in offset reflectarrays, by using a coordinate system and a set of parameters that is well suited for design and analysis of offset reflectarrays. Furthermore, it can be advantageous for complex reflectarray configurations, such as offset dual-reflectarray systems [6].

II. THE OFFSET SYSTEM CONFIGURATIONS

In any offset reflector configuration, many parameters exist that can be utilized to achieve the required design goals [2]. However, the most important reflector design parameters that can fully describe a focused offset system can be given in terms of four key parameters. The geometrical model of the offset parabolic reflector system is given in Fig. 1 (a), where the design parameters are:

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- D = reflector diameter;
- F = focal length;
- H = offset height;
- ψ_B = feed tilt angle.

Note that the first three parameters completely describe the geometrical setup of the parabolic reflector, however since in general the primary feed does not point to the reflector center (C), for a full system specification it is necessary to specify the feed tilt angle. In the general case, the feed points to point B on the reflector surface as shown in Fig. 1 (a).

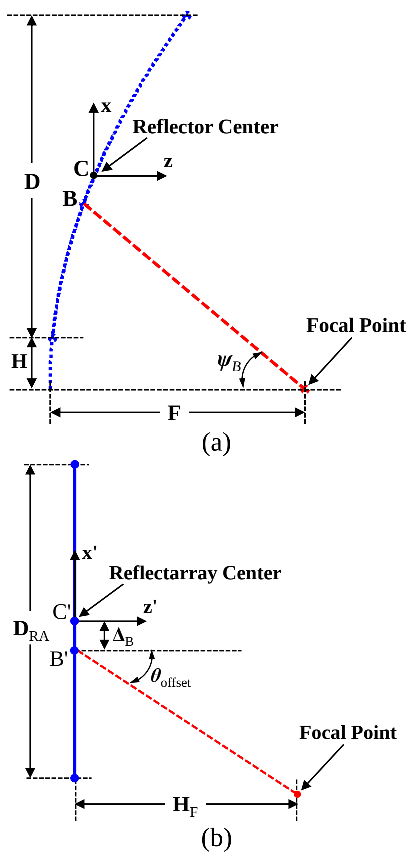

Fig. 1. The geometrical model and key parameters of the offset systems: (a) parabolic reflector, and (b) reflectarray.

Similar to the parabolic reflector, the offset reflectarray system can also be fully specified with four key parameters. The geometrical model of the offset reflectarray system is given in Fig. 1 (b), where the design parameters are:

- D_{RA} = reflectarray diameter;
- H_F = focal point to aperture distance;
- Δ_B = offset distance from reflectarray center;
- θ_{offset} = feed offset angle.

Similar to the reflector system, the primary feed points to a point on the reflectarray aperture (B') which is not necessarily the geometrical center, where the optimal position for B' is determined by efficiency analysis, as described in [7].

The two sets of parameters described here for these offset configurations make it possible to fully describe the systems. In order to establish an analogy between these two systems, one must be able to derive the relationship between the parameters in the two sets. It should be noted here that in the reflector system, the main beam of the antenna points in the z-direction in the coordinate setup of Fig. 1 (a). For the reflectarray system however, one has the option to control the beam direction by adding a progressive phase to the aperture. More discussion on this will be given in the next section.

III. The Analogous Offset Reflector and Reflectarray Systems

In the previous section, the key system design parameters for offset reflectors and reflectarrays were described. In general, in order to analyze the antennas one makes use of a coordinate system that best suits the geometrical setup, which are different for the reflector and reflectarray systems. In this section however, in order to establish the analogy between these two systems, we will use the coordinate setup of the reflector antenna. It is implicit that once the relationship between the design parameters is determined, one can describe the systems in any desired coordinate system.

A. Transformation from reflector to reflectarray system

The geometrical models of the offset configurations in the reflector coordinate setup are shown in Fig. 2. Here we consider the case where the reflector design parameters (F, D, H, ψ_B) are given. First, we derive some additional parameters for the reflector using the four main parameters that would aid in our calculations. The angle subtended to the reflector lower tip, and the distance from focal point to this point are given as:

$$\psi_L = 2\tan^{-1}\left(\frac{H}{2F}\right), \tag{1}$$

$$R_L = \frac{2F}{1+\cos\psi_L}. \tag{2}$$

Similarly, the angle subtended to the reflector upper tip, and the distance from focal point to this point are given by (3) and (4). Note that with this setup, the feed subtended angle, ψ_S, is identical in both systems and is given by (5):

$$\psi_U = 2\tan^{-1}\left(\frac{D+H}{2F}\right), \tag{3}$$

$$R_U = \frac{2F}{1+\cos\psi_U}, \tag{4}$$

$$\psi_S = \psi_U - \psi_L. \tag{5}$$

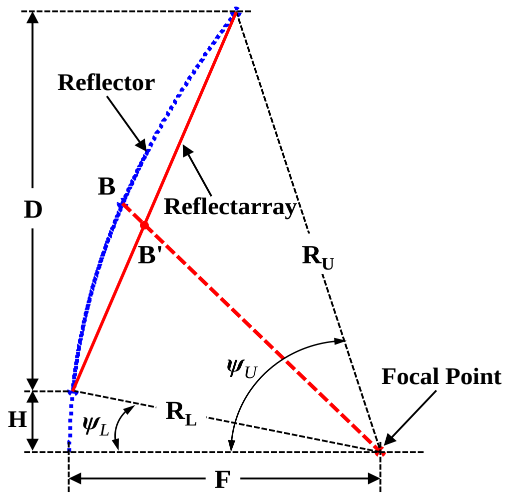

Fig. 2. The analogous geometrical setup of the offset reflector and reflectarray systems.

With these parameters defined, now let us derive the reflectarray system parameters in this coordinate setup. The system setup is given in Fig. 3, where the feed subtended angle is given as:

$$\psi_S = \theta_U + \theta_L \,. \tag{6}$$

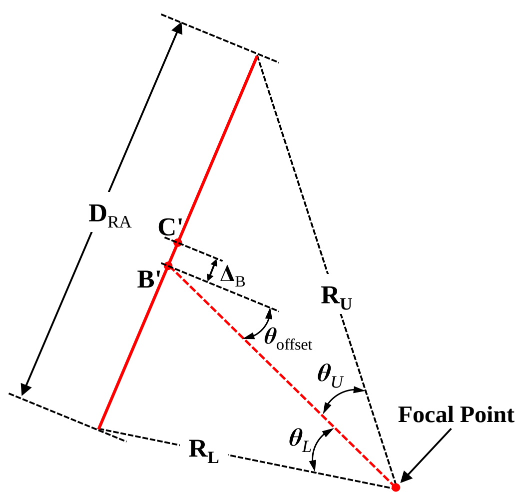

Fig. 3. The geometrical setup of the offset reflectarray in the reflector coordinates.

These angles can be given in terms of reflector parameters as:

$$\theta_L = \psi_B - \psi_L, \;\; \theta_U = \psi_U - \psi_B \,. \tag{7}$$

The diameter of the reflectarray antenna can now be calculated in terms of these parameters as:

$$D_{RA} = \sqrt{R_U^2 + R_L^2 - 2R_U R_L \cos\psi_S} \,. \tag{8}$$

As discussed earlier, with the reflectarray antenna one can direct the main beam to any desired direction. However, in the study presented here we aim at describing analogous systems, thus the main beam for the reflectarray points in the same direction as the reflector, as shown in Fig. 4. Note that in the current setup, the reflectarray aperture is tilted by a certain angle with respect to the reflector coordinates which is given by:

$$\theta_{\text{tilt}} = \cos^{-1}\left(\frac{D}{D_{RA}}\right). \tag{9}$$

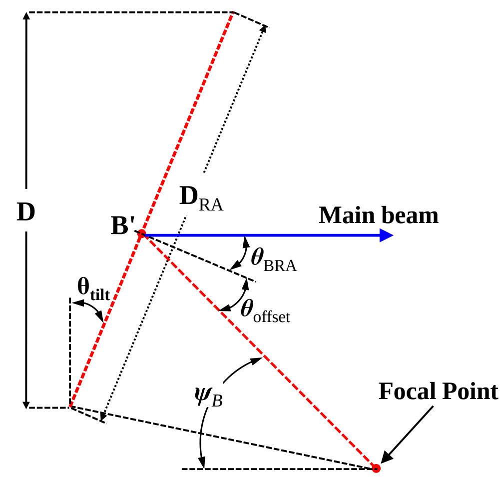

Fig. 4. The main beam direction and geometrical setup of the offset reflectarray system.

To direct the reflectarray main beam in the same direction as the reflector, one must have $\theta_{\text{BRA}} = \theta_{\text{tilt}}$, where θ_{BRA} is the angle with respect to the normal on the reflectarray aperture. The feed offset angle for the reflectarray can then be given as

$$\theta_{\text{offset}} = \psi_B - \theta_{\text{BRA}} \,. \tag{10}$$

The distance from focal point to B' can now be given as:

$$r_f = R_L \frac{\sin(\frac{\pi}{2} - \theta_L + \theta_{\text{offset}})}{\sin(\frac{\pi}{2} - \theta_{\text{offset}})} \,. \tag{11}$$

The focal point to reflectarray aperture distance, and the feed point offset distance from reflectarray center can now be given in terms of these parameters as:

$$H_F = r_f \cos(\theta_{\text{offset}}) \,, \tag{12}$$

$$\Delta_B = \frac{1}{2}\Big(\sqrt{r_f^2 + R_U^2 - 2r_f R_U \cos\theta_U} - \sqrt{r_f^2 + R_L^2 - 2r_f R_L \cos\theta_L}\,\Big) \,. \tag{13}$$

The equations given in this section can fully describe the analogous reflectarray system in terms of the reflector design parameters. In particular, the four parameters of the offset reflectarray are computed from Equations (8), (10), (12), and (13), respectively. In the next section, we will derive the equations to describe the analogous reflector system based on the reflectarray main design parameters.

B. Transformation from reflectarray to reflector system

Now let's consider the case where the reflectarray design parameters (H_F, D_{RA}, Δ_B, θ_{offset}) are given. The geometrical setup of the reflectarray is essentially identical to the previous case shown in Fig. 3. Similarly, we derive the distance from focal point to the reflectarray upper and lower tips using:

$$R_L = \sqrt{r_f^2 + (\frac{D_{RA}}{2} - \Delta_B)^2 - 2r_f(\frac{D_{RA}}{2} - \Delta_B)\cos(\frac{\pi}{2} - \theta_{offset})}, \quad (14)$$

$$R_U = \sqrt{r_f^2 + (\frac{D_{RA}}{2} + \Delta_B)^2 - 2r_f(\frac{D_{RA}}{2} + \Delta_B)\cos(\frac{\pi}{2} + \theta_{offset})}, \quad (15)$$

where r_f is the distance from focal point to B' and can be derived using (12). The feed angles subtended to the upper and lower tips are then given as:

$$\theta_L = \cos^{-1}\left(\frac{(\frac{D_{RA}}{2} - \Delta_B)^2 - R_L^2 - r_f^2}{-2r_f R_L}\right), \quad (16)$$

$$\theta_U = \cos^{-1}\left(\frac{(\frac{D_{RA}}{2} + \Delta_B)^2 - R_U^2 - r_f^2}{-2r_f R_U}\right). \quad (17)$$

Using the geometrical formula for a paraboloid, the distance from focal point to the upper and lower edges can also be given using:

$$R_L = \frac{2F}{1 + \cos\psi_L}, \quad (18)$$

$$R_U = \frac{2F}{1 + \cos(\psi_L + \psi_S)}, \quad (19)$$

where ψ_S is the total feed subtended angle as given in (5). The angle subtended to the reflector upper tip can be derived by solving these two equations simultaneously as:

$$\psi_L = \mathrm{Re}\left\{-i\ln\left(-\frac{R_L - R_U + 2i\sin(\frac{\psi_S}{2})}{R_L - R_U e^{i\psi_S}}\right)\right\}. \quad (20)$$

The reflector design parameters can now be computed using:

$$F = \frac{R_L}{2}\left(1 + \cos(\psi_L)\right), \quad (21)$$

$$H = R_L \sin(\psi_L), \quad (22)$$

$$D = R_U \sin(\psi_L + \psi_S) - H, \quad (23)$$

$$\psi_B = \cos^{-1}\left(\frac{D}{D_{RA}}\right) + \theta_{offset}. \quad (24)$$

These equations make it possible to describe an analogous offset reflector system in terms of the reflectarray design parameters. One important note here is that in the reflectarray to reflector transform, the direction of the main beam of the reflectarray cannot be defined independently. In the setup outlined here, the direction of the reflectarray main beam is in the same direction as the reflector. However as discussed earlier, the reflectarray can be designed to scan the beam in any desired direction, but in any case, all the important reflector characteristics such as focal depth, offset height, and feed tilt angle, will be identical. In other words, with the same configuration setup for both antennas (Fig. 2), the reflectarray allows for more flexibility in the system design.

IV. COMPARISON OF ANALOGOUS PARABOLIC REFLECTOR AND REFLECTARRAY ANTENNA SYSTEMS

The formulations presented in the previous section allow one to accurately determine the important parameters of offset reflectarray systems in a fashion that has been well established for parabolic reflector antennas. Here, let us study the performance of an analogous parabolic reflector and reflectarray system. A Ka-band reflectarray antenna with an aperture diameter of 190 mm, designed for the center frequency of 32 GHz is considered. We select a balanced feed antenna with a q value ($\cos^q\theta_f$ radiation pattern model) of 6.5. The optimal values of the reflectarray design parameters are determined based on efficiency analysis, and are summarized in Table 1. For the analogous reflector antenna, the design parameters are determined using the formulation presented in Section III, and are also summarized in this table.

The F/D ratio for this system is 0.83. As discussed earlier, the main beam direction of the reflectarray (θ_{BRA}) is determined based on the parabolic reflector parameters, and should be equal to 19°. The 2D-model (cross-sectional view) of this parabolic reflector and reflectarray system is similar to that shown in Fig. 2.

Table 1: Design parameters for the offset reflectarray and reflector

Reflectarray	D_{RA}	H_F	Δ_B	Θ_{OFFSET}
	190 mm	144.71 mm	6 mm	20°
Reflector	D	F	H	ψ_B
	179.64 mm	148.93 mm	12.78 mm	39°

For the parabolic reflector antenna, the design is essentially complete; however, for the reflectarray antenna, the array nature of the system requires one to also specify the element spacing. Here we set the element spacing to 4.7 mm (~λ/2 at 32 GHz). This corresponds to 1184 elements on the aperture of the reflectarray. In this study we are comparing the idealized performance of both systems, thus for the reflectarray we use ideal elements, i.e., elements with no reflection loss (|Γ| = 1), and the exact (non-quantized value) required phase shift. Also for the parabolic reflector we use perfect electric conductor. The 3D model of the parabolic reflector antenna in FEKO [8], and the required element phase shift on the aperture of the reflectarray antenna are given in Fig. 5.

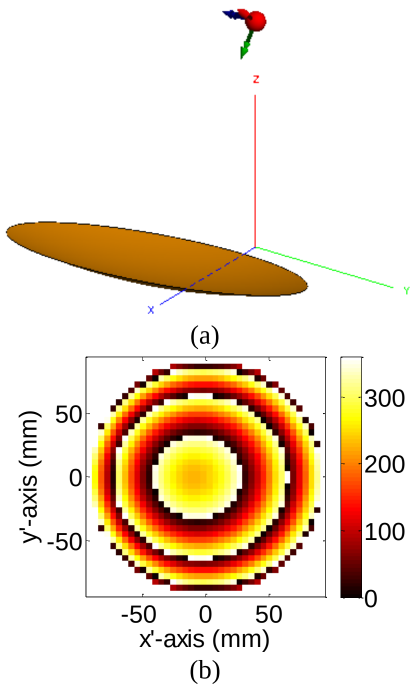

Fig. 5. (a) The parabolic reflector antenna model in FEKO. (b) The element phase distribution on the analogous reflectarray antenna aperture.

The radiation patterns of the antennas are obtained using the equivalence principle and a physical optics (PO) based analysis. For the parabolic reflector, we use the PO solver in FEKO [8]. For the reflectarray antenna we use the aperture field analysis technique as described in [9]. Note that from a computational perspective, both analysis techniques are similar and use the principle of equivalence to compute the surface currents on the antenna apertures and derive the far-field radiation characteristics. The principal plane normalized co-polarized radiation patterns for both antennas is given in Fig. 6, where it can be seen that a very similar radiation pattern is obtained with these two antennas. The simulated gain values are 34.54 dBi and 34.40 dBi for the parabolic reflector and reflectarray, respectively [10]. The slightly lower gain for the reflectarray is attributed to the discontinuous nature of the array (as opposed to the continuous aperture for the parabolic reflector); however this can be improved by using smaller element spacing.

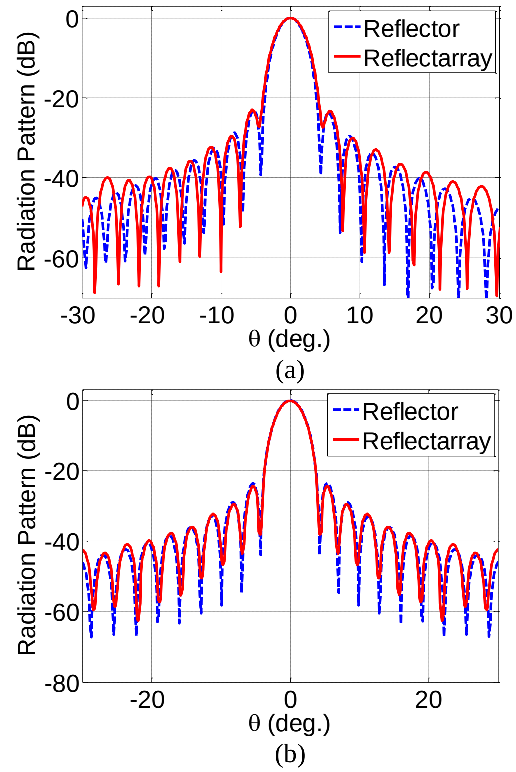

Fig. 6. Principal plane radiation patterns of the reflector and reflectarray: (a) *xz*-plane and (b) *yz*-plane.

VI. CONCLUSION

We derive the analogical relationship between offset parabolic reflectors and reflectarrays. Both the offset reflector and reflectarray systems are defined using four design parameters, and the mathematical relationship between the design parameters of these two antenna systems is determined. A comparison between the radiation patterns of an analogous Ka-band parabolic reflector and reflectarray antenna is also presented which show very similar performance. The presented study simplifies the system setup in offset reflectarrays, by using a coordinate system and a set of parameters that is well suited for reflectarray design and analysis. It can potentially be implemented in offset dual-reflectarray antennas.

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