An Efficient and Analytical Solution to the Integral on Truncated-Wedge ILDCs for Polygonal Surfaces

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Abstract **─** A new method is proposed to analytically evaluate the line integral on truncated-wedge incremental length diffraction coefficients (TW-ILDCs). By utilizing the coherence of a geometry, the trailing edges can be divided into several linear segments, and the line integrals can be reduced as the two end-point contributions for each segment based on the mathematical model derived in this paper. Thus, the efficiency is greatly enhanced in comparison with the traditional numerical techniques. Numerical results for the bistatic radar cross sections show excellent performance of the proposed method both in accuracy and efficiency.

Index Terms **─** Closed form, linear division, TW-ILDCs.

I. INTRODUCTION

It is well known that the surface integral in the physical theory of diffraction (PTD) has the following form:

$$I(k)=\int_{v_1}^{v_2}\int_{u_1}^{u_2} f(u,v)e^{jk\varphi(u,v)}dudv\,, \tag{1}$$

in which k is a large wavenumber, $f(u,v)$ represents the amplitude of the integrand, and $\varphi(u,v)$ is the phase function, both of them have two arguments u and v. The integral on u is seen as the inner integral with the lower and upper limits u_1 and u_2; while the integral on v is thought to be the outer integral with limits v_1 and v_2. As is known, this PTD surface integral was firstly introduced by Ufimtsev, who has also reduced it into point contributions. But his reduction procedure is a directly surface-to-point process, lacking the surface-to-line step, which may contribute to 'very complicated and immense equations' [1]. Thu, Ufimtsev's result is not very practical [3].

Therefore, there have been numerous contributions to better evaluate the inner integral in (1) in the past decades [2]-[10]. The theory of incremental length diffraction coefficients (ILDCs) [2] proposed by Mitzner has made significant improvement for such evaluation. In ILDCs, the two associated faces of a wedge are assumed to be two half-planes based on high frequency localization phenomenon. Hence, the inner integral limit is from zero to infinity. Meanwhile, a function v_B [11] is used to describe the fringe-wave surface current $f(u,v)$, the variable of which only contains u without v in this case, and the phase function $\varphi(u,v)$ can be expressed separately by $\varphi(u)$ and $\varphi(v)$ for the inner and outer integral. Thus, the inner integral in (1) is able to be reduced into closed form, and $I(k)$ can be evaluated analytically for straight wedges. For curved wedges, the stationary phase method [13] can be used to asymptotically reduce the outer integral into point contributions. Hence, ILDCs is an efficient algorithm, and has been successfully applied to design B-2 stealth aircraft [14].

However, some problems exist in ILDCs. Firstly, singularities emerge in some combinations of incidence and observation directions. The reason, as pointed out by Michaeli, is the inappropriate selection of the coordinate system [3]. Unlike ILDCs, who chooses the direction normal to the wedge edge as the inner integral direction, Ref. [3] has selected the grazing diffracted direction, which is a more natural way as pictured by the ray behavior in the geometrical theory of diffraction (GTD) [15]. As a result, most singularities were removed except the non-removable Ufimtsev singularity. Secondly, the half-plane assumption is in contrast to real conditions, which will influence the accuracy especially when the observation direction is close to the grazing diffracted ray [9]. This problem of ILDCs leads to the necessity to consider the second-order diffraction. In this circumstance, the upper limit of the inner integral is a finite value because the fringe wave surface current incremental strips will be truncated when hitting the second-order diffraction points. Work related to solve this kind of inner integral includes Refs. [6], [7] for the half-plane, and [8] for a right-angled wedge. Though the expression for a wedge with arbitrary angle was firstly

Submitted On: November 30, 2016
Accepted On: December 10, 2016

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gained by Michaeli [9], the non-removable singularities still exist due to the improper mathematical derivation procedure. This difficulty was finally overcome by Johansen [10] with no non-removable singularities emerged in his result. Therefore, an analytical, more accurate and robust evaluation of the inner integral in (1) than ILDCs is achieved. Thus, the method in [10], named as truncated-wedge incremental length diffraction coefficients (TW-ILDCs), has been widely used and implemented in Xpatch [16] and GRASP [17].

However, compared with ILDCs, when considering the second-order diffraction, the outer integral in (1) cannot be analytically evaluated even for a straight wedge, because the length of each incremental strip is different and depends on the geometry of an object. To calculate such an integral, the numerical quadrature method has been used [16], [18], [19]. In Ref. [16], to implement TW-ILDCs in Xpatch, sample points on a wedge's leading edge are taken, and the incremental strips of the fringe wave surface currents, emanated from each sample point, will travel until hitting a point on another discontinuous edge. Obviously, the length of the incremental strip has to be calculated again for different sample points and incident angles. In Ref. [18] the distance between every two sample points is set to be $\lambda/10$, which is a frequency-related value and a similar value is taken in [19] as well. Thus, the final result of $I(k)$ is the summation of the diffraction coefficients calculated from all sample points. Consequently, to deal with such highly oscillatory integral, the computation time will increase largely as the frequency increases. To accelerate this numerical technique, a fixed truncated length TW-ILDCs method was proposed based on the idea of rectangular strip [7]. Similar to ILDCs, the amplitude of the integrand in the outer integral in (1) is thereby constant and this line integral can be reduced to closed form resultantly. Though the efficiency was improved by this approach, the accuracy cannot be guaranteed which has been illustrated by the examples in Ref. [18].

In this paper, we propose a new method that can rigorously reduce this line integral into a closed-form expression for planar structures. For the commonly used triangular patch mesh, it is found that the trailing edges can be divided into several linear segments, and on each of them, the outer integral in (1) can be evaluated analytically. The mathematical model required in this process is derived in details. Meanwhile, the efficiency can be greatly improved in comparison with the numerical technique.

This paper is organized as follows. In Section II, a mathematical model referring to the integral on the complementary error function is introduced. Then its application to PTD is described in Section III, including details in integral reduction. Numerical results are given in Section IV to illustrate the validation of the proposed method. Finally, Section V gives the concluding remarks.

II. MATHEMATICAL MODEL

Consider the integral form given below:

$$I(a_1,b_1,a_2,b_2)=\int F\left(\sqrt{a_1z+b_1}\right)e^{jk(a_2z+b_2)}dz\,, \tag{2}$$

in which z is the integral variable, a_1,b_1,a_2,b_2 are the constant terms, and $F(x)$ is the modified Fresnel integral [9]. The aim here is to rigorously reduce the integral (2) into a closed form.

A. The general case

Note that the modified Fresnel integral has a relationship with the complementary error function [20] $F(z)=0.5erfc\left(\sqrt{j}z\right)e^{jz^2}$, then (2) can be rewritten as:

$$\begin{aligned}&I(a_1,b_1,a_2,b_2)\\&=2\int erfc\left(\sqrt{j(a_1z+b_1)}\right)e^{j\left[(a_1+ka_2)z+(b_1+kb_2)\right]}dz.\end{aligned} \tag{3}$$

By using the approach of integrating by parts, and noting that $erfc'(x)=-2e^{-x^2}/\sqrt{\pi}$, (3) can be transformed into:

$$\begin{aligned}&I(a_1,b_1,a_2,b_2)\\&=\frac{2e^{j(b_1+kb_2)}}{j(a_1+ka_2)}\Bigg[erfc\left(\sqrt{j(a_1z+b_1)}\right)e^{j(a_1+ka_2)z}\\&+\sqrt{\frac{j}{\pi}}a_1e^{-jb_1}\int\frac{e^{jka_2z}}{\sqrt{(a_1z+b_1)}}dz\Bigg].\end{aligned} \tag{4}$$

For the integral contained in (4), it has,

$$\begin{aligned}&\int\frac{e^{jka_2z}}{\sqrt{(a_1z+b_1)}}dz\\&=-\frac{\sqrt{\pi}}{a_1}\sqrt{j\frac{a_1}{ka_2}}e^{-\frac{jka_2b_1}{a_1}}erfc\left(\sqrt{-\frac{jka_2}{a_1}(a_1z+b_1)}\right).\end{aligned} \tag{5}$$

Therefore, based on the relationship between modified Fresnel integral and the error function, the final result of (2) can be obtained by taking (5) into (4) as:

$$\begin{aligned}&I(a_1,b_1,a_2,b_2)=\frac{4e^{jk(a_2z+b_2)}}{j(a_1+ka_2)}\\&\cdot\left[F\left(\sqrt{a_1z+b_1}\right)-j\sqrt{\frac{a_1}{ka_2}}F\left(\sqrt{-\frac{ka_2}{a_1}(a_1z+b_1)}\right)\right].\end{aligned} \tag{6}$$

B. The singular case

In (3), when $a_1+ka_2=0$, the integral will be changed as the following form:

$$I(a_1,b_1,a_2,b_2)=2e^{j(b_1+kb_2)}\int erfc\left(\sqrt{j(a_1z+b_1)}\right)dz. \tag{7}$$

Referring to Ref. [21], the result of integral

$\int z \cdot erfc(z)dz$ is listed in its table. Noting that $0.5\int erfc\left(\sqrt{z}\right)dz = \int z \cdot erfc(z)dz$, it is easy to get the value of (7) by variable substitution:

$$I(a_1,b_1,a_2,b_2) = 2e^{j(b_1+kb_2)} \cdot \left[\frac{1}{a_1}\left((a_1 z+b_1) + \frac{j}{2} \right) \cdot erfc\left(\sqrt{j(a_1 z+b_1)}\right) - \sqrt{\frac{j(a_1 z+b_1)}{\pi}} e^{-j(a_1 z+b_1)} \right]. \tag{8}$$

III. APPLICATION TO PTD

A. The line integral on TW-ILDCs

The geometry of a perfect conducting wedge with two finite sized polygonal planes is shown in Fig. 1. The points B and C are two points on the leading edge, from which the fringe wave incremental strips emanate, along the grazing diffracted direction $\hat{\sigma}$, hit an edge at point B' and C'. The dashed lines BB' and CC' are their propagation paths, whose lengths are l_B and l_C, respectively. And the equivalent edge currents are distributed on the polygon face AA'B'C'D. Therefore, edges A'B', B'C' and C'D are the so-called trailing edges.

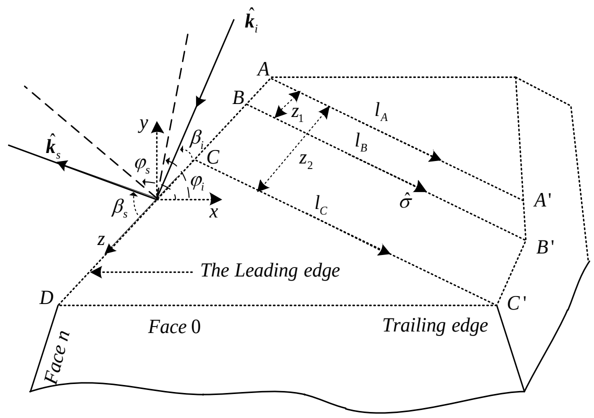

Fig. 1. The geometry of a perfectly conducting wedge. The dashed lines on the wedge represent the propagation paths of the fringe wave incremental strips. Segments AB and BC are the projections of line segments A'B' and B'C' on the leading edge AD. φ_i and φ_s are the angles from the x axis to the projections of $\hat{\boldsymbol{k}}_i$ and $\hat{\boldsymbol{k}}_s$ on the xy plane, β_i and β_s are the angles from the leading edge to $\hat{\boldsymbol{k}}_i$ and $\hat{\boldsymbol{k}}_s$, respectively.

The fringe-wave field is expressed by the radiation integral [12] as:

$$\boldsymbol{E}^{fw} = jk\int \left[ZI_T \hat{\boldsymbol{k}}_s \times \left(\hat{\boldsymbol{k}}_s \times \hat{\boldsymbol{t}}\right) + M_T \hat{\boldsymbol{k}}_s \times \hat{\boldsymbol{t}} \right] \frac{e^{-jkR}}{4\pi R} dz, \tag{9}$$

where Z_0 is the impedance, $\hat{\boldsymbol{t}} = \hat{\boldsymbol{z}}$, the direction of the leading edge, and M_T, I_T are the truncated equivalent magnetic and electric currents [10], having the expressions that $M_T = M_{ut} - M_{cor}$, $I_T = I_{ut} - I_{cor}$, in which M_{ut} and I_{ut} represent the untruncated equivalent magnetic and electric currents and their expressions can be obtained in Ref. [3], while M_{cor} and I_{cor} are the correction terms when considering partly the secondary diffraction, respectively.

Since M_{ut} and I_{ut} are constants for a wedge edge, the integrals on them can be expressed analytically. Hence, the main work here is to evaluate the integral on the correction terms M_{cor} and I_{cor}:

$$\boldsymbol{E}_{cor}^{fw} = jk\frac{e^{-jkR}}{4\pi R} \cdot \left[Z_0 \hat{\boldsymbol{k}}_s \times \left(\hat{\boldsymbol{k}}_s \times \hat{\boldsymbol{t}}\right) \int_C I_{cor} \exp\left(jk(-\hat{\boldsymbol{k}}_i + \hat{\boldsymbol{k}}_s) \cdot \boldsymbol{z} \right) dz + \hat{\boldsymbol{k}}_s \times \hat{\boldsymbol{t}} \int_C M_{cor} \exp\left(jk(-\hat{\boldsymbol{k}}_i + \hat{\boldsymbol{k}}_s) \cdot \boldsymbol{z} \right) dz \right]. \tag{10}$$

The expressions of M_{cor} and I_{cor} are obtained from Ref. [10] and listed as:

$$M_{cor} = \frac{2Z_0 \sin\varphi_s \hat{\boldsymbol{z}} \cdot \boldsymbol{H}^i \exp[jL(\mu-1)]}{jk \sin\beta_s \sin\beta_i} \cdot \left[\frac{-sign[cos(\varphi_i/2)]}{\mu + \cos\varphi_i} F\left(\sqrt{2L}\left|\cos(\varphi_i/2)\right|\right) + \left(\frac{\sqrt{1-\mu}}{\sqrt{2}(\mu+\cos\varphi_i)\cos(\varphi_i/2)} - \frac{\sqrt{2}\sin(\pi/n)}{n\sqrt{1-\mu}(\cos(\pi/n)-\cos(\varphi_i/n))} \right) \cdot F\left(\sqrt{L(1-\mu)}\right) \right], \tag{11}$$

$$I_{cor} = \frac{2sign[cos(\varphi_i/2)]\exp[jL(\mu-1)]}{jk\sin\beta_i(\mu+\cos\varphi_i)} \cdot \left[\frac{\sin\varphi_i \hat{\boldsymbol{z}} \cdot \boldsymbol{E}^i}{Z_0 \sin\beta_i} - (\cot\beta_i \cos\varphi_i + \cot\beta_s \cos\varphi_s) \hat{\boldsymbol{z}} \cdot \boldsymbol{H}^i \right] \cdot F\left(\sqrt{2L}\left|\cos(\varphi_i/2)\right|\right) + \sqrt{2(1-\mu)} \cdot \left[-\frac{\sin(\varphi_i/2)\hat{z}\cdot\boldsymbol{E}^i}{Z_0\sin\beta_i} + \frac{\hat{z}\cdot\boldsymbol{H}^i}{2\cos(\varphi_i/2)} \cdot (\cot\beta_i\cos\varphi_i + \cot\beta_s\cos\varphi_s) + \frac{\sin(\pi/n)(\mu+\cos\varphi_i)(\cot\beta_i - \cot\beta_s\cos\varphi_s)\hat{z}\cdot\boldsymbol{H}^i}{n(\cos(\pi/n)-\cos(\varphi_i/n))(1-\mu)} \right] \cdot F\left(\sqrt{L(1-\mu)}\right). \tag{12}$$

The argument L in the modified Fresnel integral has the expression:

$$L = kl\sin^2\beta_i, \tag{13}$$

in which l is the truncated length for each incremental strip, and the angle-related terms for both excitation and observation, such as β_i, φ_i, β_s, φ_s and μ, are the same as these defined in Ref. [10].

Taking (11), (12) into (10), it can be found the key point when dealing with the integrals in (10) is to calculate the two integrals below:

$$\begin{aligned} T_1 = &\int_C F\left(\sqrt{2L}\left|\cos(\varphi_i/2)\right|\right) \\ &\cdot\exp\left[jL(\mu-1)+jk(-\hat{\boldsymbol{k}}_i+\hat{\boldsymbol{k}}_s)\cdot\boldsymbol{z}\right]dz, \end{aligned} \tag{14}$$

$$\begin{aligned} T_2 = &\int_C F\left(\sqrt{L(1-\mu)}\right) \\ &\cdot\exp\left[jL(\mu-1)+jk(-\hat{\boldsymbol{k}}_i+\hat{\boldsymbol{k}}_s)\cdot\boldsymbol{z}\right]dz. \end{aligned} \tag{15}$$

B. Linear division and representation

When the mesh of an object is based on triangular patches, each edge of the meshed model is a linear segment. Therefore, within each linear segment, the truncated length of any point on the corresponding leading edge segment can be expressed as a linear function of the point's position on the leading edge. As illustrated by Fig. 1, take edge BC as an example. For face 0, the distance between points A and B is z_1, between points A and C is z_2, the distance between point A and any point on edge AD is assumed to be z, and the position vector is z. For face n, it has $\hat{z}\to-\hat{z}$; hence, the starting point of the wedge edge should also be changed. The truncated length of each point on edge BC is:

$$l = \frac{l_C - l_B}{z_2 - z_1}(z - z_1) + l_B. \tag{16}$$

Thus, the slope for this trailing edge is given as $k_l = (l_C - l_B)/(z_2 - z_1)$, then l is further written as $l = k_l z + c_l$, where c_l can be derived from (16).

As a result, the variable L in (18) and (19) can be represented as:

$$L = k_L z + c_L, \tag{17}$$

in which $k_L = kk_l\sin^2\beta_i$, $c_L = kc_l\sin^2\beta_i$.

C. Integral reduction

Taking (17) into (14) and (15), T_1 and T_2 are rewritten as:

$$\begin{aligned} T_1 = &\int F\left(\sqrt{k_L(1+\cos\varphi_i)z + c_L(1+\cos\varphi_i)}\right) \\ &\cdot e^{jk\left\{\left[\frac{k_L(\mu-1)}{k}+\cos\beta_s-\cos\beta_i\right]z+\frac{c_L(\mu-1)}{k}\right\}}dz, \end{aligned} \tag{18}$$

$$\begin{aligned} T_2 = &\int F\left(\sqrt{k_L(1-\mu)z + c_L(1-\mu)}\right) \\ &\cdot e^{jk\left\{\left[\frac{k_L(\mu-1)}{k}+\cos\beta_s-\cos\beta_i\right]z+\frac{c_L(\mu-1)}{k}\right\}}dz. \end{aligned} \tag{19}$$

Obviously, the integrals (18) and (19) have the same form as (2), so the results of them have the same form as (6) and the values of a_1, b_1, a_2, and b_2 can be determined for T_1 and T_2 in this condition. For T_1,

$$\begin{aligned} &a_1 = k_L(1+\cos\varphi_i), b_1 = c_L(1+\cos\varphi_i) \\ &a_2 = \frac{k_L}{k}(\mu-1) - \cos\beta_i + \cos\beta_s, b_2 = \frac{(\mu-1)}{k}c_L; \end{aligned} \tag{20}$$

and for T_2,

$$\begin{aligned} &a_1 = k_L(1-\mu), b_1 = c_L(1-\mu) \\ &a_2 = \frac{k_L}{k}(\mu-1) - \cos\beta_i + \cos\beta_s, b_2 = \frac{(\mu-1)}{k}c_L. \end{aligned} \tag{21}$$

Consequently, for each face of a wedge, the line integral on TW-ILDCs is successfully reduced into a closed-form expression in terms of the modified Fresnel integral.

IV. NUMERICAL RESULTS

In this section, the good performance of the proposed method will be demonstrated via calculating the bistatic radar cross sections on a trapezoid body. The results of numerical TW-ILDCs method are also exhibited as comparisons.

The geometry of the object is illustrated in Fig. 2. The incident angle is given by $\theta_i = 60^\circ$ and $\varphi_i = 30^\circ$, and the observation angles are determined by $\theta_s = 90^\circ$ and $\varphi_s = 0 \sim 360^\circ$ using an angular resolution of 0.25°. The working frequency is set to be 3 GHz. Numerical TW-ILDCs are performed using the method presented in [18], in which the $\lambda/10$ mesh on wedge edge is taken to give enough accuracy.

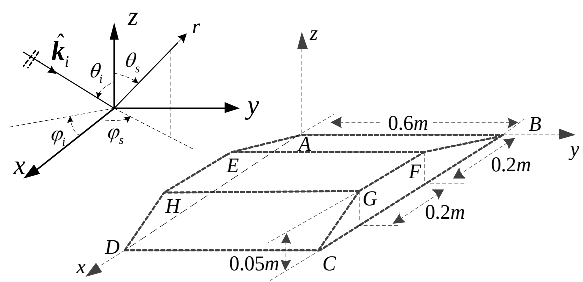

Fig. 2. Perfectly conducting object. The bottom face is ABCD, a 0.6m × 0.6m square. The upper face is EFGH, the front and back side faces are CDHG and ABFE. θ_i, θ_s are from the z axis to the incident and scattering directions, φ_i and φ_s are from the x axis to the projections of the incident and scattering directions on xy plane, respectively.

Considering the linear division process for all discontinuous edges, the trailing edges of edges AE, BF, EH and FG can only be divided into one linear segment for their two faces, thus these four wedge edges do not need to be divided. For the upper face of edge AB, the trailing edges are divided into four linear segments: EA, HE, DH, part of CD. Thus, the outer integral in (1) can be written as the summation of four sub-integrals on these four linear segments. For the bottom face of AB, the trailing edges are: AD and part of CD. Hence AB should be divided into two parts, and the outer integral in (1) is the summation of two sub-integrals on these two linear segments. The division of BC, CD, CG, EF, GH and DH are similar to AB.

As can be seen from Figs. 3-4, the results of the proposed method are almost the same as the numerical TW-ILDCs except some small discrepancies which are possibly due to the slight numerical errors in the calculation process. Moreover, the results of multi-level fast multipole method (MLFMM) minus PO are also given as a reference, in which the results of MLFMM are obtained by software FEKO and the reason for the difference between the proposed method and MLFMM minus PO is that the higher-order and vertex diffraction contributions are not considered by both numerical TW-ILDCs and the proposed method.

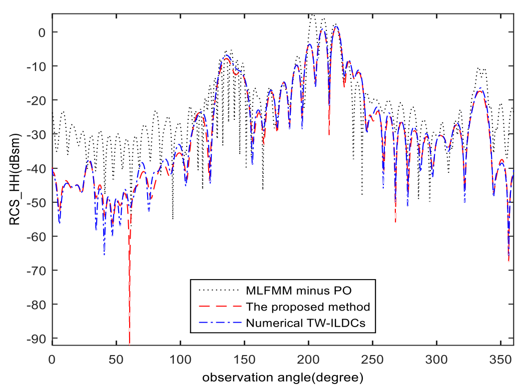

Fig. 3. HH polarization bistatic radar cross section results.

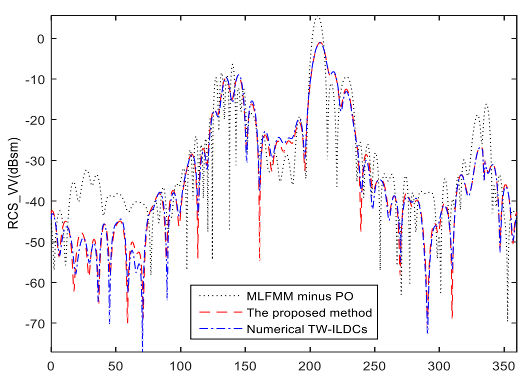

Fig. 4. VV polarization bistatic radar cross section results.

Meanwhile, to illustrate the efficiency of the proposed method and numerical TW-ILDCs, the computation time is listed in Table 1.

Table 1: Computation time consumed by diffraction

Frequency (GHz)	Computation Time (s)	
	The Proposed Method	Numerical TW-ILDCs
3.0	0.827	3.276
10.0	0.905	10.701
30.0	0.905	31.121

It can be seen that the time needed by numerical TW-ILDCs is increased as the frequency increases; while the computation time of the proposed method is relatively stable and takes very small part of that of numerical TW-ILDCs.

V. CONCLUSION

In this paper, a new method is proposed to analytically treat the line integral on TW-ILDCs. Based on the triangular patch mesh, the trailing edges corresponding to a leading edge can be divided into several linear segments, and the line integral can then be reduced to an analytical form in terms of the modified Fresnel integral among the linear segments. Thus, the efficiency has been improved significantly compared with the traditional numerical quadrature method. The future work will focus on the higher-order diffraction contributions to develop a more accurate algorithm.

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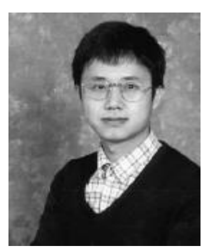

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