A Low Complexity High Performance Robust Adaptive Beamforming

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Abstract **─** In this paper, we propose a low complexity adaptive beamforming with joint robustness against array steering vector (ASV) mismatch and array manifold errors. The proposed robust beamforming is based on the diagonal loading technique, and the diagonal loading factor can be adjusted adaptively according to the input signal-to-noise ratios (SNRs). The eigenvalue corresponding to the desired signal is identified by projecting the presumed ASV of the desired signal onto the eigenvectors of the sample covariance matrix. We find that the ratio of the eigenvalues corresponding to the desired signal and the noise can be used to accurately reflect the input SNR when the input SNRs is large enough, and the ratio is also the diagonal loading factor. Then, an orthogonal subspace is obtained with the steering vectors associated with the region where the desired signal may locate at. Also, the actual steering vector is estimated as a linear combination of this orthogonal subspace. In order to further reduce the computation complexity, we use the traditional diagonal loading method in low SNRs. Simulations results demonstrate that the proposed beamformer provides strong robustness against a variety of array mismatches, and the complexity is significantly reduced compared with popular existing methods.

Index Terms **─** Adaptive arrays, diagonal loading, low complexity, robust beamforming.

I. INTRODUCTION

Adaptive beamforming has been widely used in radar, sonar, mobile communications, radio astronomy and other fields due to its ability on suppressing the interferences and noise [1-4]. The standard Capon beamformer (SCB), which is one of the representative adaptive beamformers, has excellent resolution and jammer rejection performance in ideal case. Unfortunately, the SCB lacks robustness in the presence of model mismatches, especially, when the desired signal is presented in the training data. Therefore, the behavior of the SCB significantly degraded when the mismatch existed in the array steering vectors (ASV) or the array manifold [5-10].

Many robust adaptive beamforming methods have been developed over the past several decades [11-19]. Among these methods, the diagonal loading method is the most common one due to its lowest complexity, where a fixed value is added to the diagonal of sample covariance matrix [11]. However, it doesn't provide any guidance to select the optimal diagonal loading factor, and thus it cannot provide sufficient robustness. Robust Capon beamformer (RCB) was proposed in [12], which exploited a spherical or ellipsoidal uncertainly set to limit the ASV of the desired signal. This method has been proved belongs to a kind of diagonal loading approach except that the loading factor can now be determined precisely based on the uncertainty set. However, the performance of the RCB is mainly determined by the uncertain parameter set, and uncertain parameter set is difficult to be known accurately in practice. The RCB is equivalent with the worst-case beamformer proposed in [13].

Robust adaptive beamforming based on steering vector estimation has been proposed in [14]. To achieve an accurate steering vector, one needs to maximize the beamformer output power, and the ASV prevented from converging to any interference ASVs or their linear combinations, which is a quadratically constrained quadratic programming (QCQP) problem and can be converted to semi-definite programming (SDP). Certainly, the large amount of calculation is needed and long time to converge. In [15], robust adaptive beamforming based on interference-plus-noise (IPN) covariance matrix reconstruction and ASV estimation has been proposed,

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the IPN covariance matrix was reconstructed by utilizing the Capon spectrum to integrate over a region separated from the SOI direction. This method enjoys good performance in the case of ASV direction errors. Unfortunately, the array manifold information must be known accurately in this method. As a result, this method lacks robustness in the presence of array calibration errors, especially in low signal-to-noise ratios (SNRs) [16-17]. Then, a variable loading method is proposed in [19], which can improve the robustness by deliberately preventing the weight vector converging to the noise components, where the loading factor is set in an ad hoc manner.

In this paper, we propose a novel variable diagonal loading beamforming to achieve high performance and low complexity. The diagonal loading factor is associated with the input SNRs and can be adaptively adjusted. In our proposed beamformer, the eigenvalue corresponding to the desired signal is identified by projecting the presumed ASV of the desired signal onto the eigenvectors of the sample covariance matrix. The input SNR is estimated by using the eigenvalues corresponding to the desired signal and noise, respectively, which is also the diagonal loading factor in high SNR. The traditional diagonal loading method is used in low SNR for further reducing the computation complexity. A correlation matrix of the ASV associated with the region where the desired signal may locate is constructed and an orthogonal subspace is obtained by Eigen-decomposition of this matrix. The actual ASV of the desired signal is subsequently expressed as a linear combination of the orthogonal subspace. The proposed method with low complexity can obtain a closed-form expression of the weight vector. Simulation results show the proposed beamforming can provide strong joint robustness against ASV mismatch and array manifold errors.

II. THE SIGNAL MODEL

We consider a uniform linear array (ULA) with N unidirectional antennas with spacing d. Assuming that there are $M+1$ signals arriving from the directions θ_p, $p=0,1,\cdots,M$. The received data of the array can be expressed as:

$$\mathbf{X}(k)=\mathbf{AS}(k)+\mathbf{N}(k), \tag{1}$$

where $\mathbf{X}(k)=[x_1(k)\ x_2(k)\ \cdots\ x_N(k)]^T$ is a $N\times 1$ array observations data vector. $(\bullet)^T$ denotes the transpose. k is the time index. $\mathbf{S}(k)=[s_0(k)\ s_1(k)\ \cdots\ s_M(k)]^T$, $s_p(k)$ denotes the complex waveform of the pth signal. Here, $s_0(k)$ is considered as the desired signal, while $s_i(k)$, $i=1,2,\cdots,M$ are the interferences. $\mathbf{N}(k)=[n_1(k)\ n_2(k)\ \cdots\ n_N(k)]^T$ is a vector of the additive white sensor noise, $\mathbf{A}=[\mathbf{a}(\theta_0)\ \mathbf{a}(\theta_1)\ \cdots\ \mathbf{a}(\theta_M)]$, where $\mathbf{a}(\theta_p)=[1\ \mathrm{e}^{j\beta_p}\ \cdots\ \mathrm{e}^{j(N-)1\beta_p}]^T$ represents a steering vector in the θ_p direction, and β_p is the wave number that can be represented as $\beta_p=2\pi d\sin(\theta_p)/\lambda$.

We assume that the signal and noise are statistically independent. The output of the beamformer $y(k)$ is given by:

$$y(k)=\mathbf{w}^{\mathrm{H}}\mathbf{X}(k), \tag{2}$$

where $\mathbf{w}$ is the $N\times 1$ complex weight vector and $(\bullet)^H$ represents the Hermitian transpose.

The minimum variance distortionless response (MVDR) beamformer is formulated as the following linearly constrained quadratic optimization problem:

$$\min_{\mathbf{w}}\ \mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\bar{\mathbf{a}}_0=1, \tag{3}$$

where $\bar{\mathbf{a}}_0$ is the presumed ASV of the desired signal, $\mathbf{R}_{i+n}$ is the IPN covariance matrix. In practice, $\mathbf{R}_{i+n}$ is commonly replaced by the sample covariance matrix:

$$\hat{\mathbf{R}}=\frac{1}{K}\sum_{k=1}^{K}\mathbf{X}(k)\mathbf{X}^H(k), \tag{4}$$

where K is the number of snapshots. Thus, the optimal solution to (3) is:

$$\mathbf{w}=\frac{\hat{\mathbf{R}}^{-1}\bar{\mathbf{a}}_0}{\bar{\mathbf{a}}_0^H\hat{\mathbf{R}}^{-1}\bar{\mathbf{a}}_0}. \tag{5}$$

The solution (5) is commonly referred to as the sample matrix inverse (SMI). The standard MVDR beamformer has a good interference rejection performance by producing sharp nulls at the direction of interferences in the ideal case. However, the standard MVDR beamformer lacks robustness against the ASV or manifold errors, which case seriously performance degradation.

III. THE PROPOSED ALGORITHM

A. Input SNR estimation and diagonal loading

In this section, we propose a novel low complexity variable diagonal loading beamformer. In this method, the eigenvalue corresponding to desired signal is identified firstly. The sample covariance matrix $\hat{\mathbf{R}}$ defined by (4) can be decomposed as:

$$\hat{\mathbf{R}}=\sum_{i=1}^{N}\lambda_i\boldsymbol{\upsilon}_i\boldsymbol{\upsilon}_i^H=\mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^H+\mathbf{U}_n\boldsymbol{\Lambda}_n\mathbf{U}_n^H, \tag{6}$$

where λ_i, $i=1,2,\cdots,N$ are the eigenvalues are of $\hat{\mathbf{R}}$, and $\boldsymbol{\upsilon}_i$, $i=1,2,\cdots,N$ are the corresponding eigenvectors. $\mathbf{U}_s=[\boldsymbol{\upsilon}_1\ \boldsymbol{\upsilon}_2\ \ldots\ \boldsymbol{\upsilon}_{M+1}]$ represents the signal-plus-interference (SPI) subspace, which is composed by the M interferences and a desired signal. $\mathbf{U}_n=[\boldsymbol{\upsilon}_{M+2}\ \boldsymbol{\upsilon}_{M+3}\ \ldots\ \boldsymbol{\upsilon}_N]$ represents the noise subspace, $\boldsymbol{\Lambda}_s=diag\{\lambda_1,\lambda_2,\cdots,\lambda_{M+1}\}$ are the eigenvalues of the SPI, and $\mathbf{\Lambda}_n = diag\{\lambda_{M+2}, \lambda_{M+3}, \cdots, \lambda_N\}$ are the eigenvalues of noise. As we know, the eigenvectors in $\mathbf{U}_s$ and the ASVs of the SPI lie in the same subspace. What's more, the mismatch between $\mathbf{a}(\theta_0)$ and $\bar{\mathbf{a}}_0$ is not too large in fact. We project the presumed desired signal ASV $\bar{\mathbf{a}}_0$ onto the eigenvectors to get $p(i)$, i=1,2,⋯,N. $p(i)$ can be expressed as:

$$p(i) = \left|\mathbf{v}_i^H \bar{\mathbf{a}}_0\right|^2, \quad i=1,2,\cdots,N. \tag{7}$$

The projections $p(i)$ can be arranged in descending order, as $p_{[N]} \geq p_{[N-1]} \geq \cdots \geq p_{[1]}$. Meanwhile, the corresponding eigenvectors can be arranged as $\mathbf{v}_{[N]}$, $\mathbf{v}_{[N-1]}$, ⋯ ,$\mathbf{v}_{[1]}$, and the corresponding eigenvalues can be arranged as $\lambda_{[N]}, \lambda_{[N-1]}, \cdots, \lambda_{[1]}$. Note that the eigenvalues $\lambda_{[N]}, \lambda_{[N-1]}, \cdots, \lambda_{[1]}$ are the same as (6), but they have been reordered according to $p_{[N]} \geq p_{[N-1]} \geq \cdots \geq p_{[1]}$.

As well known, when $\mathbf{v}_i$ is the eigenvector corresponds to the desired signal, the maximum of the projections $p_{[N]}$ can be obtained [8]. That is to say the eigenvector corresponding to the desired signal can be identified by $p_{[N]}$. Easy to see that $\mathbf{v}_{[N]}$ and $\lambda_{[N]}$ are the corresponding eigenvector and eigenvalue, respectively. There is no doubt that $\lambda_{[1]}$ is the eigenvalue corresponding to the noise. Subsequently, we can use the parameter γ to reflect the input SNR directly [20], which can be expressed as:

$$\gamma = 10 * \log(\frac{\lambda_{[N]}}{\lambda_{[1]}}). \tag{8}$$

When the input SNR is very small, $\gamma < 0$. On the contrary, the large value of γ can be obtained in high SNR environment. Here, we give an example to discuss the relationship between γ and the input SNR.

We consider a ULA of $N = 10$ antennas spaced at a half wavelength distance. Additive noise is modelled as independent complex Gaussian noise with zero mean and unit variance. Two independent interferences are from the directions of 30° and -50°, respectively. The interference-to-noise ratios (INRs) of the interferences are 30 dB. The desired signal is assumed from the direction of 3°. The random DOA estimation mismatch is distributed in $[-5^\circ, 5^\circ]$. The number of snapshots is K=100. 200 repetitions are executed to obtain each simulated point.

Figure 1 shows the values of γ versus input SNRs. We can observe form Fig. 1 that the values of γ are quite consistent with the optimal SINR when the input SNR> -10 dB. The relationship between γ and SNR is almost linear. We can say the parameter γ can reflect the input SNR exactly as long as SNR is large enough. When SNR ≤ -10 dB, γ failed to reflect the input SNR due to the overestimated of signal subspace. Luckily, the traditional diagonal loading method can achieve the same performance as other methods in low SNRs. Naturally, we prefer to use the traditional diagonal loading in low SNR due to its low complexity.

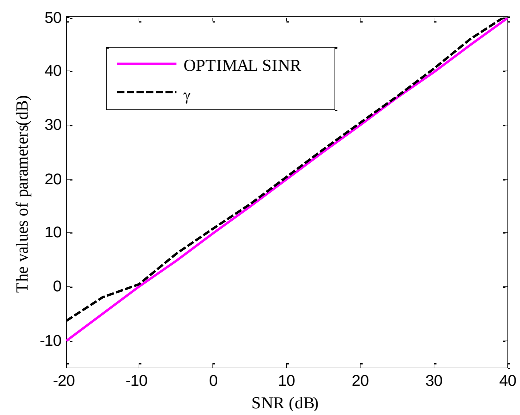

Fig. 1. Values of γ versus the SNRs.

Thus, the proposed diagonal method can be expressed as:

$$\tilde{\mathbf{R}} = \begin{cases} \hat{\mathbf{R}} + \frac{\lambda_{[N]}}{\lambda_{[1]}}\mathbf{I} & \gamma > 0 \\ \hat{\mathbf{R}} + \eta\mathbf{I} & \gamma \leq 0 \end{cases}, \tag{9}$$

where $\mathbf{I}$ is an identity matrix, and η is a fixed value. We usually set η as twice as the noise power. We can see that a new covariance matrix can be obtained by using the proposed diagonal method. The diagonal loading factor in (9) is changed according to the SNR. When $\gamma > 0$, the diagonal loading factor is positive to the SNR. The large diagonal loading factor can increase the noise power and the proportion of the desired signal in the covariance matrix is reduced. As a result, the effect of the desired signal is reduced in high SNRs. When $\gamma \leq 0$, the diagonal loading factor is set to be a fixed value.

B. Desired signal steering vector estimation

In practical applications, the presumed ASV of the desired signal may not precisely, and the mismatches cause significant performance degradation. Using the similar idea of [18], we can obtain accurate ASV of the desired signal. We assume that the Θ is the angular sector in which the desired signal is located. Define the correlation matrix of the steering vector:

$$\mathbf{C} = \int_\Theta \mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta, \tag{10}$$

where $\mathbf{a}(\theta)$ is the ASV associated with a presumed direction θ located in Θ. The matrix can be decomposed as:

$$\mathbf{C} = \sum_{n=1}^{N} \mu_n \bar{\mathbf{u}}_n \bar{\mathbf{u}}_n^H, \tag{11}$$

where μ_n are the eigenvalues of $\mathbf{C}$ in descending, and $\bar{\mathbf{u}}_n$ are the corresponding eigenvectors. The Q largest eigenvalues can be extracted, and the corresponding eigenvectors can be expressed as $\bar{\mathbf{u}}_n$, $n=1,2,\cdots,Q$. Then a column orthogonal matrix $\mathbf{U}$ can be constructed as $\mathbf{U}=[\bar{\mathbf{u}}_1,\bar{\mathbf{u}}_2,\cdots,\bar{\mathbf{u}}_Q]$. As we all know, the Q eigenvalues can contain most of the energy of $\mathbf{C}$ as long as Q is large enough. That is to say any ASV whose direction located in Θ can be expressed as a linear combination of columns of $\mathbf{U}$. So the actual ASV of the desired signal can be estimated as:

$$\tilde{\mathbf{a}}_0 = \mathbf{U}\mathbf{r}, \tag{12}$$

where $\mathbf{r}$ is defined as a rotating vector. In order to obtain the vector $\mathbf{r}$, we can maximize the output power of the desired signal. Taking into the norm constraint, the optimization problem can expressed as:

$$\begin{aligned} &\min_{\mathbf{r}} \quad \mathbf{r}^H \hat{\mathbf{R}}_U \mathbf{r} \\ &\text{subject to} \quad \mathbf{r}^H\mathbf{r} = N, \end{aligned} \tag{13}$$

where $\hat{\mathbf{R}}_U = \mathbf{U}^H \hat{\mathbf{R}}^{-1}\mathbf{U}$. The constraint $\mathbf{r}^H\mathbf{r} = N$ is aims to avoid scaling ambiguity. The problem (13) can be solved by Lagrange multiplier methodology, and the cost function is given by:

$$L(\mathbf{r},\mathbf{U}) = \frac{1}{2}\mathbf{r}^H\hat{\mathbf{R}}_U\mathbf{r} + \hat{\mu}(N - \mathbf{r}^H\mathbf{r}), \tag{14}$$

where $\hat{\mu}$ denotes the Lagrange multiplier. Computing the gradient of $L(\mathbf{r},\mathbf{U})$ with respect to $\mathbf{r}$, and then setting it equal to zero. We can get:

$$\hat{\mathbf{R}}_U\mathbf{r} = \hat{\mu}\mathbf{r}. \tag{15}$$

It is easy to understand that $\mathbf{r}$ can be regarded as the eigenvector of $\hat{\mathbf{R}}_U$ which corresponding to the smallest eigenvalue. $\mathbf{r}$ can be obtained as follows:

$$\mathbf{r} = \overline{\nu}[\hat{\mathbf{R}}_U], \tag{16}$$

where $\overline{\nu}[\bullet]$ is the operator that extracts the eigenvector corresponding to the smallest eigenvalue. Then, the estimated ASV of the desired signal can be obtained by substituting this solution into (12). Taking into consideration of the norm constraint, we obtain:

$$\tilde{\mathbf{a}}_0 = \frac{\sqrt{N}}{\|\mathbf{r}\|}\mathbf{U}\mathbf{r}\cdot \tag{17}$$

Once the $\tilde{\mathbf{a}}_0$ is obtained, according to (5), the weighting vector can be achieved. It is worth noting that this method suffers performance degeneration in low SNR due to the Eigen-decompose operator. To address this problem, we can utilize the presumed ASV $\bar{\mathbf{a}}_0$ for $\gamma \le 0$. Then the weighting vector can be expressed as:

$$\tilde{\mathbf{w}} = \begin{cases} \dfrac{(\hat{\mathbf{R}} + \frac{\lambda_{[N]}}{\lambda_{[1]}}\mathbf{I})^{-1}\tilde{\mathbf{a}}_0}{\tilde{\mathbf{a}}_0^H(\hat{\mathbf{R}} + \frac{\lambda_{[N]}}{\lambda_{[1]}}\mathbf{I})^{-1}\tilde{\mathbf{a}}_0} & \gamma>0 \\ \dfrac{(\hat{\mathbf{R}}+\eta\mathbf{I})^{-1}\bar{\mathbf{a}}_0}{\bar{\mathbf{a}}_0^H(\hat{\mathbf{R}}+\eta\mathbf{I})^{-1}\bar{\mathbf{a}}_0} & \gamma \le 0 \end{cases}. \tag{18}$$

It can be seen from (18) that the diagonal loading factor and ASV of the desired signal in the proposed method can be adjusted by introducing the parameter γ. The expression of the weight vector is an attractive closed-form without any iteration process. As a result, the complexity is significantly reduced, especially in low SNR.

C. Complexity analysis

The main computational complexity of the proposed method is the Eigen-decomposition operation and matrix inverse operation. Its overall computational complexity is $O(N^3)$. The RCB algorithm also needs Eigen-decomposition operation, which has a complexity of $O(N^3)$. The worst-case beamforming and the beamformer of [14] (SDP-RAB) have at least the complexity of $O(N^{3.5})$. The beamformer of [15] (IPN-RAB) has a complexity of $O(SN^2)$, where S is the number of sampling points in the area eliminating desired signal. Typically, $S \ge N$. The computational complexity of the beamformer in [19] is $O(N^3)$.

IV. SIMULATION

In this section, the basic simulation conditions are the same as above unless otherwise is specified. The possible angular sector of the SOI is set to $\Theta = [-5^\circ, 11^\circ]$, so the complement sector is $\bar{\Theta} = [-90^\circ, -5^\circ) \cup (11^\circ, 90^\circ]$. We set $Q = 2$. The proposed beamformer is investigated and compared with the diagonally loaded SMI (LSMI) [11], RCB [12], SDP-RAB[14], IPN-RAB[15], the beamformer of [18] and the beamformer of [19]. The optimal parameter $\varepsilon = 0.3N$ is used for the RCB, while the diagonal loading factor of LSMI is selected as twice as the noise power. CVX software is used to solve these convex optimization problems [21].

A. Simulation Example 1: The ASV was known exactly

In this example, we consider the situation that the actual ASVs are exactly known. That means the presumed ASVs and array manifold knowledge are consistent with the actual.

Figure 2 shows the output SINR of the tested beamformers versus input SNR for K=100. It can be seen from the Fig. 2 that the IPN-RAB enjoys the best performance with a high complexity when the ASV is known exactly. The proposed method outperforms the

RCB, beamformer of [18] and beamformer of [19], which all have the same computational complexities. As it can be observed that the beamformer of [18] suffers significantly performance degradation due to the subspace swap phenomenon in low SNRs. The proposed method overcomes this problem by using the traditional diagonal loading method in low SNRs without eigenvalue decomposition of the sample covariance matrix. The output SINR of the proposed beamformer exceeds beamformer of [19] 3.94 dB when the input SNR is 30 dB. Figure 3 corresponds to the output SINR performance versus the number of the snapshots with input SNR=20 dB. It can be observed that the proposed beamformer offers good performance with low computational complexity. The inaccurate eigenvalue decomposition of the covariance matrix in (6) and (16) is the main reason why the proposed method suffers little performance degradation when the number of snapshots is very small.

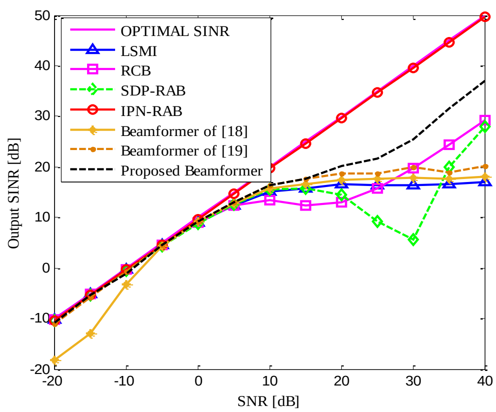

Fig. 2. Output SINR versus the input SNR when the ASVs are exactly known.

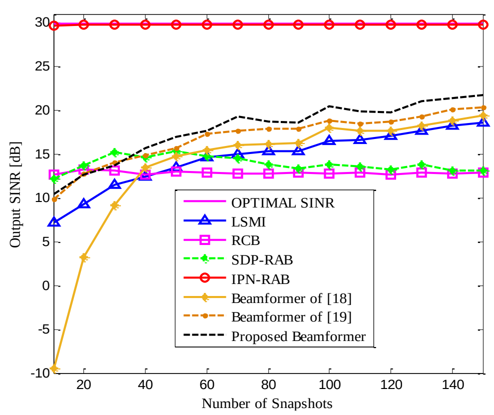

Fig. 3. Output SINR versus the number of snapshots when the ASVs are exactly known.

B. Simulation Example 2: Signal look direction mismatch

In this example, we consider the situation that the random desired signal direction error is occurred. The random DOA estimation mismatch is distributed in $[-5^{\circ},5^{\circ}]$ for each simulation run. So, the actual desired signal DOA is uniformly distributed in $[-2^{\circ},8^{\circ}]$. It is worth noting that the DOA mismatch is changed in each run but remain fixed in each snapshot.

Figure 4 displays the mean output SINR of the tested methods versus the SNR for K=100. As it can be observed, the LSMI suffers significant performance degradation in high SNRs. The performance of the proposed beamformer is only next to the IPN-RAB method. The proposed method provides better performance compared with the LSMI, RCB, beamformer of [18] and beamformer of [19]. In Fig. 5, the output SINR is shown with respect to the number of snapshots for SNR=20 dB. Similar to the previous example, the proposed beamformer has good performance except that the number of the snapshots is very small.

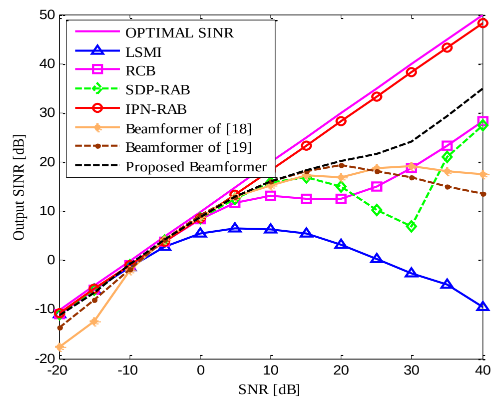

Fig. 4. Output SINR versus the input SNR in the case of look direction mismatch.

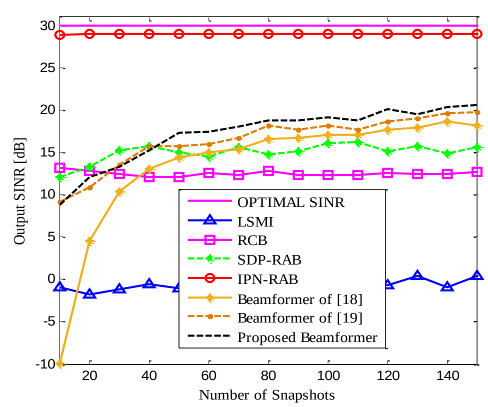

Fig.5. Output SINR versus the number of snapshots in the case of look direction mismatch.

C. Simulation Example 3: Desired ASV mismatch due to wavefront distortion

In this simulation, we consider the ASV of the desired signal is distorted by the effects of wave propagation due to the inhomogeneous medium [14]. In particular, the independent-increment phase distortions are accumulated from the components of the presumed ASV. Assuming that the phase increments are fixed in each simulation runs and are independently chosen from a Gaussian random generator with zero mean and standard deviation 0.04. Figure 6 shows the output SINR of the beamformers versus input SNR for K=100. Figure 7 shows the output SINR of the beamformers versus the number of snapshots for fixed input SNR=20 dB.

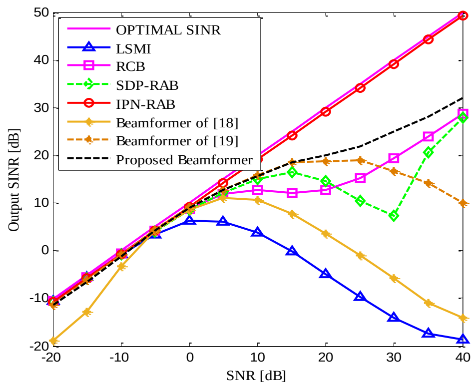

Fig. 6. Output SINRs versus input SNR in the case of wavefront distortion.

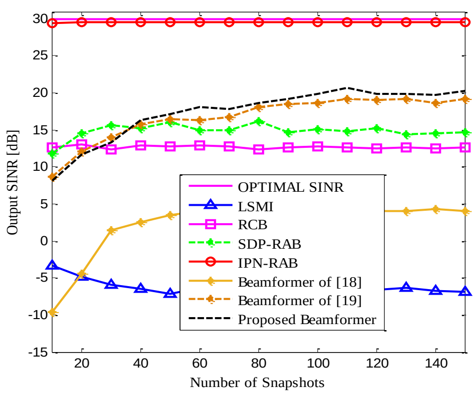

Fig. 7. Output SINRs versus the number of snapshots in the case of wavefront distortion.

It can be observed from the Fig. 6 that the beamformer of [18] lacks robustness against the wavefront distortion, and the SDP-RAB method suffers performance degradation when the $10\,\text{dB} \le \text{SNR} \le 30\,\text{dB}$. The IPN-RAB method outperforms the proposed method at high SNRs, but significantly more complicated. The proposed method is superior to the beamformer of [19] without increasing the calculation load. In particular, we can find from Fig. 7 that the output SINR of the proposed method exceeds the beamformer of [19] 1.15 dB when K=150. From the discussions mentioned above, the performance of our proposed method has little deterioration in small number of snapshots due to the inaccurate estimation of the SOI component.

D. Simulation Example 4: Effect of the error in the knowledge of the array geometry

In this simulation, the effect of the element position errors on the performance of the tested beamformers is investigated. We assume the difference between the presumed and actual positions of each element is modelled as a uniform random variable distributed in the interval $[-0.075\lambda, 0.075\lambda]$, where λ represents the wavelength. The actual DOA of SOI is 5°, and hence, the DOA mismatch is 2°.

Figure 8 shows the output SINR of the beamformers versus input SNR for K=100. It can be seen from the Fig. 8 that the IPN-RAB suffers serious performance degradation in low SNRs due to the inaccurate array manifold information. The proposed beamformer provides strong robustness in the presence of the element positions errors both at low and high SNRs. Figure 9 displays the output SINR performance of all the tested beamformer versus the number of training snapshots for SNR=20 dB. As is shown in the picture, we can see clearly that the proposed method enjoys the best performance when $K \ge 60$.

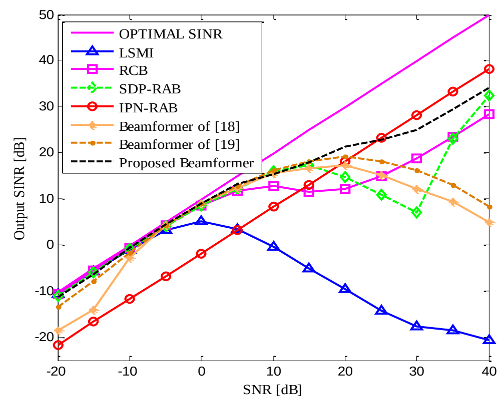

Fig. 8. Output SINR of beamformers versus input SNR for the case of element position errors.

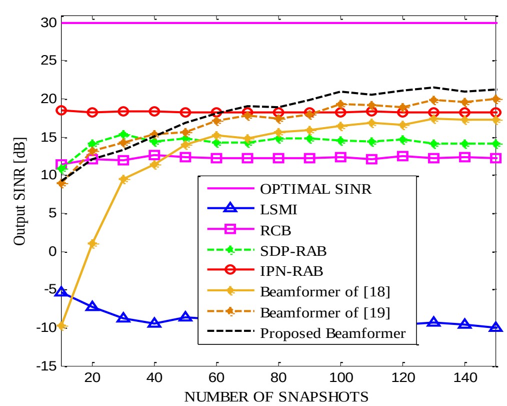

Fig. 9. Output SINR versus the number of snapshots for the case of element position errors.

E. Simulation Example 5: Mismatch due to arbitrary ASV errors

In this simulation, we study the performance of the proposed beamformer when arbitrary ASV errors are considered. Here, the ASV mismatch is comprehensive and arbitrary-type, which may be caused by direction errors, calibration errors, gain and phase perturbations, and so on. The actual ASVs of can be modelled as [17]:

$$\bar{\mathbf{a}}(\theta) = \mathbf{a}(\theta) + \hat{\mathbf{e}}, \qquad (19)$$

where $\bar{\mathbf{a}}(\theta)$ denoted the presumed ASVs, $\hat{\mathbf{e}}$ is a zero-mean complex random vector with the variance σ_e^2. In this example, all the array imperfections are generated as Gaussian variables with the given variance, $\sigma_e^2 = 2$.

The output SINR of the beamformers versus input SNR for K=100 is displayed in Fig. 10. We can notice that the proposed method can improve the output SINR of the beamformer efficiently, and fit to be used in complex environment. This means the proposed beamformer is effective in the presence of the arbitrary ASV errors. Figure 11 displays the output SINR performance of all the tested beamformer versus the number of training snapshots for SNR=20 dB. Obviously, the proposed beamformer has the best output SINR among all the tested beamformers when $K \geq 60$.

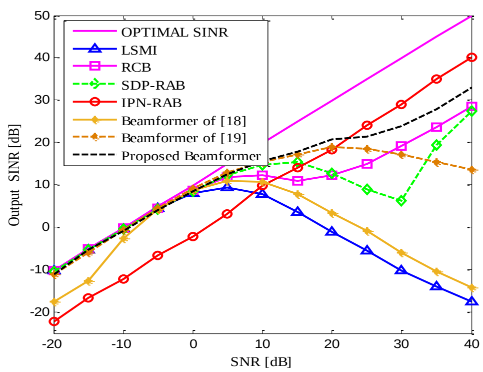

Fig. 10. Output SINR of beamformers versus input SNR with arbitrary ASV errors.

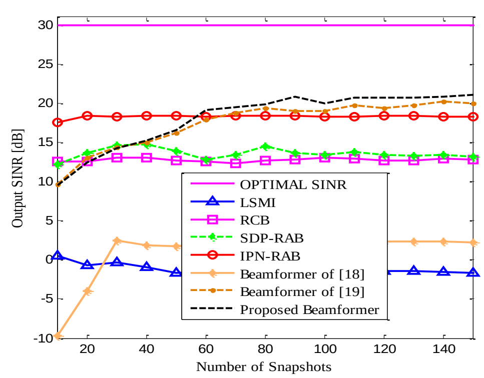

Fig. 11. Output SINR of beamformers versus snapshots with arbitrary ASV errors.

V. CONCLUSION

In this paper, a robust adaptive beamforming method with low complexity has been proposed and its performance has been investigated. The proposed beamformer is realized based on variable diagonal loading method, and the diagonal loading factor is selected according to the input SNR. The expression of the weighting vector in a closed-form has been provided. The proposed method has a low complexity and possesses good operability and excellent performance. The simulation results demonstrated that the proposed beamformer can provide superior performance against unknown arbitrary-type mismatches compared to the existing popular methods.

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