Multiplication Theory for Prediction of the Scattering Grating-lobe of Array Antenna

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Abstract **─** Experimental results reported in many literature have demonstrated that an antenna array should have two or more grating lobes in its scattering pattern; however, it is not clear how these grating lobes occur and vary. In this paper, we propose a new method to predict the number and location of scattering grating lobes generated by an array antenna. In the proposed method, the radar cross section (RCS) of array antenna can be decomposed into multiplication of the array RCS factor and the element RCS factor. The array RCS factor bears universal applicability so that it can be used to directly determine scattering properties of array antenna. Compared with the full-wave simulation techniques, the new method requires much less computation time and memory storage so that it is more suitable to be employed as the basis of a synthesis method to predict the desired RCS pattern of a large array antenna. As examples, the scattering properties of the dipole antenna arrays are investigated to validate the new method, numerical results demonstrate that the number and location of scattering grating lobes predicted by using the proposed method coincide with those simulated by using a MoM based commercial; software FEKO.

Index Terms **─** Array antenna, array factor, decomposition, Radar Cross Section (RCS), scattering grating lobe.

I. INTRODUCTION

As the radar cross section (RCS) of military platform itself has been reduced to realize a low observation level along with the development of stealth techniques, there is still an increasing interest to reduce the RCS of array antenna mounted on the platform. To this end, the principal task is how to efficiently predict the scattering feature of an antenna array; therefore, many efforts have been given to the full-wave simulation techniques to simulate scattering patterns for different types of array antennas such as microstrip patch antenna array [1, 2], waveguide slot antenna array [3, 4], dipole antenna array [5, 6], and so on. Both experimental data and numerical simulation results have shown that the scattering grating lobes arise if the inter-element spacing is larger than a half wavelength considering the two way transit of the radar signal, which may be significant in terms of the increased detectability. Hence, an important work is to suppress the grating lobes in the scattering patterns to improve the stealth property of an array antenna.

Since existing of the grating lobes is determined by the spatial arrangement of array elements [7], they cannot be independently suppressed using the traditional stealth techniques such as geometrical shaping optimization [8, 9], applications of radar absorbing materials [10, 11] and other electromagnetic materials [12, 13]. One practicable way is to properly adjust the element positions to break the periodicity. As shown in [14], Haupt applied the hybrid genetic algorithm (GA) to optimize the position of each perfectly conducting strip to generate prescribed backscattering pattern, in which the method of moments (MoM) is employed to evaluate the objective function. This kind of synthesis concept for scattering pattern optimization is also employed by people else to tune other parameters. For example, to obtain the desired RCS pattern of resistive strips, Choi [15] proposed to optimize the resistance distribution varying transversely on the strip to generate corresponding distribution of the induced surface current density on the strip. In [16], Coe successfully realized the desired null in the RCS pattern of a two dipoles array by optimizing the terminal impedance loads. The similar concept was used by Thors and Josefsson [17] to design the low RCS and high radiation performance conformal array antenna.

In the aforementioned RCS synthesis methods, the scattering patterns of various arrays are calculated using full-wave numerical methods, therefore these synthesis methods are time consuming for large arrays or even infeasible when the structure of antenna element is complex. Hence, it is significant to look for an approximation technique that is more efficient to be employed as the basis of a synthesis method to generate the desired RCS pattern of a large and complex array antenna.

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For this purpose, approximation closed-form expressions based on the multiplication theory are derived to calculate the scattering patterns of an array antenna. Extra computational cost for the proposed method increases slightly with an increase of the number of array elements, and it is worth to mention that there is no relation of the simulation time with the antenna type. Moreover, the number and location of scattering grating lobes can be directly determined by using this analytical method, therefore, it can help set up the RCS optimization goal in advance. Utilizing the proposed new method, the relationship between the scattering grating lobes and the inter-element spacing of dipole antenna arrays are discussed.

II. THEORY AND FORMULAS FOR THE SCATTERING FROM ARRAY ANTENNA

Traditionally, the scattered field of an antenna can be decomposed into two components, which are called the antenna and structural modes [18]. The multiplication theory derived in this paper is extracted from the structural mode and antenna model RCS of the array antenna, respectively, which can be used to clarify the physical mechanism of array scattering and to guide the stealth technique choice. At first, the theory of scattered field from single element is briefly reviewed.

A. Scattered field of single element in the array

As the references [6, 18] discussed, when the antenna load impedance Z_L is equal to the characteristic impedance Z_c of the transmission line that connects the antenna and load, the reflection seen from the transmission line to the load is $\Gamma^l = (Z_L - Z_c)/(Z_L + Z_c) = 0$. It is obvious that no energy is reflected back to the antenna and only induced current is on the antenna body. Therefore, the antenna acts as a general passive scattering object. A planar array with $M\times N$ elements is shown in Fig. 1, in which each square represents one element is composed of only metallic conductors. Applying the conclusion to the element in the array, the scattered field of single antenna (m, n) in the array is given as follows:

$$\vec{E}^s_{mn}(Z_L) = \vec{E}^s_{mn}(Z_c) + \vec{E}^a_{mn}(Z_L), \tag{1}$$

where the first term $\vec{E}^s_{mn}(Z_c)$ on the right side is called the structural mode scattering, and the second term $\vec{E}^a_{mn}(Z_L)$ on the right side is called antenna mode scattering.

B. Extracting the array scattering factor from the structural mode scattering of array antenna

$\vec{E}^s_{mn}(Z_c)$ in (1) represents the scattered field generated by the induced current on the antenna physical structure. This scattered field can be calculated by using the MoM method as follows [19]:

$$\begin{aligned}\vec{E}^s_{mn}(Z_c) &= -\frac{j\omega\mu_0}{4\pi r_{mn}} e^{-j\vec{k}_0\cdot\vec{r}_{mn}} \\ &\cdot \sum_{l=1}^{L} I_{l(mn)} \int_{s'_{mn}} (\hat{\theta}\hat{\theta} + \hat{\varphi}\hat{\varphi})\, \vec{f}_l(\vec{r}^{\,'}_{mn}) e^{j\vec{k}_0\cdot\vec{r}^{\,'}_{mn}} dS',\end{aligned} \tag{2}$$

where $\vec{k}_0$ is the vector wave number of the free space, ω is the angular frequency of scattered field, the permeability of free space is μ_0, $\vec{r}_{mn}$ represents the observation point with spherical coordinates (r, θ, φ), $\vec{r}^{\,'}_{mn}$ is the location of the induced current on the element (m, n) and S'_{mn} and $\vec{f}_l(\vec{r}^{\,'}_{mn})$ represent the source range and basis function of element (m, n), respectively. Ignoring the mutual coupling between elements, the current coefficients should have an equal value $I_{l(mn)} = I_l$ (l=1, 2, $\cdots$, L). Considering the far field observation and referring to Fig. 1, we have:

$$\vec{r}_{mn} = \vec{r}_{11} - \vec{d}_{mn}, \tag{3}$$

$$\vec{r}^{\,'}_{mn} = \vec{r}^{\,'}_{11} + \vec{d}_{mn}, \tag{4}$$

$$r_{mn} \approx r_{11}. \tag{5}$$

The Equation (2) reduces to:

$$\vec{E}^s_{mn}(Z_c) = \vec{E}^s_{11}(Z_c) \cdot e^{2j\vec{k}_0\cdot\vec{d}_{mn}}, \tag{6}$$

where

$$\begin{aligned}\vec{E}^s_{11}(Z_c) &= -\frac{j\omega\mu_0}{4\pi r_{11}} e^{-j\vec{k}_0\cdot\vec{r}_{11}} \\ &\cdot \sum_{l=1}^{L} I_l \int_{s'_{11}} (\hat{\theta}\hat{\theta} + \hat{\varphi}\hat{\varphi})\, \vec{f}_n(\vec{r}_{11}{'}) e^{j\vec{k}_0\cdot\vec{r}_{11}{'}} dS'\end{aligned} \tag{7}$$

$$\vec{d}_{mn} = \hat{x}\cdot dx_m + \hat{y} dy_n,$$

where dx_m and dy_n are the distances from the element (m, n) to the element (1, 1) at the origin point in terms of the x-and y-axes, respectively. The total monostatic structural model scattering of the antenna array is obtained by summing over all the elements:

$$\vec{E}^s(Z_c) = \vec{E}^s_{11}(Z_c) \sum_{m=1}^{M}\sum_{n=1}^{N} e^{2j\vec{k}_0\cdot\vec{d}_{mn}}. \tag{8}$$

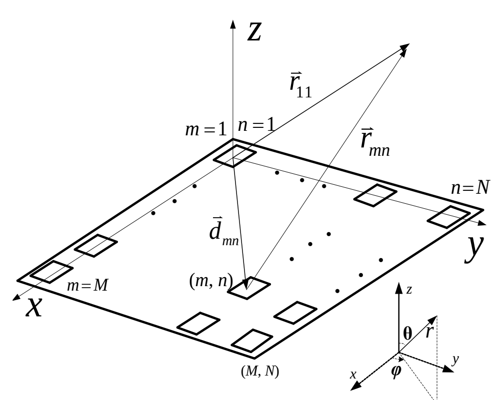

Fig. 1. Configuration of an array antenna with $M\times N$ elements.

C. Extracting the array scattering factor from antenna mode scattering of array antenna

When the incident wave frequency falls into the antenna operating band, the incident energy collected by the antenna travels through a feed network, and is radiated. The radiation field, namely, antenna mode scattering, is expressed as [20]:

$$\vec{E}^{a}_{mn}(Z_L) = \frac{\Gamma^{l}_{mn}}{1-\Gamma^{A}_{mn}\Gamma^{l}_{mn}} b_{mn}\vec{E}_r, \tag{9}$$

where $\Gamma^{l}_{mn} = (Z_L - Z_c)/(Z_L + Z_c)$, $\Gamma^{A}_{mn} = (Z_A - Z_c)/(Z_A + Z_c)$, $\vec{E}_r$ is the radiation field of the element excited by a unit amplitude source with a=1 (watt)$^{1/2}$, b_{mn} is the receiving amplitude of the element (m, n) when terminated with a match load. Then b_{mn} is given as follows [17]:

$$b_{mn} = \frac{1}{2a}\oint_{s}(\vec{E}^{i}_{mn}\times\vec{H}^{r}_{mn} - \vec{E}^{r}_{mn}\times\vec{H}^{i}_{mn})\cdot d\vec{S}, \tag{10}$$

where $(\vec{E}^{i}_{mn}, \vec{H}^{i}_{mn})$ and $(\vec{E}^{r}_{mn}, \vec{H}^{r}_{mn})$ are the incident field and scattered field of the antenna element, respectively. Substituting (3)-(5) into (10), b_{mn} can be expressed in terms of b_{11} as follows:

$$b_{mn} = b_{11}\cdot e^{j2\vec{k}_0\cdot\vec{d}_{mn}}. \tag{11}$$

Substituting (11) into (9), the antenna mode scattering of antenna element (m, n) is given by:

$$\vec{E}^{a}_{mn}(Z_L) = \frac{\Gamma^{l}_{mn}}{1-\Gamma^{A}_{mn}\Gamma^{l}_{mn}} e^{2j\vec{k}_0\cdot\vec{d}_{mn}} b_{11}\vec{E}_r. \tag{12}$$

Then the total antenna mode scattered field of the array is obtained by summing over all the elements:

$$\vec{E}^{a}(Z_L) = b_{11}\vec{E}_r\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{\Gamma^{l}_{mn}}{1-\Gamma^{A}_{mn}\Gamma^{l}_{mn}} e^{2j\vec{k}_0\cdot\vec{d}_{mn}}. \tag{13}$$

Assuming that each element in the array is connected with the identical feed structure and ignoring the mutual coupling effects, the elements should have the equal load impedance so that reflections from them should have the same amplitude and phase. Consequently, (13) can be expressed as follows:

$$\vec{E}^{a}(Z_L) = \vec{E}^{a}_{11}(Z_L)\sum_{m=1}^{M}\sum_{n=1}^{N} e^{2j\vec{k}_0\cdot\vec{d}_{mn}}. \tag{14}$$

D. New expression for RCS of array antenna

Based on (1), (8) and (14), the monostatic scattered field of an array antenna can be written in a multiplication form:

$$\vec{E}^{s}(Z_L) = E^{s}_{a}(Z_L)\cdot\vec{E}^{s}_{e}(Z_L), \tag{15}$$

where $E^{s}_{a}(Z_L) = \sum_{m=1}^{M}\sum_{n=1}^{N} e^{2j\vec{k}_0\cdot\vec{d}_{mn}}$ is defined as array scattering factor and $\vec{E}^{s}_{e}(Z_L)$ is defined as element scattering factor.

III. SCATTERING GRATING LOBES OF ARRAY ANTENNA

It can be found in literature that an antenna array can generates two or more grating lobes in the RCS pattern [3-5], but it is not clear how these grating-lobes occur and vary. In fact, these grating lobes appear due to the certain inter-element spacing of the array, and this will be proved in the following. Hence, in this section the properties of these grating lobes are analyzed utilizing the array RCS factor. In addition this discussion can also be used to validate (15).

For the sake of mathematic simplification, an M-element dipole antenna with an uniform linear distribution, as shown in Fig. 2, is taken as an example. As (15) indicated, once the element is chosen the element scattering factor is determined, and the scattering pattern of array antennas depends mainly on the array scattering factor that includes the inter-element spacing information of array. The array scattering factor in (15) can be rewritten as below:

$$E^{s}_{a}(Z_L) = e^{j[(M-1)/2]\psi}\cdot\sin(\frac{M}{2}\psi)\Big/\sin(\frac{1}{2}\psi). \tag{16}$$

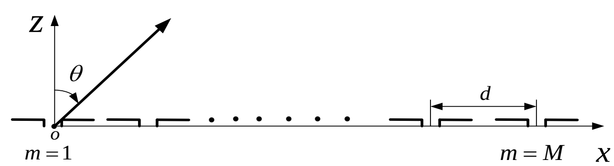

Fig. 2. Configuration of a uniform linear array consists of M half-wave dipole antennas.

Then the array MRCS (monostatic RCS) factor can be written as:

$$\sigma_a(\theta,\varphi) = \left|\frac{\vec{E}^{s}_{a}(Z_L)}{\vec{E}^{i}}\right|^2 = \left|\sin(\frac{M}{2}\psi)\Big/\sin(\frac{1}{2}\psi)\right|^2, \tag{17}$$

where $\psi = 2\vec{k}_0\cdot\vec{d}_{mn} = 2k_0 d\sin\theta\cos\varphi$, d is the inter-element spacing. The maximum values of (17) occur in the following condition:

$$\frac{\psi}{2} = k_0 d\sin\theta\cos\varphi = \pm l\pi,\ (l = 0,\ 1\cdots), \tag{18}$$

where the integer l has a constraint condition of $l \le |2d/\lambda|$.

Considering the scattering pattern in the plane of $\varphi = 0^{\circ}$ and the range of $-90^{\circ} \le \theta \le -90^{\circ}$, (18) reduces to:

$$\begin{aligned} &\frac{\psi}{2} = k_0 d\sin\theta|_{\theta=\theta_m} = \pm l\pi \\ &\Rightarrow \theta_m = \sin^{-1}[\pm\frac{\lambda l}{2d}], \end{aligned} \quad (l = 0,\ 1\cdots). \tag{19}$$

Utilizing (19), we can simply predict the number and location of the grating lobes in the RCS pattern.

Considering a 11-dipole antenna linear array with an inter-element spacing of 0.6λ, the maximum scattering will occur when l equals to 0 and 1 according to (19). For l=0 (19) has only one solution of $\theta_m = 0^{\text{o}}$ and there are two solutions of $\theta_m = \pm 56.4^{\text{o}}$ for l=1. A good agreement between the calculated and FEKO full-wave simulated MRCS patterns is evident, as shown in Fig. 3. From Fig. 3, one of the maximum scattering grating lobes occurs at the angle $\theta_m = 0^{\text{o}}$ due to the specular scattering of the array aperture. The other two maximum scattering grating lobes are lower than that of specular scattering because the amplitudes of the element RCS factor at angles $\theta_m = \pm 56.4^{\text{o}}$ is lower than that at $\theta_m = 0^{\text{o}}$, as shown in Fig. 4.

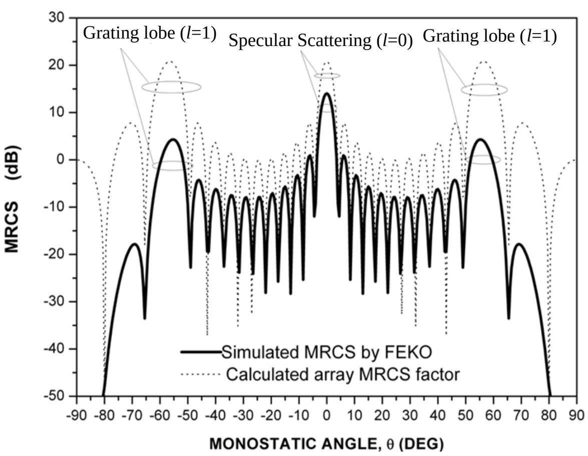

Fig. 3. MRCS of 11-dipole linear array ($d = 0.6\lambda$) simulated by using FEKO and the array MRCS factor calculated by using (17).

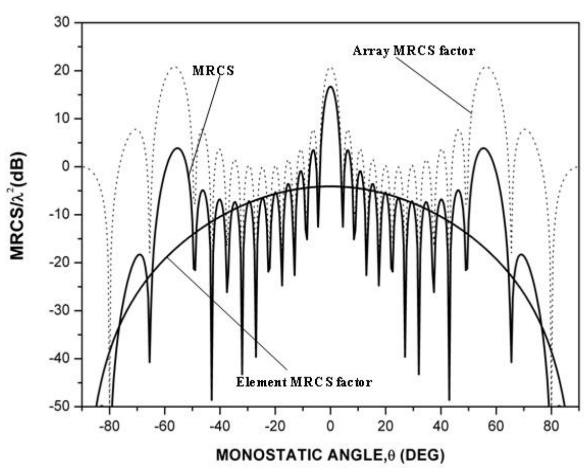

Fig. 4. Calculated array MRCS factor, element MRCS factor and the MRCS of 11-dipole linear array ($d = 0.6\lambda$) by using (15) and (17).

To further investigate the relationship between the scattering grating lobes and the inter-element spacing, the inter-element spacing of the 11-dipole antenna linear array discussed above is extended to $d = 1.4\lambda$. According to (19), the integer l should have three values, i.e., 0, 1 and 2, and there are five maximum scattering angles in the scattering pattern. Figure 5 depicts the FEKO full-wave simulated MRCS pattern and calculated MRCS factor pattern of the array antenna, and it can be observed from Fig. 5 that the five maximum scattering grating lobes correspond to different values of the integer l.

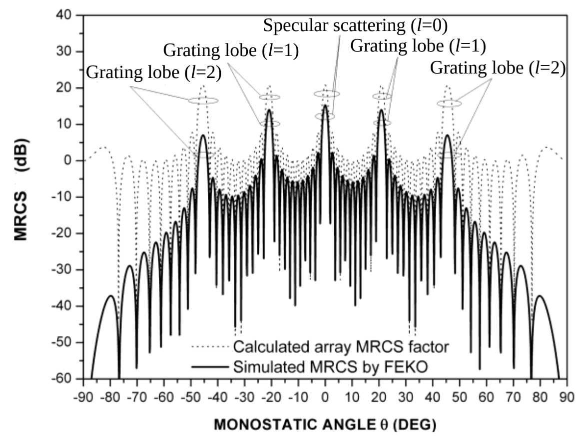

Fig. 5. FEKO full-wave simulated MRCS of 11 half-wave dipoles linear array ($d = 1.4\lambda$) and the calculated array MRCS factor by using (17).

As discussed above, we can conclude that different inter-element spacing makes the grating-lobes in the MRCS pattern of array antenna appear in different incident directions. With the increase of inter-element spacing, more grating lobes will occur in the MRCS pattern and move toward the normal incident direction. This is similar to the radiation of array antenna [19, 21].

IV. CONCLUSION

The scattering grating lobes of an array antenna can be represented by the multiplication of array RCS factor and the element RCS factor. The relationship between scattering grating lobes and inter-element spacing of a typical uniform array antenna were analyzed in this paper. FEKO full-wave simulation results and the calculation results based on the proposed method demonstrated that the new method can fast and accurately determine the number and location of the scattering grating lobes of a practical array antenna. Since the maximum scattering grating lobes of an array antenna depends mainly on the array RCS factor, the conclusions obtained in this paper are also applicable for other types of antennas with the same array distribution (array RCS factor). To eliminate the scattering grating lobes, one can take the formulation here as the base of synthesis methods to optimize the element positions.

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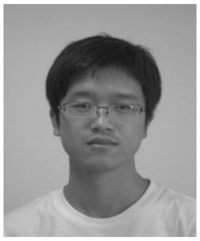

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