Nonlinear Position-Flux Zero-Bias Control for AMB System with Disturbance

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Abstract — This study presents two novel nonlinear controllers for a single one-degree-of-freedom (1–DOF) active magnetic bearing (AMB) system operated in zero-bias mode with externally bounded disturbance. Recently developed controllers are complicated and inherently difficult to implement. The simple and low-order controllers proposed in this paper are designed using nonlinear feedback tools, including Lyapunov-based techniques and control Lyapunov functions (CLFs). The control objective is to globally stabilize the mass position of the nonlinear flux-controlled AMB system with control voltage saturation. The zero-bias AMB control model is derived from the voltage switching strategy. The developed CLF-based controllers are verified by numerical calculations.

Index Terms — Active magnetic bearing, control Lyapunov function, nonlinear flux controller, zero-bias control.

I. INTRODUCTION

The active magnetic bearing (AMB) control system with classical large bias current is a well-known linear control problem, and as a result, PID controllers, $\mathcal{H}_\infty$–based control and μ-synthesis methods can be applied, e.g., see author references: [1, 2, 3]. However, large bias-current or a bias-flux implies power loss, where the loss mechanisms are generally proportional to the square of the electromagnetic force. Moreover, a large bias causes heat dissipation and further changes the electromagnets' parameters. In order to improve the energy efficiency of the AMB system, zero-bias flux control can be applied. In this system, the dynamics become strongly nonlinear. Therefore, nonlinear control methods can be applied in order to design a stable AMB system with zero-bias or low-bias [4÷11]. All of the aforementioned approaches are fundamentally based on position-current or position-flux state feedbacks.

In particular, a nonlinear and uncertain flux-controlled AMB system operated with zero-bias was considered in paper [10]. The major parametric uncertainties of the AMB such as: magnetic saturation perturbation, bias flux (premagnetization) and uncertain losses increase the nonlinearity of the AMB system. In response to this problem, paper [10] presents the robust stability and robustness analyses of a nonlinear closed-loop AMB system with inherent uncertainties. The so-called *small gain theorem* can be used to calculate the robust stability of an uncertain AMB system [10].

Flux-based control with zero-bias increases the nonlinearity of an AMB system. Nonlinear control approaches intended for AMBs have been developed [12, 13]. In the last century, stability concepts pertaining to nonlinear systems were formulated by Lyapunov and were first expounded upon by Malkin in 1952 [14]. Later, Lyapunov functions were applied, for example, to the passivity theorem and to dissipative systems in 1972 [15] as well as to solving optimal and inverse optimal control problems. The Lyapunov technique has been extended to control systems in [16÷19], for example. Since characterizing stability in terms of the smooth Lyapunov function is not possible in some cases, the stabilizing feedback design should be used. This is the main reason for using the so-called control Lyapunov function (CLF). Its concept was introduced by Artstein and Sontag in 1983 [20, 21]. The idea of CLF-based control is to select a Lyapunov function $V(x)$ and then to try to find a feedback control $u(x)$ that renders $\mathrm{d}V(x,u)/\mathrm{d}t$, defined negatively. Thus, by choosing a suitable $V(x)$, and when $V(x)$ is the CLF, we can find a stabilizing control law $u(x)$ for the system feedback [22]. The CLF-based control concept was extended to dynamic systems with known disturbance [23÷25], where $V(x)$ is the RCLF (a robust CLF), if, for a bounded disturbance, ω ensures that $\dot{V}(x,u,\omega) < 0$ [17, 26]. The linear $\mathcal{H}_\infty$ control method was used to solve a disturbance attenuation problem in a nonlinear system which is analogous to the RCLF [27, 28].

The main aim of the present work is to show simple nonlinear controllers that contribute improvements to flux-controlled AMB systems operated in zero-bias mode in comparison with existing approaches. The proposed nonlinear control laws are based on the control Lyapunov function (CLF) and are effective in AMB zero-bias control systems with control voltage saturation. However, the control law based on Artstein-Sontag's theorem includes Lie derivative terms and leads to a complex solution [29]. The main advantages of the

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proposed controllers, if compared with CLF based on Artstein-Sontag's theorem, are that they are simpler and inherently easier to implement in low-power micro-controller AMB hardware. Performed simulations show that simple low-order controllers based on CLF give satisfactory results in comparison with complex solutions based on Artstein-Sontag's theorem [29]. In comparison with previous solutions [29, 30], the obtained control laws ensure similar or even better transient responses and better disturbance attenuation.

The paper is organized as follows. Section 2 presents a simplified one-dimensional active magnetic bearing (AMB) system. Section 3 formulates conditions for zero-bias flux-feedback control and flux-switching strategy. Section 4 proposes Lyapunov-based controllers and describes control law design functions. Section 5 provides numerical examples which prove the control laws proposed in Section 4. Finally, Section 6 closes the paper with some concluding remarks.

II. THE 1-DOF AMB MODEL

Let us consider the simplified 1–DOF (one-degree-of-freedom) AMB model that consists of two opposite and presumably identical electromagnetic actuators (electromagnets), which generate attractive forces, F_1 and F_2, on the rotor [31]. To control the position x of the rotor mass m to the stable state x=0, the voltage inputs of the electromagnets, v_1 and v_2, are used to design the control law, see Fig. 1.

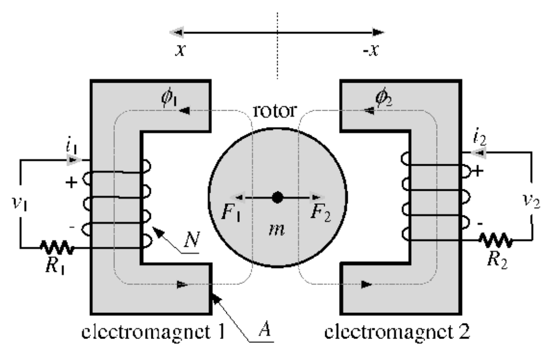

Fig. 1. Simplified one-dimensional AMB.

The 1-DOF model of the AMB is nonlinear where mechanical and electrical dynamics are coupled. Consider Fig. 1, in which, neglecting gravity (for the horizontal rotor control direction), the dynamic equation is given by [31]:

$$F_j = \frac{\cos\alpha}{\mu_0 A}\Phi_j^2, \quad \text{for } j = 1,2, \tag{1}$$

where F_j is the total force generated by each electromagnet, Φ_j is the total magnetic flux through each active coil, A is the cross-sectional area of each electromagnet pole, α is the angle at which electromagnetic force acts, and μ_0 is the permeability of free space (=1.25 × 10^{-6} H/m).

The total flux generated by the j-th electromagnet is $\Phi_j = \Phi_0 + \phi_j$. In the case of zero-bias operation, the bias flux Φ_0 equals zero, and the total flux equals control flux ϕ_j. Then, we define the *generalized control flux* as:

$$\begin{gathered}\phi := \phi_1 - \phi_2 \\ \phi := \frac{1}{N}\left(\int_0^t (v_1 - Ri_1)dt - \int_0^t (v_2 - Ri_2)dt\right),\end{gathered} \tag{2}$$

where N denotes the number of turns of the coil of each electromagnet.

If $\Phi_0 = 0$, then according to (1), the mass motion equation is given by:

$$\frac{d^2}{dt^2}x = \frac{\cos\alpha}{\mu_0 mA}\left(\phi_1^2 - \phi_2^2\right). \tag{3}$$

The electrical dynamics of the AMB system are given by the governing equation [31]:

$$v_j = N\frac{d\phi_j}{dt} + Ri_j, \ j = 1, 2, \tag{4}$$

where R is the electromagnet's resistance. Then Eq. (4) can be rewritten in an equivalent form as:

$$\dot{\Phi}_j = \dot{\phi}_j = \frac{1}{N}\left(v_j - Ri_j\right), \ j = 1, 2. \tag{5}$$

III. ZERO–BIAS FLUX-FEEDBACK CONTROL

In the case of zero-bias control, the nonlinear flux/force characteristic has a dead zone near the origin (low dynamic response of the AMB) [32]. This means that the slope of the magnetic force vs. flux curve near the origin is zero, and we need a large change in flux in order to generate a small control force. According to (5), the flux depends on the control voltage and current. Voltage commands are limited in real applications and voltage saturation is another problem. In short, zero-bias nonlinear control with voltage saturation is a challenging task.

In zero-bias control, the control force F_j depends on control flux ϕ_j which fulfils the following condition of the switching scheme [29, 30]:

$$\begin{gathered}\phi = \phi_1, \quad \phi_2 = 0 \text{ when } \phi \geq 0 \\ \phi = -\phi_2, \ \ \phi_1 = 0 \text{ when } \phi < 0\end{gathered}. \tag{6}$$

The ϕ described by (6) is called a *generalized flux*. The switching scheme allows us to minimize control fluxes ϕ_1 and ϕ_2, since at least one of the control fluxes is zero at the starting time. This means that at least one of the electromagnets is inactive at any given instant of time. The system minimizes energy and power losses [31, 33].

For zero-bias, based on (3), according to the generalized complementary flux condition (8), the total generalized attractive force is given by:

$$F(\Phi) = \frac{\Phi|\Phi|}{\mu_0 mA}, \tag{7}$$

where generalized attractive force F=F_1-F_2. The system's nonlinearity in (7) is given by *non-decreasing function*

$\eta(\Phi)=\Phi|\Phi|$. The general form of the flux-based control law is given by:

$$u_\phi = -f_\phi(\phi_r - \phi), \tag{8}$$

where ϕ_r is the flux reference and f_ϕ is a nonlinear control function which also ensures bounds of ϕ_j such that:

$$\lim_{t\to\infty}\phi_j(t) = \min\{\phi_1(0),\ \phi_2(0)\}. \tag{9}$$

Fluxes ϕ_1 and ϕ_2 remain bounded, and condition (9) represents convergence of ϕ_j, which is ensured if system (3)–(4) is asymptotically stable. The nonlinear and fast dynamic flux controller generates the required fluxes in the AMB's structure due to nonlinear characteristics of controlled flux ϕ versus generated total force F. Typically, when cascaded control is applied, the linearizing flux controller works in the inner control flux loop. The transfer function for the low level control feedback rule in the s-domain is given by:

$$P(s) = \frac{\phi(s)}{\phi_r(s)}. \tag{10}$$

The AMB closed-loop system (10) is used in the case of local force control in electromagnets. However, in this work, we present not a local, but a global nonlinear rotor position controller.

From the simple analysis presented above, it follows that, for the dynamics of system (3) with the *generalized control flux* given by (2), under switching strategy (6), and with state coordinates defined as:

$$x_1 = x,\ x_2 = \dot{x},\ x_3 = \phi, \tag{11}$$

then the state-space AMB dynamic model is given by:

$$\begin{cases} \frac{d}{dt}x_1 = x_2 \\ \frac{d}{dt}x_2 = \frac{\cos\alpha}{\mu_0 mA}|x_3|x_3, \\ \frac{d}{dt}x_3 = \frac{1}{N}(v - Ri) \end{cases} \tag{12}$$

where $v = v_1 - v_2$ is the *generalized control voltage* and $i = i_1 - i_2$ is the *generalized current*.

IV. LYAPUNOV-BASED CONTROL

A. Problem statement - AMB model with disturbance

In this section we will find the CLF that will make the AMB system globally stable with respect to additive measurement disturbances. It is well known that bounded disturbances in a nonlinear system can cause severe forms of instability [24]. Moreover, a nonlinear control law that guarantees global stability of a nonlinear system under perfect state feedback will not ensure global robustness to state measurement disturbances. There are many classes of systems for which stabilizability is preserved in the presence of state measurement disturbances, e.g., strict feedback systems [34].

In order to simplify notation, and to work with a system having the minimum number of parameters, let us introduce the following non-dimensionalized state and control variables along with a non-dimensionalized time [29, 30]:

$$\begin{gathered} x_1 := \frac{x}{g_0},\ x_2 := \frac{\dot{x}}{\Phi_{\text{sat}}\sqrt{g_0/\mu_0 mA}},\ x_3 := \frac{\phi}{\Phi_{\text{sat}}} \\ u := \frac{v\sqrt{g_0\mu_0 mA}}{N\,\Phi_{\text{sat}}^2},\ \tau := t\frac{\Phi_{\text{sat}}}{\sqrt{g_0\mu_0 mA}},\ w = \frac{\omega}{\omega_{max}}, \end{gathered} \tag{13}$$

where g_0 is the nominal air gap (clearance), u – the non-dimensionalized control variable, Φ_{sat} – the saturation flux, τ denotes non-dimensionalized time, w is an external non-dimensionalized input, and ω is the bounded disturbance with its maximum value ω_{max}.

Importantly, the AMB system parameters in (13) are constant and their nominal values andabsolute boundary values are given in Table 1.

Let us assume that w is a known bounded disturbance and impact via state x_1 to the AMB system. Then, in accordance with (13), the model of the AMB system with disturbance input $w \in \mathbb{R}$ is written in the state-space:

$$\begin{cases} \frac{d}{d\tau}x_1 = x_2 + x_1 w \\ \frac{d}{d\tau}x_2 = x_3|x_3|\ , \\ \frac{d}{d\tau}x_3 = u \end{cases} \tag{14}$$

where x_1, x_2, x_3 are defined by (13) and u is a control input. In this way, variables x_1, x_2 and x_3 indirectly relate to the position x [m] of the rotor mass, velocity $\dot{x}$ [m/s] and electromagnetic flux ϕ [Wb], respectively.

However, the disturbance w and the control voltage are always limited in the AMB system. Moreover, in AMB applications, since the electromagnet coils are typically driven by power amplifiers, these amplifiers must be configured to operate in voltage mode or current mode with saturation. In a real AMB system, the voltage input is bounded as $u(t) = \text{sat}(v(t))$, where $\text{sat}(v(t))$ is the saturation function of voltage $v(t)$ defined here as:

$$\text{sat}(v(t)) = \begin{cases} -v_{\text{lim}} & \text{if } v(t) < -v_{\text{lim}} \\ v(t) & \text{if } -v_{\text{lim}} \le v(t) \le v_{\text{lim}}, \\ v_{\text{lim}} & \text{if } v(t) > v_{\text{lim}} \end{cases}$$

where v_{lim} is the voltage input limit and refers to v_{sat} (saturation voltage value) given in Table 1.

B. CLF for AMB with disturbance

Note that system (14) is the control affine system of the form:

$$\dot{x} = f(x) + g(x)u + h(x)w, \tag{15}$$

where $u \in \mathbb{R}$ – control input, w – bounded independent disturbance input, and vector fields f: $\mathbb{R}^3 \to \mathbb{R}^3$ and g: $\mathbb{R}^3 \to \mathbb{R}^3$, h: $\mathbb{R}^3 \to \mathbb{R}^3$ are given by $f(x) = \begin{bmatrix} x_2 & x_3^{[2]} & 0 \end{bmatrix}^T$, $g(x) = [0 \quad 0 \quad 1]^T$, $h(x) = [x_1 \quad 0 \quad 0]^T$ with $x_3^{[2]} := x_3^2\text{sgn}(x_3) = x_3|x_3|$.

Recall that system,

$$\dot{x} = f(x) + g(x)u, \tag{16}$$

is asymptotically stabilizable with respect to the equilibrium pair (x_0, u_0), where $x_0 = x(0)$, if there exists a feedback law $u = \alpha(x)$, $\alpha(x_0) = u_0$, defined on a neighbourhood U_{x_0} of x_0 such that α is continuously differentiable on $U_{x_0} \setminus \{x_0\}$, for which the closed-loop system,

$$\dot{x}(t) = (f + \alpha g)(x(t)), \tag{17}$$

is locally asymptotically stable (with respect to x_0). Recall also that (see [16, 21]) a real continuous function defined on open set $X \subset R^n$ is a local control Lyapunov function for system (17) (relative to the equilibrium state x_0), if it satisfies the following properties:

(i) V is proper at x_0, i.e., $\{x \in X: V(x) \le \varepsilon\}$ is a compact subset of some neighborhood U_{x_0} of x_0 for each sufficiently small $\varepsilon > 0$.

(ii) V is positive defined on U_{x_0}: $V(x_0) = 0$ and $V(x) > 0$ for each $x \in U_{x_0}, x \neq x_0$.

(iii) $L_f V(x) < 0$ for each $x \neq x_0,$ $x \in U_{x_0}$, such that $L_g V(x) = 0$, where $L_g V(x) \coloneqq \nabla V(x) \cdot g(x)$ denotes the Lie derivative of V with respect to g, and $L_f V(x)$ is the Lie derivative of V with respect to f.

The pair (f, g) of vector fields f and g given by (16) that satisfies conditions (i)-(iii) is called *a control Lyapunov pair*. If the origin of (15) has CLF, then there exists a control law that renders the system asymptotically stable.

Proposition 1 [10]:

If the system (15) is stabilized by a feedback $u = \alpha(x) + k^T x$, where $k = (k_1, \ldots, k_m)$, $k_i, i = 1, \ldots, m$, are roots of a Hurwitz polynomial p, and α is continuously differentiable on $U_0 \setminus \{0\}$, then the pair (f, g) satisfies the Lyapunov condition (i.e., conditions (i) and (ii) given above) at the origin.

After applying the control law $u = \alpha(x) + k^T x$ to (15), we obtain the system:

$$\dot{x} = f(x) + g(x)(\alpha(x) + k^T x) + h(x)w, \tag{18}$$

with external disturbance input w.

Case 1

Let us assume that the nominal system $\dot{x} = f(x) + g(x)u$ is stabilizable and the CLF for nominal system (17) is known. We assume that for all $x \neq 0$ there is a positive, proper function $V \in \mathbb{R}_+$ such that,

$$\nabla V(x)[f(x) + g(x)u] < 0. \tag{19}$$

Then, this nominal control law must be redesigned to account for disturbance w in the actual system. Let us emphasize that the nominal CLF is chosen independently of any knowledge of the disturbance input matrix $h(x)$. Then after including function $h(x)$ in inequality (19), and to keep system (15) (with disturbance w) stable, function V must satisfy:

$$\nabla V(x)[f(x) + g(x)u + h(x)w] < 0, \forall x \neq 0. \tag{20}$$

Let us assume that CLF describes the kinetic energy of system (14), i.e.,

$$V = \frac{1}{2}(3x_1^2 + 2{x_2}^2 + x_3^2). \tag{21}$$

Then,

$$\nabla V(x)[f(x) + g(x)u + h(x)w] = 3x_1^2 w + 2x_2 x_3 |x_3| + 3x_1 x_2 + x_3 u, \tag{22}$$

and the control law, which fulfils condition (20), is chosen as:

$$u = -\mathrm{sat}(3x_1^2 x_3 + 2x_2|x_3| + 3x_1 x_2 x_3 + x_3 - u_0), \tag{23}$$

where $u_0 = -k_1 x_1 - k_2 x_2$ with k_1, k_2 – roots of some Hurwitz polynomial, and saturation function is given according to saturated control voltage defined as $\mathrm{sat}(v(t))$ in order to enforce the constraint on the maximum voltage allowed.

In this way, one obtains a globally stable closed-loop system with $|x_3| > \xi \ge x_1 x_2$, and for bounded disturbance $w < x_2(x_3^2 - 1)/x_1$, where ξ is a positive design constant. In fact, note that AMB system (14), with non-dimensional variables $[x_1, x_2, x_3]$ given by (13) and for the absolute maximum values of the physical AMB parameters collected in Table 1, is on the stability border. Then, the complementary sensitivity function S for these values also has its maximum value and system (14) is the most sensitive to disturbance w. Therefore, the inequality $w < x_2(x_3^2 - 1)/x_1$ should be met for maximum system variables, and it is easy to check that it holds true if w<0.1377. Then, including the non-dimensionalized value in (13) and maximum value of $|\omega|_{max}$ = 0.0001 [m] (see Table 1), we get that ω<0.00001377. Thus, it is implied that the above inequality is always true.

Note that in this case, the condition: $|x_3| > \xi$ follows from the fact that, in the case of an AMB system operated in zero-bias mode, we need a large change in flux resulting in large voltage commands (7) in order to produce a small control force. Design coefficient ξ is a part of the AMB control system and its value depends on the parameters of the AMB system (which are given in Table 1). The condition $\xi \ge x_1 x_2$ is always met in the flux-controlled AMB.

Case 2

The stabilization problem for system (15) is solved if we can assign negative value to the time derivative of function V, thus the stability condition is given by:

$$L_f V(x) + L_g V(x) + L_h V(x) < 0, \tag{24}$$

where we suppose that function V is given by (21).

Following (24) and for CLF given by (21), with condition: $|x_3| > \xi \ge x_1 x_2$, the stable feedback loop can be written as $L_f V + L_g V + L_h V = 3x_1^2 w + 2x_2 x_3 |x_3| + 3x_1 x_2 + x_3 u$. Then, the second control law is selected as:

$$u = -\mathrm{sat}\left(\frac{1}{2}(-3x_1^2 - 2x_2 x_3|x_3| - 3x_1 x_2 + x_3) - u_0\right), \tag{25}$$

with, as previously, $u_0 = -k_1 x_1 - k_2 x_2$ where k_1, k_2 are roots of some Hurwitz polynomial, and the saturation function in (23) is given according to saturated control voltage defined as $\mathrm{sat}(v(t))$ in order to enforce the constraint on the maximum voltage allowed.

V. NUMERICAL EXAMPLES

This section presents results obtained for AMB system (12) after applying zero-bias flux control with switching scheme (6) and with external disturbance ω. In this way, the first equation of AMB system (12) is replaced with $\frac{d}{dt}x_1 = x_2 + x_1\omega$. The possibilities of compensating for disturbance ω are investigated with control laws (23) and (25). The simplified 1–DOF model of the AMB (given in Fig. 1) was extended by magnetic saturation, coil resistance, voltage saturation and geometrical specifications such as: nominal air gap, number of coil turns over a single pole of the AMB stator, pole area, permeability of air, and electromagnetic force acting angle. The data for these AMB specifications are collected in Table 1. Variable x is the rotor displacement from the centre point (when x=0), and g_0 is the nominal width of the air gap.

Table 1: AMB specification

Symbol	Value	Meaning
$\lvert x\rvert_{max}$ [m]	0.00025	Maximum rotor position
$\lvert \dot{x}\rvert_{max}$ [m/s]	0.05	Maximum speed
$\lvert \phi\rvert_{max}$ [Wb]	0.0005	Maximum control flux
$\lvert \omega\rvert_{max}$ [m]	0.0001	Maximum rotor position disturbance
g_0 [m]	0.00058	Nominal width of air gap
m [kg]	2.5	Rotor mass
N	108	Number of coil turns
R [Ω]	0.5	Coil resistance
A [m^2]	0.0014	Electromagnet pole area
α [deg]	22.5	Electromagnetic force acting angle
Φ_{sat} [Wb]	0.0022	Saturation flux
B_{sat} [T]	1.6	Saturation flux density
v_{sat} [V]	±150	Saturation voltage
i_{sat} [A]	±5	Saturation current

The AMB model detailed above, with dynamics (14) and switching scheme (6), was applied in Matlab/Simulink® software. Numerical simulations were performed for position-flux zero-bias control, for bias flux Φ_0 equalling zero. The system's trajectories and control input are illustrated for the given nonlinear controllers with zero-bias and voltage constraints. For this purpose, the initial conditions are assumed to be as follows: $\{\phi_1(0),\ \phi_2(0)\} = \{0{,}0\}$ and $\{x(0),\ \dot{x}(0),\ \phi(0)\} = \{0, 0, 0\}$. All simulations are performed with optimized gains k_1 and k_2 equal to 0.92 and 9.94, as previously done in work [10]. The amplitude of step disturbance w equals 0.1 [mm] in all simulations.

The AMB system's responses to disturbance w, in zero-bias mode, for selected controller gains: $k_1 = 0.92, 1, 1.5$ and $k_2 = 9.94, 5, 5$ are presented in Fig. 2. Disturbance ω is successfully compensated with zero overshoot where the control voltage amplitude does not exceed 100 [V].

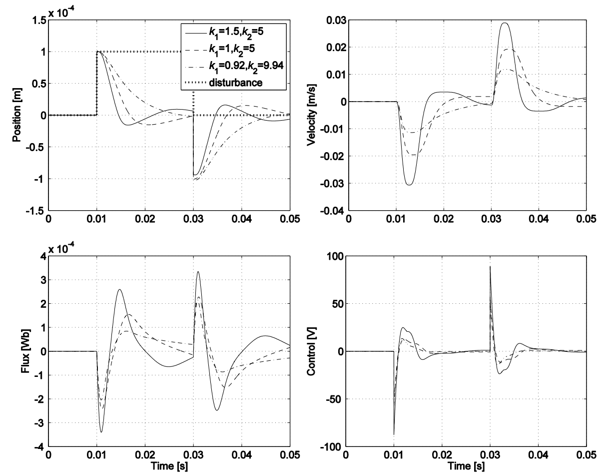

Fig. 2. Responses of closed-loop system with zero-bias to disturbances employing control law (23) for selected gains k_1 and k_2.

Figures 3 and 4 show the results of simulations using control laws (23) and (25), with optimized controller gains: $k_1 = 0.92$ and $k_2 = 9.94$. Figure 3 shows the AMB system's responses to disturbance ω, and Fig. 4 shows voltage v_1, v_2 and flux ϕ_1, ϕ_2 trajectories according to each active electromagnet.

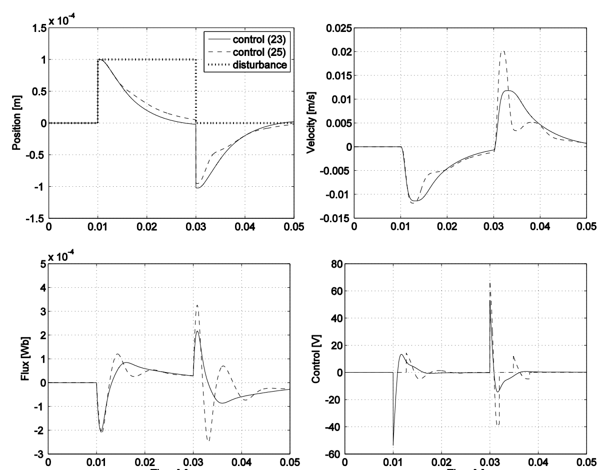

Fig. 3. Comparison of step responses between closed-loop systems employing (23) and (25) controllers for k_1=0.92, k_2=9.94 with zero-bias.

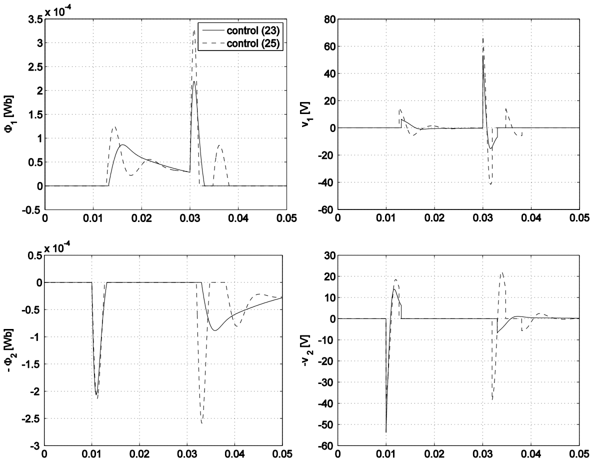

Fig. 4. Voltage switching rule with zero-bias and control laws (23) and (25) for k_1=0.92, k_2=9.94.

Figure 4 presents the idea of the voltage switching strategy used in zero-bias control with CLF and the flux and voltage signal responses to disturbance ω. In the results given (see Figs. 3 and 4), the maximum voltage is about 60 [V], and the settling time is equal to 0.02 [s].

For example, the saturation level in [33] and [35] is set to 30 [V], and in [30] $v_{\max}$=10 [V]. But in [30], the settling time for rotor position is equal to 0.2 [s], which is 10 times longer than in our simulation results. However, in light of the given results, one may conclude that higher values of control voltage lead to shorter settling times. As observed in Figs. 2 and 3, the settling time decreases as voltage saturation level increases, as expected.

VI. CONCLUSIONS

In this paper, nonlinear CLF–based controllers have been proposed and effectively applied to the AMB flux-controlled system with zero-bias and control voltage saturation. Specifically, when using the switching voltage rule with zero-bias operation, one must preclude the singularities present in the control law. The stability of the two designs has been discussed. The desired control performance was achieved despite control voltage saturation. Simulation results have shown that the novel and simple low-order controllers based on CLF gave equivalent results compared to high-order complex control, e.g., based on Artstein-Sontag’s theorem [29] or as given in [30]. The dynamic performance of the proposed control laws as well as the AMB system's responses are similar to the ideal model case.

Future investigations into this topic will focus on a Lyapunov-based nonlinear dynamic output feedback control method for a 5-DOF AMB system. The fabricated test rig of the whole system and its details can be found in work [3, 36]. In the first step, the rotor will be assumed to be rigid for simplicity. In the second step, the nonlinear Lyapunov controller will be considered for control of the 5-DOF flexible rotor.

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