Error Estimators in Self-Adaptive Finite Element Field Calculation

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Abstract: The aim of this paper is to present different error estimates to improve accuracy in linear and nonlinear self- adaptive finite element field calculation. The first estimator is based on the polynomial theory, the second one makes an estimation of the flux density divergence, the third one is linked to a magnetomotive force associated to elements sides, and the fourth one is based on the use of the bilinear element. All methods were implemented in our software named LMAG2D developed at "Escola Politécnica da Universidade de São Paulo", Brazil.

I. INTRODUCTION

The design of an electromagnetic device has always been a hard task for both electrical and electronic engineers. The development of the Finite Element Method (FEM) and Computer Aided Design (CAD) techniques have changed several topics associated to the design of electromagnetic devices.

The Finite Element Method is reliable when the domain is wisely divided and self-adaptive schemes can greatly improve the quality of the mesh.

Self adaptive schemes [1] provide an adequate mesh to analyze the electromagnetic phenomena. The solution of the problem is more accurate, therefore more reliable. Consequently some relevant electromagnetic quantities such as flux, force and torque become more reliable.

A self-adaptive scheme is always based on an error evaluation. Several methods have been proposed to estimate the error on finite element analysis. Usually, the error estimators are based either on complementary methods, or on approximated estimation using field derivatives [2][3].

In this work, four estimators are proposed. The first one based on the polynomial theory is a modification of the estimator proposed by Fernandes et al [3]. This error evaluation was modified so as to allow for non-homogeneous domains. The second method is associated to a local error estimator and makes an evaluation of the flux density divergence for every element within the mesh. The third one is based on the inter-element discontinuity of the magnetic field intensity, when the magnetic potential vector is used. In the fourth method, the error estimation is based on the difference of two unlike fields: one is calculated with first order finite element triangular calculation and the other with a bilinear quadrilateral element.

II THE ERROR ESTIMATORS

All estimators here proposed are associated to a bidimensional magnetostatic problem linked to a finite element model, where the magnetic vector potential is applied.

The first error estimator is based on polynomial interpolation theory. According to Dhatt and Touzot[4], for first order triangular element, the error can be written as:

$$e = C_0 l^2 \mathrm{Max}\left(\frac{\partial^2 A}{\partial x^2};\frac{\partial^2 A}{\partial x \partial y};\frac{\partial^2 A}{\partial y^2}\right) \qquad (1)$$

where C_0 is a constant
A is the magnetic vector potential
l is the element biggest side

The procedure for calculating the second derivative of the potential is similar to the one proposed by Fernandes [3], i.e.,

Step 1: The flux density in each vertex is the average of the flux density vectors in triangles that contain this vertex.

In this calculation the triangles must have the same magnetic property; thus, in non-homogeneous problems, the adopted procedure to evaluate the error is based on a multi-valued flux density at the interfaces.

This change makes Fernandes's error estimation more reliable.

Step 2: The flux density calculated in Step 1 leads the calculation of the second potential derivative by the use of the following hypothesis: the flux density is a linear interpolation of the nodal flux density. This hypothesis is assumed only for error calculation.

The second proposed error estimator is based on the divergence of the flux density. Firstly, the flux density vector is calculated in each node as the average of the flux density vectors in triangles that contain the vertex. The same procedure has already been adopted in the first error estimator (Step 1). To calculate the flux density vector within the triangles, the same shape functions applied to the magnetic vector potential were used. The determination of this vector divergence can be, in the whole domain, assumed as an evaluation for the solution error.

$$e = \mathrm{div}\vec{B} = \mathrm{div}(N_i \vec{B}_i) = \sum_{i=1}^{3} [(\mathrm{grad}N_i) \bullet \vec{B}_i] \qquad (2)$$

Both estimators use the same flux density nodal evaluation, whereas the error evaluation reached by each method is different.

The third proposed method calculates the tangential magnetic field vector discontinuity on the sides of the triangles. Such discontinuity is associated to the solution error when the magnetic potential vector is used.

The discontinuity is only due to the numerical solution and it is linked to a magnetomotive force on the element side, because around this side Ampere's law is not satisfied by the numerical solution.Thus, in each triangle side, there is a magnetomotive force (J_{12}), which can be understood as a "side error" evaluation:

$$J_{12} = |H_{t1} - H_{t2}| l_{12} \qquad (3)$$

On each side of the mesh, Equation (3) shows that side error (J_{12}) is associated to the jump of the tangential component of the magnetic intensity, multiplied by the side length.

In this work, the error is associated to the nodes because this procedure does not require significant changes in the main code, or in the data structure. The nodal error can be written as:

$$e_i = \left(\sum_{j=1}^{NV} (J_{ij}/2) \right) / n_i \qquad (4)$$

Where: NV is the total number of vertex and n_i is the total number of the sides that contain the node i.

So the nodal error e_i is the arithmetic mean of all side errors associated to the node.

The fourth method to evaluate the error is based on the use of a bilinear element. In a first order triangular mesh, a set of quadrilateral elements can be build (Fig.1). Each element has three neighbor triangles, from which it is possible to have three different quadrilateral elements built. If, however, two neighbor triangles have different magnetic proprieties, the quadrilateral can not be created, for the quadrilateral element must have only one magnetic propriety.

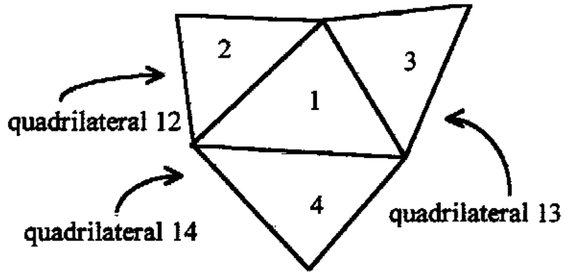

Figure 1 Three neighbor triangles

The local error can be written as:

$$e = \int_{\Omega} \left| \vec{B} - \vec{B}_q \right| d\Omega \qquad (5)$$

where $\vec{B}$ is the flux density, calculated by the finite element method, using a first order triangular element; $\vec{B}_q$ is the flux density, calculated using quadrilateral elements and Ω is one of the quadrilateral elements.

Thus, in the general case, a set of three error values can be computed for each triangular element. The local error in each element is assumed as being the highest value in the set.

For all proposed estimators, the applied adaptive refinement is a combination of a bisection, a Delaunay triangulation and an optimization of the nodal coordinates.

III TEST CASES

The efficacy of an electromagnetic field solution with a self-adaptive procedure can be measured either in cases where the analytic solution is known, or in problems where the numeric solution has been exhaustively tested by an electromagnetic field solution software, or in cases where experimental results are available.

Three test cases were then analyzed. The first one has an analytical solution through conformal transformations. It consists on the L-shaped region as can be seen in Figure 2.

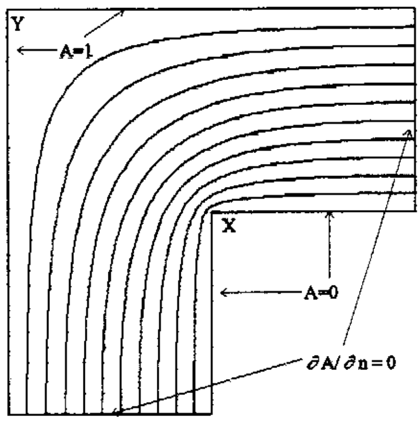

Figure 2- Geometry and Field Distribution for the First Problem

Simkin[5] has proposed a problem, where the main objective is the force computation in an iron part. The geometry of the problem is shown in Figure 3. Lowther [6] suggests solutions to this problems.

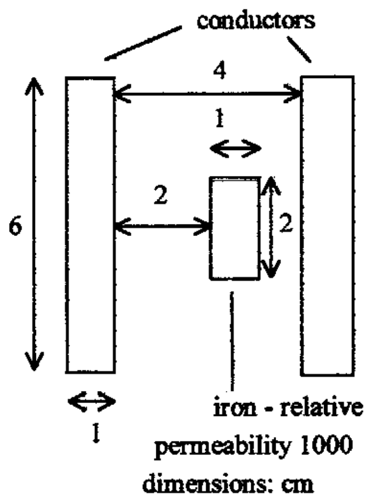

Figure 3 Geometry for the second case

The third case is a 75 kW permanent-magnet electric motor, which has a nonlinear behavior. This case was divided into three different subcases: an open-circuit test, a test to evaluate the inductance per phase, and one on-load condition test.

IV RESULTS

The indicators showed a satisfactory performance, concerning precision and convergence ratio related to local and global quantities for the analyzed cases.

Figure 4 shows the energy convergence of the first case for the first three adaptive proposed procedures. They are also compared to a regular mesh in figure 4 and all estimators have a high convergence ratio and provide a minimum of energy with fewer nodes than the regular method.

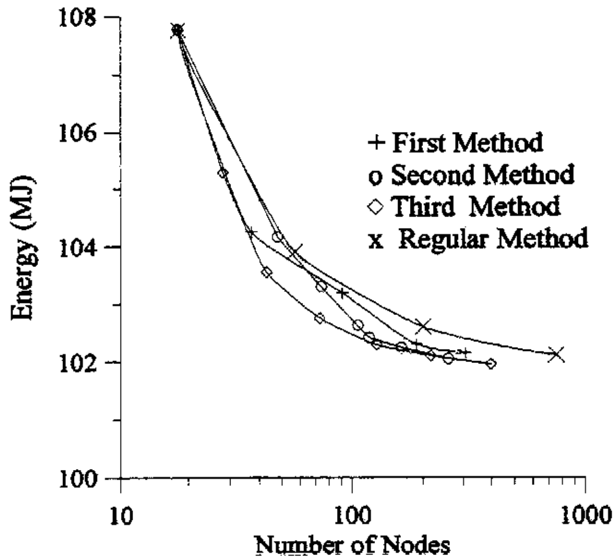

Figure 4 Energy Convergence

To calculate the precision on the magnetic vector potential along XY segment (Figure 2), analytical and numerical solutions are compared, and the deviation between them both was computed. Figure 5 shows the deviation on the magnetic vector potential along this segment, when the estimator based on the divergence is applied. High errors were reached only on the vicinity of the singular point (X) [7].

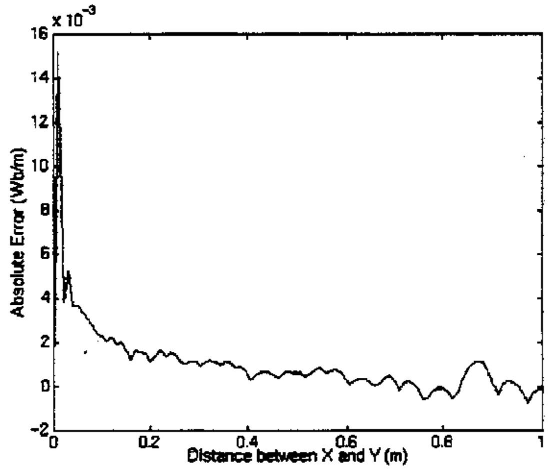

Figure 5 Error on the Potential – Case 1

In the second case, more complex than the first one, satisfactory results were obtained for the calculated force, applying the virtual work principle. Figure 6 shows the field distribution, and Figure 7 shows the obtained mesh when the third method was used.

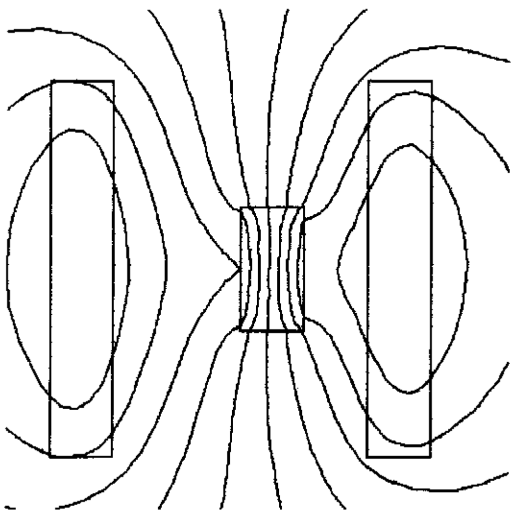

Figure 6 Field Distribution for the Second Case

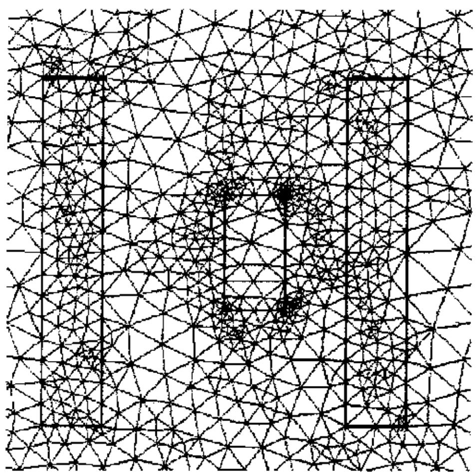

Figure 7 Mesh for the second problem

Table 1 shows some important results linked to the second case.

TABLE 1 COMPARATIVE RESULTS FOR THE SECOND CASE: FORCE CALCULATION

Method	Force (N)	Nodes	Lowther's Result (N)	Iterations
1	6170	3010		8
2	6280	2892	6146	6
3	6130	1445		7
4	6158	2215		6

High accuracy was obtained by methods 1, 3 and 4 because these error estimators identify more elements for refinement on the corners of the iron part and on the conductors.

According to Simkin [5], a refinement on the corners of the ferromagnetic part is the key to achieve a high precision force calculation.

The deviation of method 2 is the highest because the iron-air interface is well divided in triangles, but the conductors and the surrounding air around the part do not have a proper discretization.

The third case presents a nonlinear behavior of the ferromagnetic material. The self-adaptive scheme provides a good mesh and good results, compared to the prototype, for the four error estimators proposed.

Figure 8 shows the field distribution for the PM motor at no-load, using method 1. The self-adaptive scheme provides a minimization of the errors and symmetry can be observed in the figure.

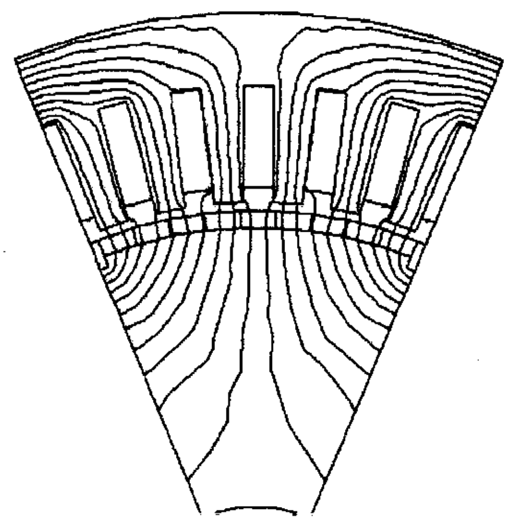

Figure 8 Field Distribution for the PM Motor at no-load

An electromotive force at no-load in an auxiliary winding was calculated and compared to the experimental data and a good agreement was reached. Table 2 shows, for the four estimators, the main data for the self-adaptive processes.

TABLE 2 COMPARATIVE RESULTS: THIRD CASE – E.M.F.

Method	Number of Nodes	Iterations	Calculated e.m.f. (V)	Measured e.m.f. (V)
1	1276	4	3.42	
2	1779	3	3.40	3.61
3	769	6	3.41	
4	1651	4	3.37	

The calculation of the inductance per phase was also performed, and the end-winding inductance was considered using analytical methods. Figure 9 shows the mesh obtained and Figure 10 shows the field distribution, using method 2. There is a high density mesh around the energized conductor and also in the air-gap, because both regions have high density energy.

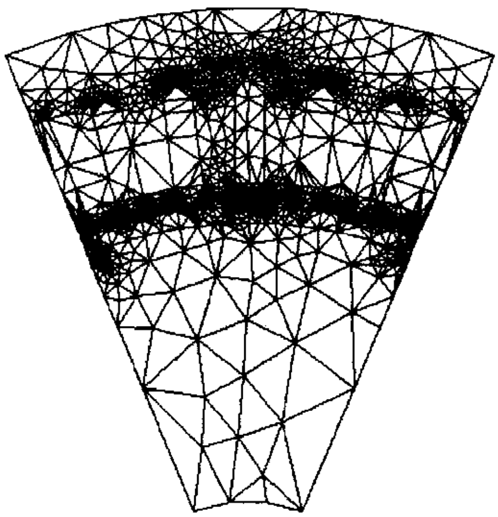

Figure 9 Mesh for the third problem: inductance calculation

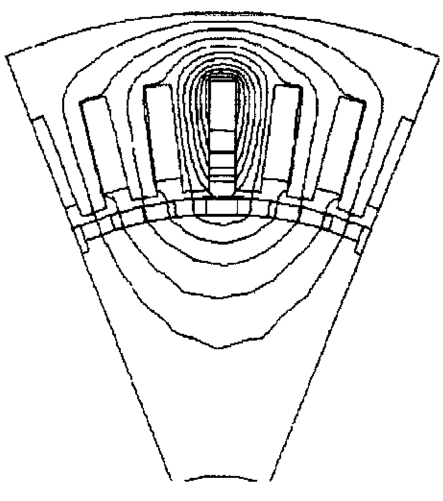

Figure 10 Field Distribution: inductance calculation

Table 3 shows some relevant data related to the self-adaptive processes, using the four proposed estimators. A good agreement between experimental and calculated data was achieved.

TABLE 3 COMPARATIVE RESULTS – INDUCTANCE CALCULATION

Method	Number of Nodes	Iterations	Calculated Inductance (mH)	Measured Inductance (mH)
1	1589	5	7.12	
2	2015	3	7.17	7.09
3	2369	3	7.21	
4	1043	4	7.13	

A calculation of an on-load condition of the PM-motor was performed. Figure 11 shows the obtained mesh, and Figure 12 shows the field distribution when the fourth method was used.

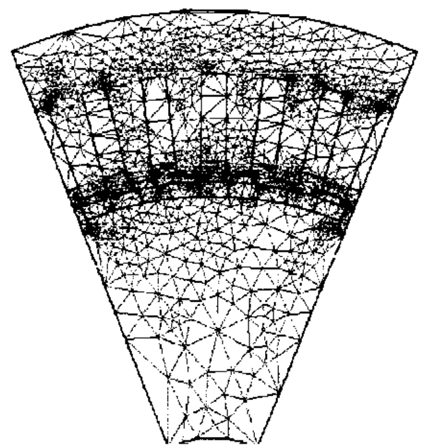

Figure 11 Mesh for the third problem: PM-motor on-load

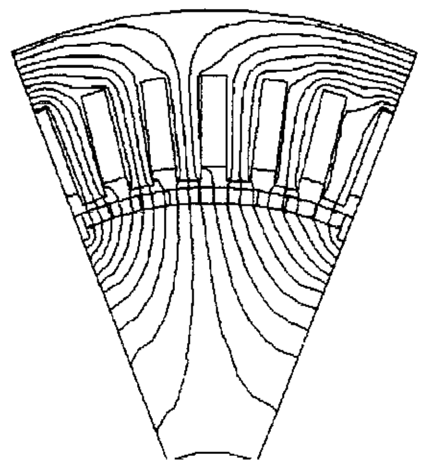

Figure 12 Field distribution for the third problem: PM motor on load

Table 4 shows a comparison between the experimental and computed torque for the four proposed estimators. A good agreement was reached for all methods because the air gap was wisely subdivided in elements. The Finite Element Method can model accurately the armature reaction and the developed torque, only if the air-gap is wisely subdivided.

TABLE 4 COMPARATIVE RESULTS – TORQUE CALCULATION

Method	Nodes	Computed Result (N.m)	Experimental Result (N.m)	Iterations
1	1075	320		4
2	1809	329	328	3
3	1961	332		3
4	1823	327		4

V CONCLUSIONS

This paper analyzes four error estimators. The first and the fourth error estimators provide adequate element mesh to analyze electromagnetic field phenomena.

Relevant to say that the first error estimator implies in shorter CPU times than the fourth one due, to its simplicity. The use of a multi-value density flux at interfaces made it more effective in calculation with heterogeneous media. However, the fourth error estimator is also reliable.

The second and the third error estimator produce a reability that is lower if compared to the other two estimators. Even though, the obtained results show better accuracy for most analyzed cases. The CPU time for this estimators is usually shorter.

Results show that a self-adaptive scheme is a powerful tool to improve accuracy in a finite element field calculation, even with nonlinear cases.

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