A Finite-Difference Frequency Domain Solver for Quasi-TEM Applications

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Abstract **─** A finite-difference frequency-domain (FDFD) solver applicable to quasi-TEM applications is defined. The FDFD solver is applicable to a wide variety of transmission line structures where the cross-sectional dimensions govern the frequency range over which the quasi-TEM approximation is valid. The quasi-TEM FDFD solver provides an efficient solution for conductor current distributions involving both skin effect and proximity effect. Simulation results obtained using the quasi-TEM FDFD solver are compared to measurements and other numerical methods.

Index Terms **─** Finite-difference frequency-domain, quasi-TEM approximation.

I. INTRODUCTION

Low-frequency electromagnetic simulation presents a variety of computational challenges, irrespective of the general solution technique employed. The accuracy of low-frequency solutions using the method of moments (MOM), the finite-element method (FEM) and the finite-difference time-domain (FDTD) technique all suffer due to ill-conditioned matrices for the frequency-domain schemes, and the exorbitant number of time-steps required for time-domain schemes.

A transmission line structure, operating under the quasi-TEM approximation, generates small longitudinal fields internal and external to the transmission line conductors. Under the quasi-TEM approximation, these longitudinal fields are assumed negligible in comparison to the transverse fields, such that the transverse fields are accurately approximated by the true TEM fields of the corresponding static problem. A low-frequency 2D FDFD scheme that exploits the quasi-TEM approximation and avoids the numerical pitfalls described above is defined in the next section.

II. QUASI-TEM FDFD SOLVER

A simple transmission line model is used to demonstrate the formulation of the quasi-TEM FDFD scheme. Non-ideal ($\sigma_c < \infty$) and nonmagnetic ($\mu_c = \mu_o$) transmission line conductors are assumed along with an ideal (μ_i, ϵ_i, $\sigma_i = 0$) insulating medium. Under quasi-TEM operation, the displacement current is assumed to be negligible relative to the conduction current, and the fields are assumed to be invariant in the direction of wave propagation (the transmission line is assumed to be short relative to wavelength).

The transmission line structure carries a true TEM mode only if the conductors are assumed ideal. The transverse fields of the true TEM mode are identical to the static fields for the given conductor configuration. Low-level longitudinal fields are present throughout the system when realistic conductors are assumed, leading to a quasi-TEM mode (designated here as qTEM). According to the quasi-TEM approximation, the qTEM transverse fields are essentially identical to those of the corresponding static fields. Both electrostatic and magnetostatic solutions are not required to determine the complete qTEM transverse fields, however, since the transverse electric and magnetic fields of a TEM mode are related by a simple cross-product relationship. Thus, the electrostatic problem can be solved to determine the transverse electric field, and the resulting transverse magnetic field is found by implementing the TEM cross-product relationship. The qTEM transverse magnetic field on the surface of the non-ideal conductors is equated to the TEM transverse magnetic field on the surface of the ideal conductors, based on the quasi-TEM approximation [1]. One can then determine the surface qTEM magnetic field on the surface of the non-ideal conductors using Maxwell's equations, and solve the governing differential equation for the magnetic field internal to each non-ideal conductor. The basic steps of the FDFD scheme based on the quasi-TEM approximation are summarized as follows.

Step 1. Solve for the quasi-TEM electric field surrounding the conductors, which is approximated by the electrostatic solution, according to:

$$\nabla^2 \tilde{V}_{qTEM} = 0 , \tag{1}$$

and

$$\tilde{\boldsymbol{E}}_{qTEM} = -\nabla \tilde{V}_{qTEM} , \tag{2}$$

where $\tilde{V}_{qTEM}$ and $\tilde{\boldsymbol{E}}_{qTEM}$ are the phasor potential and electric field, respectively, surrounding the conductors.

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Step 2. Determine the quasi-TEM magnetic field surrounding the conductors ($\widetilde{\boldsymbol{H}}_{qTEM}$) according to [1]:

$$\widetilde{\boldsymbol{H}}_{qTEM} = \frac{1}{\eta_i}\hat{\boldsymbol{z}} \times \widetilde{\boldsymbol{E}}_{qTEM} , \tag{3}$$

where η_i is the wave impedance of the insulating medium surrounding the conductors and $\hat{\boldsymbol{z}}$ is the unit normal to the transverse plane.

Step 3. Use the quasi-TEM magnetic field surface boundary condition to solve the governing magnetic field PDE internal to each non-ideal conductor via FDFD as given by:

$$\nabla^2 \widetilde{\boldsymbol{H}}_{qTEM} - \gamma_{\mathbf{c}}^{\mathbf{2}} \widetilde{\boldsymbol{H}}_{qTEM} = 0 , \tag{4}$$

$$\gamma_{\mathbf{c}}^{\mathbf{2}} = j\omega\mu_o(\sigma_c + j\omega\varepsilon_c) \approx j\omega\mu_o\sigma_c , \tag{5}$$

where γ_c is the propagation constant of the respective conductor.

The aforementioned scheme accounts for both proximity effect and skin effects. The transverse components of the qTEM magnetic field in step 2 account for the proximity effects for the conductor configuration. These transverse qTEM fields are large in regions where the currents tend to crowd on the surface of the transmission line conductors. The governing PDE of the non-ideal conductor accounts for the skin effect within the conductor at the given frequency of operation, based on the proximity effect embedded in the surface field boundary conditions.

III. RESULTS

The qTEM FDFD technique is applied to an isolated square copper conductor of 2.54cm×2.54cm cross-section, in order to compare results with computed and measured values in a case where only skin effect is present. The ratio of the AC resistance (R_{ac}) to DC resistance (R_{dc}) is computed via qTEM FDFD and compared to the measured result. The AC resistance determined using qTEM FDFD is based on the conductor current distribution, and is given by:

$$R_{ac} = \frac{1}{\sigma_c} \frac{\iint \left|\widetilde{\boldsymbol{J}}_{qTEM}\right|^2 dxdy}{\left|\iint \widetilde{\boldsymbol{J}}_{qTEM}\, dxdy\right|^2} . \tag{6}$$

The value of R_{ac}/R_{dc}=1.72 computed using qTEM FDFD agrees favorably with the measured result of R_{ac}/R_{dc}=1.75 from [2]. The axial electric field distributions determined via qTEM FDFD and MOM [3] also agree well, as shown in Fig. 1. It should be noted that the qTEM FDFD results are obtained using a simulation model defined by the square conductor within a large enclosure (side length = 25.4 cm) as opposed to an isolated conductor.

The qTEM FDTD solver is applied to two diverse applications having similar skin effect and proximity effect characteristics, based on comparable quasi-TEM properties. A bus conductor in proximity to its enclosure (low frequency) and a microstrip conductor in proximity to its ground plane (high frequency) are shown in Fig. 2. The bus frequency is chosen such that the bus conductor dimensions and skin depth scale by the same constant (3175) relative to the corresponding values for the microstrip. Given that the conductor shapes, aspect ratios and spacings exhibit the "principle of similitude" as defined in [2], the large low frequency bus and the small high frequency microstrip should result in identical values for the R_{ac}/R_{dc} ratio.

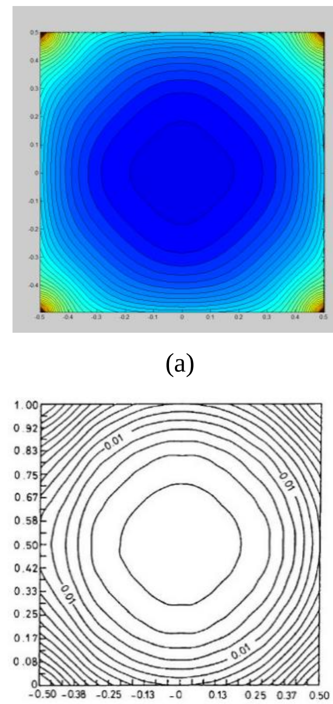


Fig. 1. Comparison of the axial electric field for a 2.54×2.54cm copper conductor using: (a) qTEM FDTD (R_{ac}/R_{dc} = 1.72), and (b) method of moments [3]. The measured value of R_{ac}/R_{dc} = 1.75 [2].

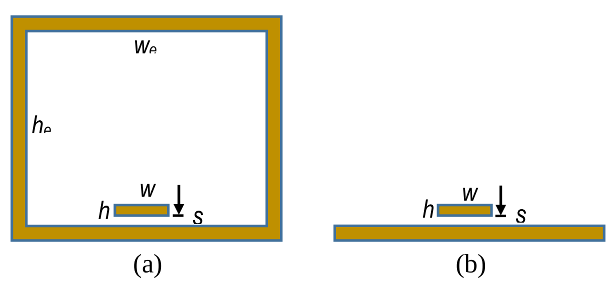
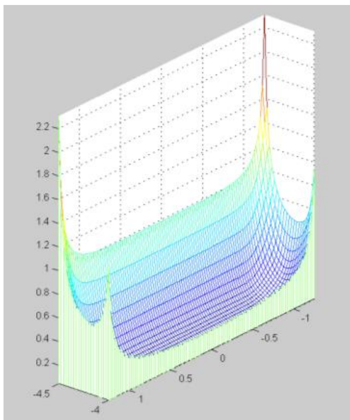
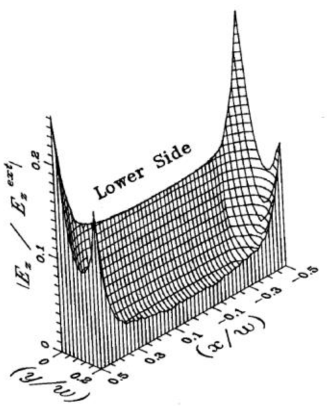


Fig. 2. (a) Bus conductor in proximity to the wall of a large enclosure (copper conductors, h = 1.27cm, w/h = w/s = 5, f = 689 Hz), (b) microstrip conductor in proximity to its ground plane (gold conductors, h = 4μm, w/h = w/s = 5, f = 9.83 GHz).

The bus configuration of Fig. 2 (a) was analyzed using the quasi-TEM FDFD scheme and compared with results for the microstrip configuration of Fig. 2 (b), which was analyzed in [4] using the method of moments. The current/field distributions and conductor resistance ratios R_{ac}/R_{dc} given in Fig. 3 are shown to agree well for the bus conductor at 689 Hz and the microstrip conductor at 9.83 GHz.



(a)



(b)

Fig. 3. Axial electric field for: (a) the bus conductor of Fig. 2 (a) (qTEM FDTD, R_{ac}/R_{dc} = 3.05), and (b) the microstrip conductor of Fig. 2 (b) (method of moments [4], R_{ac}/R_{dc} = 2.95).

VI. CONCLUSION

A two-dimensional quasi-TEM FDFD scheme has been formulated which is simple to implement for transmission line geometries and computationally efficient. The fundamental physics of proximity and skin effects in transmission lines have been demonstrated through comparisons with experimental measurements and results from other computational techniques.

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