# On the Correction of the Probe Positioning Errors in a Non-Redundant Bi-Polar Near to Far-Field Transformation

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***Abstract*** **─** An effective procedure allowing one to correct the positioning errors in a bi-polar near to far-field transformation (NTFFT) technique, that requires a minimum number of near-field (NF) data, has been here assessed from the experimental viewpoint. This NTFFT utilizes an optimal sampling interpolation formula, got by considering the antenna under test as contained in an oblate spheroid and applying the non-redundant sampling representation to the probe measured voltage, to precisely determine the NF data needed by the standard NTFFT with plane-rectangular scan from the voltages at the points prescribed by the sampling representation. These voltages are not known and are accurately recovered from the positioning errors affected measured ones by applying an efficient singular value decomposition based technique.

***Index Terms*** **─** Antenna measurements, bi-polar near to far-field transformation, non-redundant sampling representations, positioning errors correction.

## I. INTRODUCTION

As well-known, the far-field (FF) distance requirements cannot be satisfied in an anechoic chamber when the dimensions of the antenna under test (AUT) are large with respect to the wavelength. In this case, only near-field (NF) measurements can be performed and, accordingly, the AUT radiated far field must be reconstructed by applying near to far-field transformation (NTFFT) techniques [1-4]. These techniques generally exploit an expansion of the AUT near field in plane, cylindrical, or spherical waves, whose expansion coefficients are determined from the complex voltages acquired by the probe and rotated probe on a proper lattice of the scanning surface, that will be a plane, a cylinder, or a sphere, respectively. Then, the AUT far field is reconstructed by substituting the so determined coefficients in the corresponding wave expansion valid in the FF region. When the AUT radiates a pencil beam pattern, a NTFFT technique using a planar scan, such as the plane-rectangular (PR) [5, 6], the plane-polar (PP) [7-10], or the bi-polar (BP) scan [11-15], is usually adopted. The PR NTFFT is surely the most simple of them from the analytical and computational points of view. The PP NTFFT has several advantages compared to the PR one, such as a larger scanning zone for a fixed size of the anechoic chamber, a mechanically simpler scanning system, and etc. [7]. Even more convenient is that adopting the BP scanning. In fact, this scanning retains the benefits of the PP one, but makes use only of rotational motions and, as well-known, turntables provide a greater accuracy with respect to linear positioners. Moreover in the horizontal mounting, it makes easier to preserve the planarity, since the probe is attached to one of the extremities of the arm, whose other end is anchored to the rotator, so that the bending of the arm does not change throughout the acquisition. It is worth noting that the NF data necessary to reconstruct the antenna far field using the standard PR NTFFT were accurately recovered from the collected BP ones in [11, 12] by employing optimal sampling interpolation (OSI) expansions. However, such an approach did not exploit the non-redundant sampling representations of electromagnetic (EM) fields [16, 17] so that an unnecessarily large number of NF data was required. Conversely, by assuming the antenna as contained in an oblate spheroid or in a double bowl (a surface made by two bowls having the same aperture), these representations have been applied in [13-15] to the voltage detected by the scanning probe, thus making available 2-D OSI formulas, which allow one to accurately recover the needed PR NF data from a minimum number of the BP ones.

Unfortunately, as a consequence of a not precise control of the positioniers as well as of their finite resolution power, it could not be possible to collect the NF data at the points established by the non-redundant sampling representation. Anyhow, the actual locations of the collected NF data can be accurately determined by means of laser interferometric techniques. For this reason, the availability of an effective and robust technique, enabling the accurate retrieval of the NF data necessary to execute the traditional PR NTFFT from the inaccurately positioned (non-uniform) BP ones, results

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to be of primary importance. A procedure based on the conjugate gradient iteration technique and exploiting the fast Fourier transform for non-equispaced data [18] has been adopted for correcting known errors of probe positioning in the classical NTFFTs with planar [19] and spherical [20] scans. Such a procedure is, in any case, not tailored to the non-redundant BP NTFFTs [13-15]. As stressed in [21], wherein a more comprehensive analysis on the non-uniform sampling is reported, the direct recovery of the NF data required to carry out the NTFFT from the not evenly distributed ones is not opportune. A suitable and viable strategy [21] is to first retrieve the evenly distributed (uniform) samples from the non-uniform ones and afterward evaluate the necessary NF data via a precise and robust OSI expansion. Two diverse approaches have been developed for achieving such a purpose. The former makes use of an iterative technique, which turns out to be convergent only if a biunique relation linking each uniform sampling point to the nearest non-uniform one exists, and has been exploited to retrieve the uniform samples in a PR grid [21]. The latter, that does not suffer from the above shortcoming, employs the singular value decomposition (SVD) method and has been adopted to retrieve the uniform samples from the inaccurately positioned ones in the non-redundant NTFFTs with PP [22], cylindrical [23], and BP [24] scannings. Anyhow, to usefully apply this last procedure, it is necessary that the uniform samples recovery can be split into two separate 1-D problems, otherwise a remarkable computational effort is required owing to the large dimensions of the involved matrix. These procedures have been compared via simulations and experimentally validated with reference to a spherical NTFFT using a minimum number of NF data [25], while their experimental assessment in the NTFFTs with the cylindrical and the PP scannings has been provided in [26] and [27, 28], respectively.

Goal of the paper is to give the experimental validation of the SVD based approach [24], which allows one to correct known probe-positioning errors in the non-redundant BP NTFFT adopting an oblate spheroidal surface to model a quasi-planar AUT (see Fig. 1).

## II. NON-REDUNDANT VOLTAGE REPRESENTATION ON A PLANE FROM NON-UNIFORM BP SAMPLES

### A. Uniform samples representation

An efficient representation of the voltage, measured on a plane $d$ away from the AUT by a probe with a non-directive pattern, using a non-redundant number of its BP samples is summarized in this subsection. In the following, a generic observation point is specified by the spherical coordinates ($r$, $\vartheta$, $\varphi$), while a point $P$ belonging to the plane can be also identified by the BP ones ($\alpha$, $\delta$), where $\delta$ is the rotation angle of the BP arm and $\alpha$ that of the AUT (Fig. 1). As it can be easily shown, the following relations link the polar coordinates ($\rho$, $\varphi$) to the BP ones:

$$\rho = 2L\sin(\delta/2); \qquad \varphi = \alpha - \delta/2 \, , \tag{1}$$

where $L$ is the arm length. As shown in [29], the voltage acquired by a non-directive probe is characterized by practically the same spatial bandwidth of the AUT EM field and, hence, the non-redundant sampling representations of EM fields [16] can be conveniently applied to it. Accordingly, the AUT must be modeled by a rotational surface $\Sigma$, which bounds a convex domain containing it and fits well its shape, the scanning plane must be represented by means of rings and diameters (as in the PP case), an optimal parameterization $\eta$ has to be employed to describe each of these curves, and an appropriate phase factor $\mathrm{e}^{-\mathrm{j}\psi(\eta)}$ must be extracted from the acquired voltage. The so introduced "reduced voltage":

$$\tilde{V}(\eta) = V(\eta)\,\mathrm{e}^{\mathrm{j}\psi(\eta)}, \tag{2}$$

wherein $V$ denotes the voltage $V_\alpha$ or $V_\delta$ collected by the probe or by the rotated probe, is spatially almost bandlimited to $W_\eta$ [16]. The error made when approximating it by a function bandlimited to $\chi' W_\eta$ results to be negligible when an appropriate excess bandwidth factor $\chi' > 1$ is chosen [16]. Since the antennas characterized in a BP NF facility typically have a quasi-planar geometry, an oblate spheroid (with semi-minor and semi-major axes equal to $b$ and $a$) can be suitably adopted as modeling surface $\Sigma$. In this case, the bandwidth $W_\eta$, the parameter $\eta$, and the function $\psi$ relevant to a diameter are [13, 14]:

$$W_\eta = \frac{2\beta a}{\pi} E\left(\frac{\pi}{2}\,|\varepsilon^2\right); \quad \eta = \frac{\pi E\left(\sin^{-1}u\,|\,\varepsilon^2\right)}{2E\left(\pi/2\,|\,\varepsilon^2\right)}, \tag{3}$$

$$\psi = \beta a\left[v\sqrt{\frac{v^2-1}{v^2-\varepsilon^2}} - E\left(\cos^{-1}\sqrt{\frac{1-\varepsilon^2}{v^2-\varepsilon^2}}\;\Big|\;\varepsilon^2\right)\right], \tag{4}$$

wherein $\beta$ is the free-space wavenumber, $E(\bullet\,|\,\bullet)$ the second kind elliptic integral, $\varepsilon = f/a$ the eccentricity of the spheroid, $2f$ its focal distance, and $u = (r_1 - r_2)/2f$, $v = (r_1 + r_2)/2a$ the elliptic coordinates, $r_{1,2}$ being the distances between the observation point $P$ and the foci.

When the considered curve is a ring, the angle $\varphi$ can be properly adopted as optimal parameter, the phase function $\psi$ is constant, and the bandwidth $W_\varphi$ is [13, 14]:

$$W_\varphi(\eta) = \beta a \sin\vartheta_\infty(\eta)\,, \tag{5}$$

$\vartheta_\infty = \sin^{-1}u$ being the angle between the asymptote to the hyperbola through the point $P$ and the $z$-axis.

The following OSI expansion,

$$V(\eta(\rho),\varphi) = \mathrm{e}^{-\mathrm{j}\psi(\eta)} \sum_{n=n_0-q+1}^{n_0+q} \tilde{V}(\eta_n,\varphi)\; S(\eta,\eta_n,\bar{\eta},N,N''), \tag{6}$$

allows the fast and accurate evaluation of the voltage $V$

at any point $P(\rho,\ \varphi)$ on the plane. In (6), $n_0 = n_0(\eta) = \lfloor \eta/\Delta\eta \rfloor$, $2q$ is the number of the retained nearest intermediate samples $\tilde{V}(\eta_n, \varphi)$, namely, the reduced voltages at the intersections of the sampling rings with the diameter through $P$:

$$\eta_n = n\Delta\eta = 2\pi n/(2N''+1), \tag{7}$$

$$N'' = \lfloor \chi N' \rfloor + 1; \quad N' = \lfloor \chi' W_\eta \rfloor + 1, \tag{8}$$

$\lfloor x \rfloor$ stays for the greatest integer less than or equal to $x$, and $\chi$ is an oversampling factor necessary for the control of the truncation error [16]. Moreover,

$$S(\eta, \eta_n, \bar{\eta}, N, N'') = \Omega_N(\eta - \eta_n, \bar{\eta}) D_{N''}(\eta - \eta_n), \tag{9}$$

is the interpolation function of the OSI expansion, with

$$\Omega_N(\eta, \bar{\eta}) = \frac{T_N\left[2\cos^2(\eta/2)/\cos^2(\bar{\eta}/2) - 1\right]}{T_N\left[2/\cos^2(\bar{\eta}/2) - 1\right]}, \tag{10}$$

and

$$D_{N''}(\eta) = \frac{\sin[(2N''+1)\eta/2]}{(2N''+1)\sin(\eta/2)}, \tag{11}$$

being the Tschebyscheff and Dirichlet sampling functions [16]. In (10), $T_N(\eta)$ is the Tschebyscheff polynomial of degree $N = N'' - N'$ and $\bar{\eta} = q\Delta\eta$.

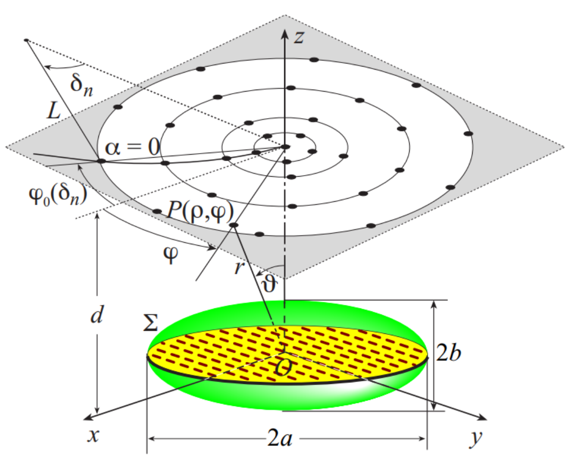


Fig. 1. BP scanning for a quasi-planar AUT.

A quite similar OSI expansion along the rings allows the effective evaluation of the intermediate samples. As shown in [13, 14], it results:

$$\tilde{V}(\eta_n, \varphi) = \sum_{m=m_0-p+1}^{m_0+p} \tilde{V}(\eta_n, \varphi_{m,n}) S(\varphi, \varphi_{m,n}, \bar{\varphi}_n, M_n, M_n''), \tag{12}$$

where $m_0 = m_0(\varphi) = \lfloor (\varphi - \varphi_0(\eta_n))/\Delta\varphi_n \rfloor$, $2p$ is the retained samples number, and,

$$\varphi_{m,n} = \varphi_0(\eta_n) + m\Delta\varphi_n = -\delta_n/2 + 2\pi m/(2M_n''+1), \tag{13}$$

$$M_n'' = \lfloor \chi M_n' \rfloor + 1; \ M_n' = \lfloor \chi^* W_\varphi(\eta_n) \rfloor + 1; \ \bar{\varphi}_n = p\Delta\varphi_n, \tag{14}$$

$$\chi^* = 1 + (\chi' - 1)[\sin\vartheta_\infty(\eta_n)]^{-2/3}; \ M_n = M_n'' - M_n'. \tag{15}$$

It must be noticed that the shift $\varphi_0(\eta_n)$ is a consequence of the different way of the NF data acquisition with respect to that adopted in the PP scanning (Fig. 1).

By properly matching the 1-D OSI expansions (6) and (12), the 2-D OSI one is easily attained. This last allows the accurate reconstruction of the voltages $V_\alpha$ and $V_\delta$ at the points needed for the classical PR NTFFT [5, 6] falling in the measurement circle. It is worthy to stress that the probe-compensation formulas in [6] (which in the employed reference system take the form reported in [9, 30]) require that the probe axes are kept parallel to the AUT ones during the scanning, so that the probe must properly co-rotate with it. In any case, such a co-rotation can be avoided by performing a "software co-rotation" when a probe whose far field exhibits a first-order $\varphi$ - dependence is utilized. If this is the case, it is possible to determine the voltages $V_y$ and $V_x$, which would be collected when co-rotating the probe and rotated probe, from the knowledge of $V_\alpha$ and $V_\delta$ via the relations:

$$V_y = V_\alpha \cos(\varphi - \delta/2) - V_\delta \sin(\varphi - \delta/2), \tag{16}$$

$$V_x = V_\alpha \sin(\varphi - \delta/2) + V_\delta \cos(\varphi - \delta/2). \tag{17}$$

To this end, an open-ended rectangular waveguide can be conveniently employed as scanning probe. In fact, when the fundamental mode $TE_{10}$ is propagating, the corresponding far field radiated in the forward hemisphere has practically such a $\varphi$-dependence [31].

### B. Uniform samples retrieval

Let us assume that all samples, different from the one at the pole $\rho = 0$, are not evenly distributed on rings, which in turn are irregularly spaced on the plane. This is a reasonable assumption when the BP NF data are collected along the rings by exploiting the AUT rotation and the acquisition ring is changed by rotating the arm. This way of operation is mandatory to profit from the reduction in the number of the required samples on the central rings, when applying the described non-redundant representation. In this hypothesis, the 2-D uniform samples retrieval problem is reduced to the solution of two independent 1-D ones, so that the SVD based technique can be conveniently adopted. The former problem concerns the retrieval of the uniform $2M_k''+1$ reduced voltages samples $\tilde{V}(\xi_k, \varphi_{m,k})$ on any non-uniform ring at $\rho(\xi_k)$ from the knowledge of the $J_k \geq 2M_k''+1$ non-uniform ones $\tilde{V}(\xi_k, \phi_j)$. By applying (12), the non-uniform samples $\tilde{V}(\xi_k, \phi_j)$ are expressed as a function of the unknown uniform ones $\tilde{V}(\xi_k, \varphi_{m,k})$, thus attaining the linear system:

$$\underline{B}\,\underline{X} = \underline{C}, \tag{18}$$

wherein $\underline{C}$ is the vector of the known non-uniform samples, $\underline{X}$ is that of the unknown uniform ones, and $\underline{B}$ is a matrix of dimensions $J_k \times (2M_k''+1)$, whose elements are:

$$b_{jm} = S(\phi_j, \varphi_{m,k}, \bar{\varphi}_k, M_k, M_k''), \tag{19}$$

with $\varphi_{m,k} = m\Delta\varphi_k = 2m\pi/(2M_k''+1)$ and $\bar{\varphi}_k = p\Delta\varphi_k$. Since only the samples nearest to the output point are retained in the OSI expansion, the elements $b_{jm}$ of the matrix $\underline{\underline{B}}$ are zero, when the index $m$ is outside the range $[m_0(\phi_j) - p+1, m_0(\phi_j)+p]$. The best least square approximated solution of (18) is then obtained by applying the SVD method. The latter problem deals with the recovery of the uniform intermediate samples $\tilde{V}(\eta_n, \varphi)$, required by the OSI expansion (6) to evaluate the voltage at $P(\rho, \varphi)$, from the non-uniform ones $\tilde{V}(\xi_k, \varphi)$, obtained using the OSI expansion (12) in correspondence of the intersections between the non-uniform rings and the diameter through $P$. The non-uniform samples are then expressed using (6) in terms of the unknown uniform ones, thus getting a linear system, which is again solved via the SVD method. To avoid the ill-conditioning of the above linear systems, it has been supposed that both the distances from the non-uniform rings to the corresponding uniform ones and those between the non-uniform sampling points and the associated uniform ones on them are less than one half of the related uniform spacings. Moreover, to reduce the computational effort, the same number $N_\varphi$ of uniform PP samples, coincident with that needed for the outer uniform ring, have been retrieved on any non-uniform ring. In this way, the number of systems to be solved is minimized being the samples aligned along the diameters.

The so retrieved PP uniform samples are then interpolated via the OSI expansions (6) and (12) (this last suitably adapted to take into account the lack of the shift $\varphi_0$ and the redundancy of the samples on the rings) to efficiently recover the voltages $V_\alpha$ and $V_\delta$ at the points necessary for the PR NTFFT [5, 6].

## III. EXPERIMENTAL TESTING

The experimental validation of the described approach for correcting known positioning errors, which affect the BP NF data, has been carried out through the PP NF measurement facility existing in the anechoic chamber of the Antenna Characterization Laboratory of the University of Salerno. In this facility, the probe is mounted on a vertical linear positioner and the AUT on a turntable having its axis of rotation perpendicular to the vertical positioner. The measurement of the BP NF data is made possible due to the presence of another turntable, located between the probe and the positioner. A vector network analyzer is utilized to measure the complex voltage acquired by the employed probe, an open-ended WR-90 rectangular waveguide. The AUT considered in the following experimental results is a X-band flat-plate slotted array (AUT1), manufactured by Rantec Microwave Systems Inc., having a roughly circular shape with a radius of about 23 cm and working at 9.3 GHz. It is placed on the plane $z$ = 0 and is modeled by an oblate spheroid with $b$ = 8.1 cm and $a$ = 23.2 cm. The uniform and the non-uniform NF BP samples considered in these results have been acquired on a circle with radius 110 cm on a plane, whose distance from the AUT is 16 cm. In particular, the positions of the uniform sampling points are those required by the previously described non-redundant sampling representation when the BP arm length $L$ is 120 cm and $\chi'$ = $\chi$ = 1.25. These chosen values of $\chi'$ and $\chi$ ensure low aliasing and reconstruction errors. As regards the acquired non-uniform samples, they have been deliberately not regularly spaced along non-uniform rings in such a way that the shifts from the positions of the non-uniform to related uniform rings and those from the non-uniform to associated uniform sampling points are random variables uniformly distributed in $(-\Delta\eta/2, \Delta\eta/2)$ and $(-\Delta\varphi_k/2, \Delta\varphi_k/2)$, respectively, thus avoiding the ill-conditioning of the linear systems involved in the uniform samples retrieval.

The amplitude and phase of the voltage $V_\alpha$ along the diameter at $\varphi = 0°$, recovered through the SVD procedure from the positioning error affected BP samples, are compared in Figs. 2 and 3 with those directly measured (references), whereas the comparison of the retrieved amplitude and phase of $V_\delta$ along the diameter at $\varphi = 90°$ with the directly measured ones are shown in Figs. 4 and 5. A further NF reconstruction example relevant to the comparison of retrieved and measured amplitudes of $V_\alpha$ and $V_\delta$ along the diameter at $\varphi$ = 30° is reported in Fig. 6. In other words, the solid line patterns refer to the voltage acquired at close spacing on the considered diametral lines, whereas those shown with crosses are relevant to the patterns obtained by interpolating, via the 2-D OSI expansion, the uniform NF samples reconstructed from the positioning errors affected ones through the SVD based approach. As can be noticed, notwithstanding the considerable values of the positioning errors, all the recoveries are very precise save for the zones where the voltage levels are very low. It is worthy to note the smoother behavior of the recovered voltages amplitudes, due to the OSI functions features to filter out the noise sources harmonics greater than the AUT spatial bandwidth. The effectiveness of proposed procedure to correct the positioning errors is further validated by com-paring the FF patterns in the principal planes E and H (Figs. 7 and 8) recovered from the non-uniform BP NF data with those obtained from the non-redundant uniform BP NF samples (references). The reconstructed FF patterns attained from the non-uniform BP NF data without applying the positioning errors correction technique are shown, for sake of comparison, in Figs. 9 and 10. As can be clearly seen, they appear severely deteriorated thus confirming the efficacy of the approach. Such an efficacy is even more evident from the comparison between the very low errors in the reconstructed amplitudes reported in Figs. 7 and 8 and the significantly greater ones in Figs. 9 and 10. These errors have been evaluated as differences between the reconstructed and reference amplitudes, normalized to the maximum of the reference patterns and expressed in dB. Other laboratory results, which validate the effectiveness of the developed technique and relevant to a different antenna, are reported in [32].

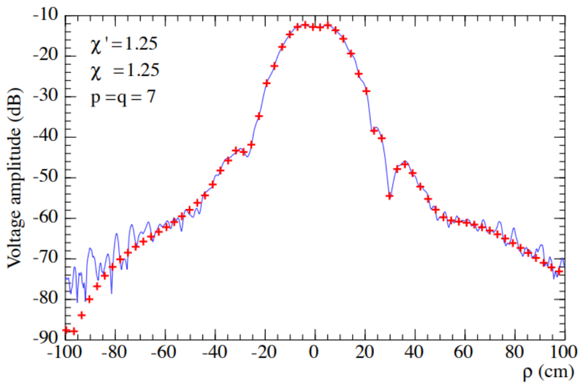


Fig. 2. $V_\alpha$ amplitude along the diameter at $\varphi = 0°$. Line: reference. Crosses: retrieved from the non-uniform BP NF samples.

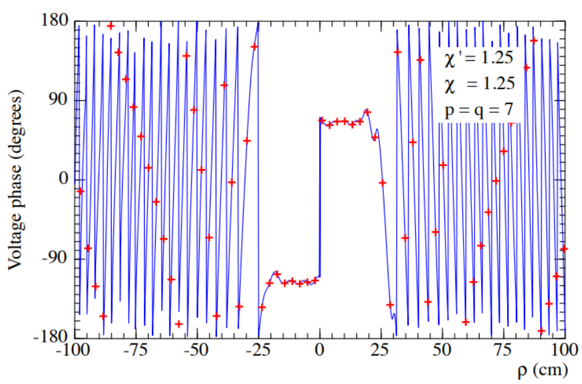


Fig. 3. $V_\alpha$ phase along the diameter at $\varphi = 0°$. Line: reference. Crosses: retrieved from the non-uniform BP NF samples.

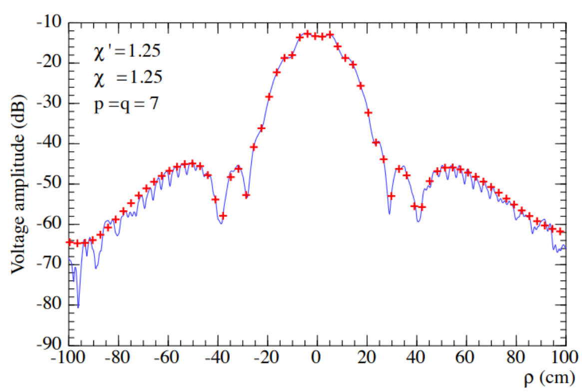


Fig. 4. $V_\delta$ amplitude along the diameter at $\varphi = 90°$. Line: reference. Crosses: retrieved from the non-uniform BP NF samples.

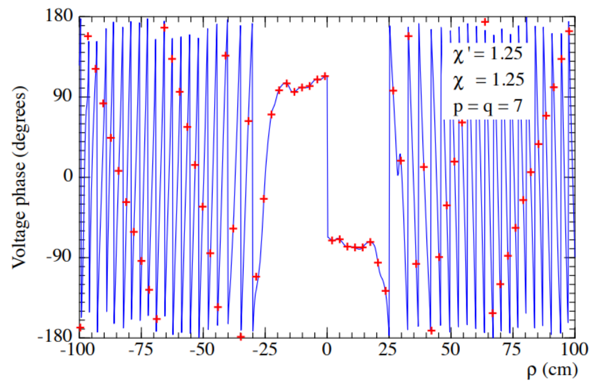


Fig. 5. $V_\delta$ phase along the diameter at $\varphi = 90°$. Line: reference. Crosses: retrieved from the non-uniform BP NF samples.

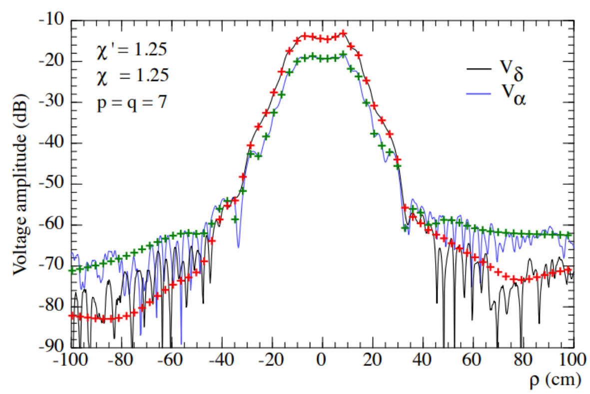


Fig. 6. $V_\alpha$ and $V_\delta$ amplitudes along the diameter at $\varphi = 30°$. Lines: reference. Crosses: retrieved from the non-uniform BP NF samples.

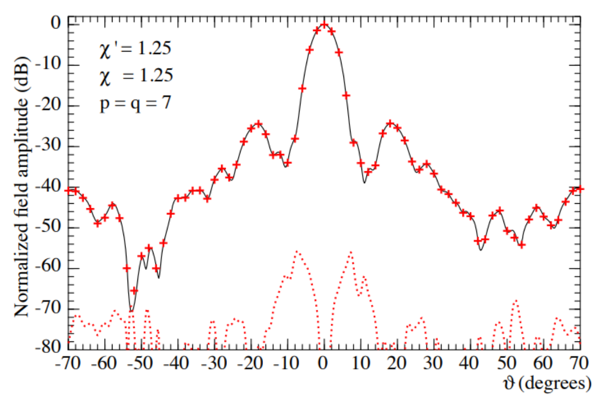


Fig. 7. E-plane pattern. Line: reference. Crosses: obtained from the non-uniform BP NF samples via the SVD procedure. Dashes: reconstruction error.

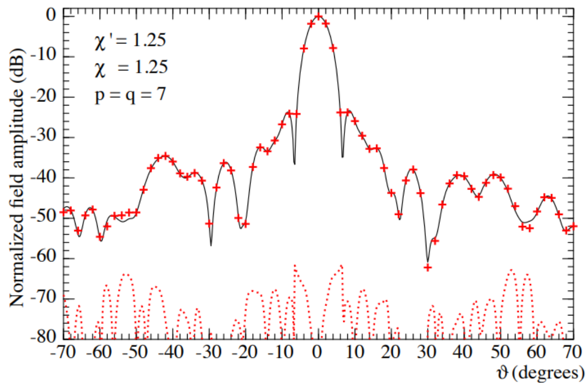


Fig. 8. H-plane pattern. Line: reference. Crosses: obtained from the non-uniform BP NF samples via the SVD procedure. Dashes: reconstruction error.

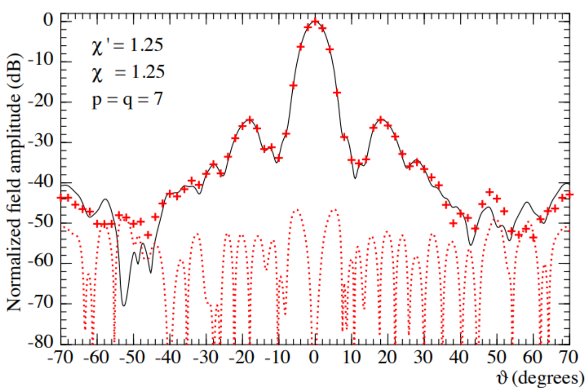


Fig. 9. E-plane pattern. Line: reference. Crosses: obtained from the non-uniform BP NF samples without using the SVD procedure. Dashes: reconstruction error.

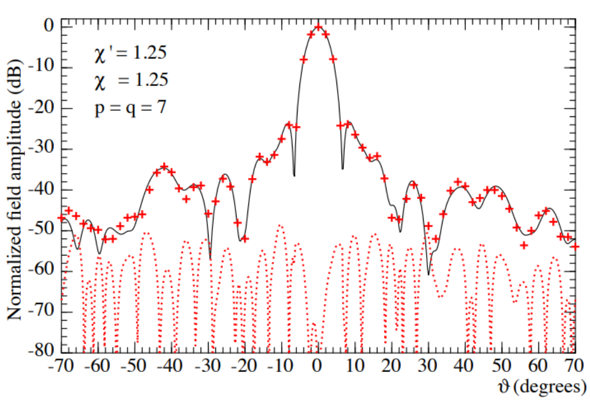


Fig. 10. H-plane pattern. Line: reference. Crosses: obtained from the non-uniform BP NF samples without using the SVD procedure. Dashes: reconstruction error.

A further assessment of the capability of the proposed procedure to compensate also remarkable positioning errors is provided by the experimental results shown in Figs. 11-14, relevant to the recovery of the far field radiated by a dual pyramidal horn antenna (AUT2), polarized in the vertical plane, operating at 10 GHz, and situated on the plane $z$ = 0 of the reference system. The distance between the centers of the horn apertures ($8.9\text{cm} \times 6.8\text{cm}$ sized) is 26.5 cm. This AUT has been modeled by an oblate spheroid with $b$ = 6.3 cm and $a$ = 18.6 cm. Unlike the previous case, $\chi'$ and $\chi$ are 1.35 and 1.25, respectively and the scanning plane distance is 16.5 cm. As can be seen, the reconstructions attained by using the SVD procedure result to be much more accurate than those directly obtained from the non-uniform BP NF samples and exhibit a remarkably smaller reconstruction error.

It is interesting a comparison between the number (2098 for the AUT1 and 1836 for the AUT2) of the acquired BP samples and that (23346 for the AUT1 and 19441 for the AUT2) of the NF data needed by the BP NTFFT [11, 12] for covering the same scanning zone.

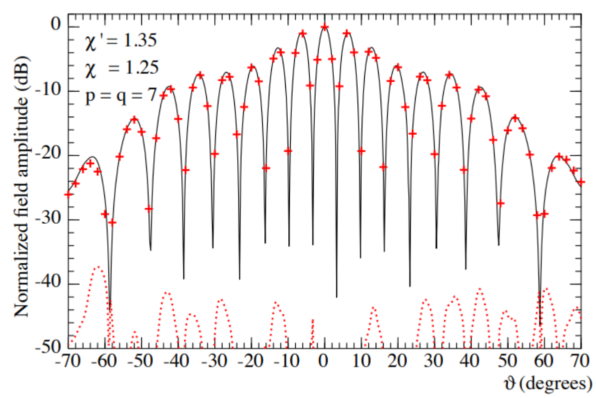


Fig. 11. E-plane pattern (AUT2). Line: reference. Crosses: got from the non-uniform BP NF samples via the SVD procedure. Dashes: reconstruction error.

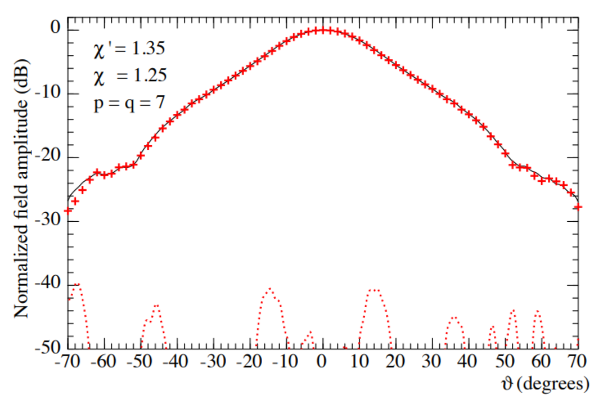


Fig. 12. H-plane pattern (AUT2). Line: reference. Crosses: got from the non-uniform BP NF samples via the SVD procedure. Dashes: reconstruction error.

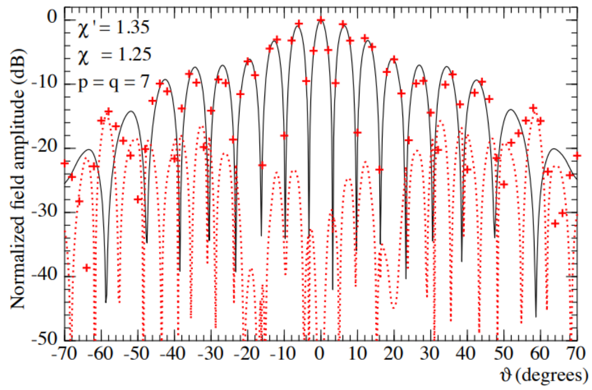


Fig. 13. E-plane pattern (AUT2). Line: reference. Crosses: got from the non-uniform BP NF samples without using the SVD procedure. Dashes: reconstruction error.

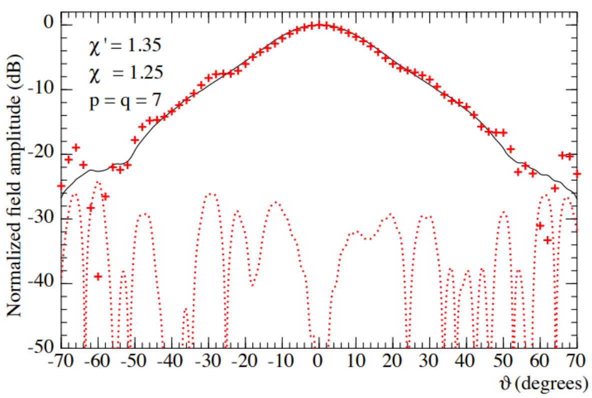


Fig. 14. H-plane pattern (AUT2). Line: reference. Crosses: got from the non-uniform BP NF samples without using the SVD procedure. Dashes: reconstruction error.

## IV. CONCLUSION

In this paper, an efficient SVD based procedure, allowing the correction of known positioning errors in the non-redundant NTFFT with bi-polar scanning adopting an oblate spheroidal AUT modeling, has been further assessed from the experimental viewpoint. Its effectiveness has been confirmed by the accurate NF and FF reconstructions obtained when it is applied even to correct large and pessimistic positioning errors and by the comparison with the severely worsened FF reconstructions directly achieved from the positioning errors affected NF data without exploiting it.

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