LF Sky Wave Propagation Excited by a Horizontal Electric Dipole Towards Understanding of Its Radiation Mechanism

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Abstract **─** In this paper, the electromagnetic pulses have been investigated based on the theory of wave-hop propagation. The approximate formulas are obtained for the electromagnetic field of a vertical magnetic dipole in the presence of an Earth-ionosphere waveguide. Based on the results obtained, the approximate formulas are derived readily for the ground wave and sky waves due to horizontal electric dipole excitation by using a reciprocity theorem. The corresponding computations and discussions are carried out specifically by analogy with those generated by a vertical electric dipole with the formulas in CCIR's recommendation. It is shown that the sky wave pulses would interfere at certain distances and occur by different time-delays.

Index Terms **─** Horizontal electric dipole, LF wave, wave-hop theory.

I. INTRODUCTION

Theoretical investigations of the electromagnetic wave propagation have led to a clearer understanding of radiation mechanism. During the last decades, the electromagnetic fields of vertical and horizontal electric dipoles in the Earth-ionosphere waveguide have been investigated for applications [1,2].

Early in the 1960's, the analytical formulas were derived for the ground wave and sky waves generated by horizontal and vertical electric dipoles in the Earth-ionosphere cavity. The details of the research findings are well summarized in a classic book [3]. It was demonstrated that the total field is a summation of ground wave and sky waves.

In the last decades, many researchers had revisited the problem and some new progresses had been made [4]. Recently, the ELF wave generated by a horizontal electric dipole was treated analytically in the Earth to anisotropic ionosphere cavity in former research by Li et al. [5]. The details of the study and ELF wave propagation in the presence of Earth-ionosphere cavity were summarized in the book by Pan and Li [6].

Based on these investigations, the LF (Low Frequency: from 30Hz to 300Hz) wave propagation is to be investigated in the present study for evaluations caused by a horizontal electric dipole source [7,8].

Despite calculations and measurements of the magnetic induction signal due to electromagnetic pulse had been presented earlier being utilized on radio detection of the radiation [9], it is necessary to investigate the radiation mechanism by analytical approximations. In the treatment of this problem, the transient field is also analyzed by the inverse Fourier transformation where an electric dipole moment is adopted here to represent the radiating source.

According to CCIR's recommendation, it is known that the total field on or near the spherical surface of the Earth is determined primarily by sky wave at long distances [10]. However, the explicit formulas have not been recorded in literature for ground wave and sky waves by horizontal electric dipole excitation. Therefore, the LF wave propagation due to horizontal electric dipole excitation is addressed in the present study.

In what follows, the computational scheme is developed when both the observation point and the transmitting antennas are located on the ground due to a horizontal electric dipole excitation. The influence of radiation mechanism radiation is investigated in time-domain for each ground pulse and sky pulses, respectively. When the observation point and the radiating source are located above the ground, the LF waves in variant of propagating paths are considered by the changes of the height factor.

II. FORMULATIONS

The geometry under consideration is shown in Fig. 1. The horizontal electric dipole representing for the radiating source is located on the upward-directed z-axis at a distance z_s from the origin of coordinates above the spherical Earth.

A. The electromagnetic field excited by a vertical magnetic dipole

Following the same manner in [6], we started by using '$\boldsymbol{V}$' as the magnetic type potential function to represent LF waves which is generated by a vertical

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magnetic dipole. They are:

$$\left.\begin{aligned} E_\varphi &= \frac{-\mathrm{i}\omega\varepsilon_0}{r}\frac{\partial(rV)}{r\partial\theta} \\ H_r &= -\frac{\dfrac{\partial}{\partial\theta}\sin\theta\dfrac{\partial V}{\partial\theta}}{r\sin\theta} \\ H_\theta &= \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}(rV) \end{aligned}\right\}, \tag{1}$$

where the wave equation is written as:

$$\nabla^2 V + k^2 V = 0\,. \tag{2}$$

By the method of separation of variables [11], we can derive the electromagnetic wave vectors in the earth-ionosphere waveguide in air as follows:

$$E_\varphi = E_0'\sqrt{\pi x}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\sum \Lambda_m' G_m(z_s)G_m(z_r)\mathrm{e}^{\mathrm{i}t_m' x}\,, \tag{3}$$

$$\eta H_r = E_0'\sqrt{\pi x}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\sum \Lambda_m' G_m(z_s)G_m(z_r)\mathrm{e}^{\mathrm{i}t_m' x}\,, \tag{4}$$

$$\eta H_\theta = iE_0'\sqrt{\pi x}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}\sum \Lambda_m' G_m(z_s)\frac{\partial G_m(z_r)}{k\partial z}\mathrm{e}^{\mathrm{i}t_m' x}\,, \tag{5}$$

where $x = (ka/2)^{1/3}\theta$, and

$$E_0' = \frac{\omega\mu_0 I\mathrm{d}a}{\lambda a}\frac{\mathrm{e}^{\mathrm{i}ka\theta}}{\sqrt{\theta\sin\theta}} \qquad (\mathrm{V\cdot m^{-1}})\,. \tag{6}$$

Unlike the traditional WKB approximation [12] Airy function is employed here in order to guarantee the higher accuracy of solution at the higher frequencies in LF band. The coefficients are derived correspondingly with the excitation factor expressed by:

$$\Lambda_m' = \left[t_m - (q^{he})^2 - (t_m - (q_i^{he})^2 - y_0) \times \left(\frac{W_1'(t_m) - q^{he}W_1(t_m)}{W_1'(t_m - y_0) + q_i^{he}W_1(t_m - y_0)}\right)^2\right]^{-1}, \tag{7}$$

and the height gain factor defined by:

$$G_t(r) = AW_1(t-y) + BW_2(t-y)\,, \tag{8}$$

where

$$A(t_m) = -\frac{W_2'(t_m - y_0) + q_i^h W_2(t_m - y_0)}{W_1'(t_m - y_0) + q_i^h W_1(t_m - y_0)}\,, \tag{9}$$

$$B(t_m) = -\frac{W_1'(t_m) - q^h W_1(t_m)}{W_2'(t_m) - q^h W_2(t_m)}\,, \tag{10}$$

and t_m are the roots of modal equation of magnetic type $A(t_m)B(t_m)=1$. Airy functions $W_1(t-y)$ and $W_2(t-y)$ are defined by:

$$W_{1,2}'(x) = \frac{1}{\sqrt{\pi}}\int_{\Gamma_{1,2}} \mathrm{e}^{ux - \frac{u^3}{3}}\mathrm{d}u\,. \tag{11}$$

B. Approximate formulas for ground wave and sky waves excited by a horizontal electric dipole

Based on the results derived, the electromagnetic components of LF wave are readily derived by using a reciprocity theorem. The six wave components are summarized as:

$$E_r^{he} = -\frac{iIds^{he}\eta}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}e^{i\frac{\pi}{4}}\cos\varphi \times i\sum\Lambda_n' Z_n(z_s)\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_r}\left[1+\left(\frac{k_0 a}{2}\right)^{2/3}\frac{\mathrm{t}_n}{2}\right]^{-1}e^{it_n x}, \tag{12}$$

$$E_\theta^{he} = -\frac{Ids^{he}\eta}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}e^{i\frac{\pi}{4}}\cos\varphi \times\left\{-i\sum\Lambda_n'\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_r}\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_s}e^{it_n x} + \frac{1}{k_0 a\sin\theta}\sum\Lambda_m'\left.\frac{\partial G_m(z)}{k_0\partial z}\right|_{z=z_r}\left.\frac{\partial G_m(z)}{k_0\partial z}\right|_{z=z_s}e^{it_m x}\right\}, \tag{13}$$

$$E_\varphi^{he} = \frac{iIds^{he}\eta}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}e^{i\frac{\pi}{4}}\sin\varphi \times\left\{\frac{i}{k_0 a\sin\theta}\sum\Lambda_n'\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_r}\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_s}\times\left[1+\left(\frac{k_0 a}{2}\right)^{2/3}\frac{\mathrm{t}_n}{2}\right]^{-1}e^{it_n x} + \sum\Lambda_m'\left.\frac{\partial G_m(z)}{k_0\partial z}\right|_{z=z_r}\left.\frac{\partial G_m(z)}{k_0\partial z}\right|_{z=z_s}e^{it_m x}\right\}, \tag{14}$$

$$H_\theta^{he} = \frac{Ids^{he}}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}e^{i\frac{\pi}{4}}\sin\varphi \times\left\{\frac{i}{k_0 a\sin\theta}\sum\Lambda_n' Z_n(z_r)\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_s}\times\left[1+\left(\frac{k_0 a}{2}\right)^{2/3}\frac{\mathrm{t}_n}{2}\right]^{-1}e^{it_n x} - \sum\Lambda_m'\left.\frac{\partial G_m(z)}{k_0\partial z}\right|_{z=z_r}G_m(z_s)e^{it_m x}\right\}, \tag{15}$$

$$H_\varphi^{he} = -\frac{iIds^{he}}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}e^{i\frac{\pi}{4}}\cos\varphi \times\left\{i\sum\Lambda_n' Z_n(z_r)\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_s}e^{it_n x} + \frac{1}{k_0 a\sin\theta}\sum\Lambda_m'\left.\frac{\partial G_m(z)}{k_0\partial z}\right|_{z=z_r}G_m(z_s)e^{it_m x}\right\}, \tag{16}$$

$$H_r^{he} = -\frac{iIds^h}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}e^{i\frac{\pi}{4}}\cdot\sin\varphi\sum\Lambda_m' G_m(z)G_m(z_0)e^{it_m x}, \tag{17}$$

where the superscript ' he ' designates the horizontal electric dipole for the radiating source.

Therefore, the approximate formulas for the ground wave and sky waves are correspondingly obtained by invoking of Equations (12)-(17) due to a horizontal electric dipole excitation. For simplicity, we take the radial electric field as an example. The ground wave is approximated by the fundamental term in Equation (12) expressed by:

$$E_r = E_0 V^{he}, \tag{18}$$

where

$$E_0 = -\frac{iIds^{he}\eta}{\lambda a}\frac{e^{ika\theta}}{\sqrt{\theta\sin\theta}} \qquad (\mathrm{V}/m), \tag{19}$$

$$V^{he} = \sqrt{\pi x}e^{i\frac{\pi}{4}}\cos\varphi i\sum\Lambda_n^{'}Z_n(z_s)\left.\frac{\partial Z_n(z)}{k_0\partial z}\right|_{z=z_r} \times\left[1+\left(\frac{k_0 a}{2}\right)^{2/3}\frac{t_s}{2}\right]^{-1}, \tag{20}$$

in which $t_s (s=1,2,3,......)$ are the sequential roots of modal equation defined by:

$$W_2^{'}(t_s) - qW_2(t_s) = 0. \tag{21}$$

In the meanwhile, the first two sky waves are expressed by:

$$E_1^{he} = \frac{Ids^{he}\eta}{\lambda}\cos\varphi 2\mathrm{e}^{ik_0 L_1}\cos^2\psi_{1\,//}R_{//}DF_tF_r / L_1, \tag{22}$$

in which L_1 is the total ray length of the first-hop sky wave; ψ_1 is the arriving angle on the ground of the first-hop sky wave; D is the convergence coefficient due to the spherical curvature of the ionosphere; F_t and F_r represent for the antenna background factors of the launch point and observation point, respectively, caused by the earth curvature and the finite conductivity; ${}_{//}R_{//}$ is the ionospheric reflection coefficient where the subscript // indicates that the electric vectors of the incident wave and the reflected wave are parallel to the incident plane, respectively;

$$E_2^{h} = \frac{Ids^{he}\eta}{\lambda}\cos\varphi 2\mathrm{e}^{ik_0 L_2}(\cos\psi_2)^2 D^2 D_g F_t F_r \times\left({}_{//}R_{//}\cdot R_{g//}\cdot{}_{//}R_{//} + {}_{//}R_{\perp}\cdot R_{g\perp}\cdot{}_{\perp}R_{//}\right)/L_2, \tag{23}$$

in which L_2 is the total ray lengths of second-hop sky wave; ψ_2 is the arriving angle on the ground of the second-hop sky wave; D_g is the convergence coefficient due to the spherical curvature of the ground; ${}_{//}R_{\perp}$, ${}_{\perp}R_{//}$ are also the ionospheric reflection coefficients where the pre-subscripts indicate that the electric vector of the incident wave is parallel or perpendicular to the incident plane while the subscripts indicate that of the reflected wave is perpendicular or parallel to the incident plane, respectively.

The computational scheme of the above parameters in this paper is not developed in detail for simplicity which is similar to the case excited by a vertical electric dipole. The related calculation methods are found in the literature [10].

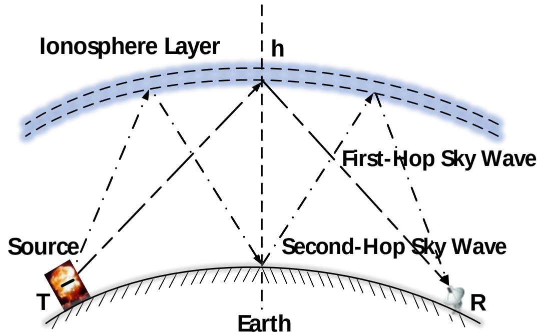


Fig. 1. The geometry of wave propagation routines due to a horizontal electric dipole excitation.

III. COMPUTATIONS AND DISCUSSIONS

Based on Equations (12)-(17), the evaluations are carried out for LF waves generated by the radiating source by horizontal electric dipole excitation in time-domain and frequency-domain, where z_s and z_r represent for the heights of radiating source and receiving point, respectively.

A. Radiation source located on the ground

To illustrate the general formulas for the field components obtained by horizontal electric dipole excitation, the computations are carried out at f = 200 KHz and f = 300 KHz in Fig. 2 for summer daytime where the ionosphere parameters taken as follows: the equivalent height of the ionosphere assumed to be h = 70 km, electron density N = 10^9 m^{-3}, electron collision frequency ν = 10^7 s^{-1}, the ground which is characterized by the relative dielectric constant $\varepsilon_r = 80$, electrical conductivity σ = 5 S·m^{-1} and $z_r = z_s = 0$ km.

It is seen that when the receiving point and the radiating source are placed on the surface of the Earth, the ground wave decays along the propagation path, but for each sky wave the maximum value occurs after a certain propagation distance.

B. Radiation source located above the ground

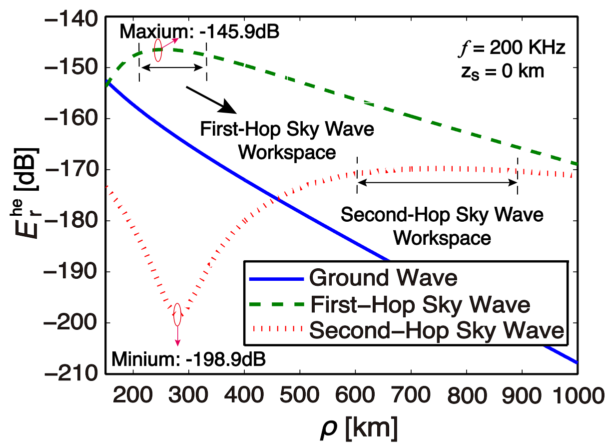
Assuming the receiving point is on the ground and the radiating source is located at a certain distance from the origin of coordinates above the Earth, the computation for the ground wave is similar to that excited by a horizontal electric dipole on the ground except for the enhanced gain factor. But for the sky waves, corrections of angular distances should be additionally taken into consideration by the change of propagation paths.

(1) For the first-hop sky wave, the routine from the radiating source T to the receiving point R in Fig. 3 (a) experiences a reflex by path $\overline{TAP}$ before the hop path $\overline{APR}$. This can be equivalent by the contribution of the image source B which is identical to the source B on the ground in Fig. 3 (b)

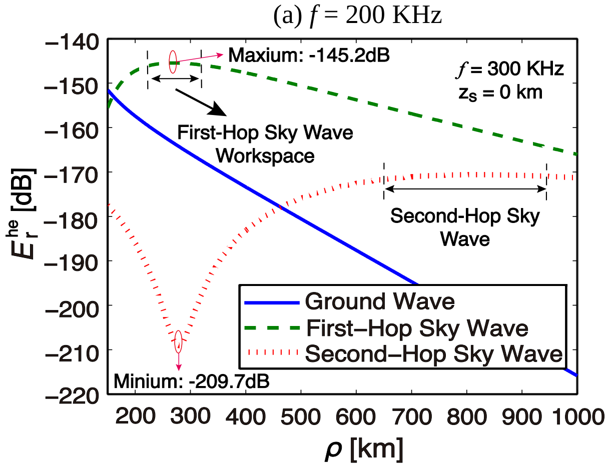
with the equivalent Earth radius |OC|. Considering the equivalent Earth radius |OC| ≈ a, the propagation path is approximated in Fig. 3 (b) with the equivalent angular distance θ. By the triangle similarity of $\overline{PBA}$ and $\overline{PBC}$,

$$\frac{h}{h+d}=\frac{\rho-a\theta_1}{(a-d)(\rho/a+\theta_1)}, \tag{24}$$

$$\theta_1=\frac{\rho d+\rho h d/a}{2ah+ad-hd}. \tag{25}$$



(a) f = 200 KHz



(b) f = 300 KHz

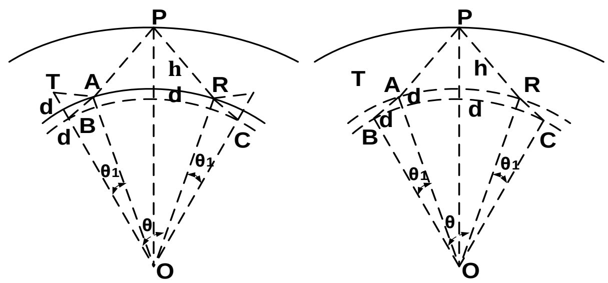
Fig. 2. Amplitudes of the electric field $\left|E_r^{he}(r,\theta)\right|$ in dB versus the propagating distances ρ due to unit horizontal electric dipole excitation; a = 6370 km, ε_r = 80, σ = 5 S·m^{-1}, and $z_r = z_s$ = 0 km; f = 200 KHz, 300 KHz, respectively.

(2) Similarly, for the second-hop sky wave, the propagation path $\overline{TAP_1BP_2R}$ in Fig. 4 (a) can be equivalent by the hop path $\overline{CAP_1BP_2R}$ excited by the image source C. Therefore, the propagation path is approximated in Fig. 4 (b) with the equivalent angular distance θ. By the triangle similarity of $\overline{PBA}$ and $\overline{PBC}$,

$$\frac{h-d}{h}=\frac{(\rho/a+\theta_1/2-2\theta_1)(a+d)}{a(\rho/a+\theta_1/2)}, \tag{26}$$

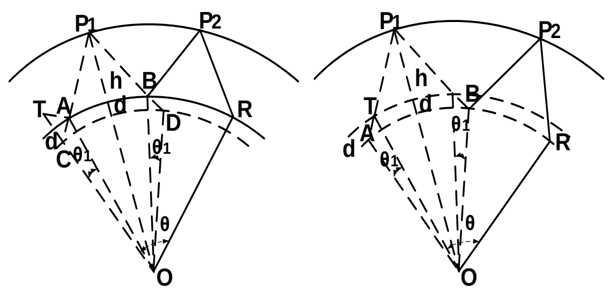
It leads to:

$$\theta_1=\frac{\rho d+\rho h d/a}{4ah-ad+3hd}. \tag{27}$$



(a) First-hop sky wave (b) Equivalent model

Fig. 3. Propagation path for the first-hop sky wave when the receiving point and radiating source are located above the ground.

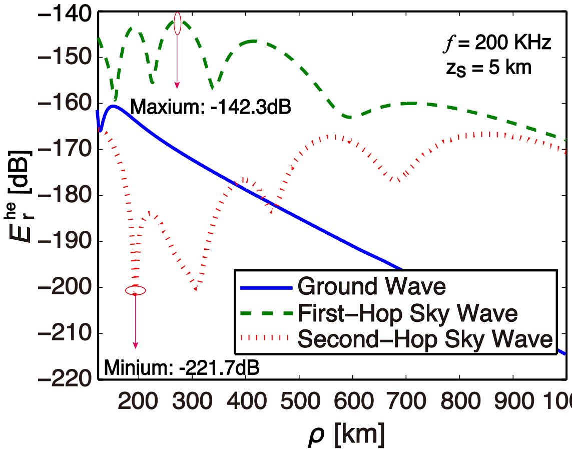


(a) Second-hop sky wave (b) Equivalent model

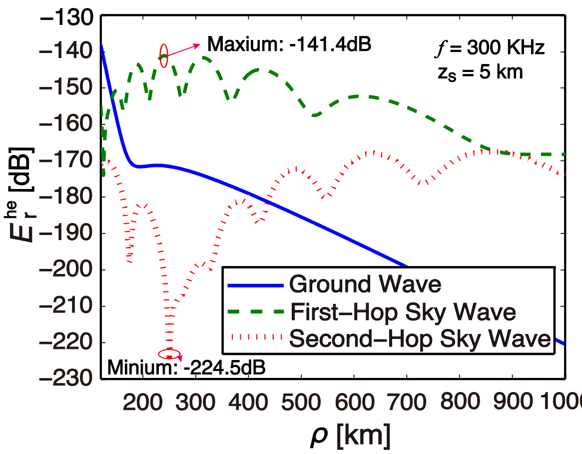
Fig. 4. Propagation path for the second-hop sky wave when the receiving point and radiating source are located above the ground.

(3) For the higher orders of sky waves, the equivalent angular distances will be more complicated to obtain. But for each sky wave vector, limited observable heights are requested for each hop wave due to the altered propagation path. In Table 1, the lowest observable heights are listed for the first-hop sky waves with propagation distances ρ = 100 km, ρ = 200 km, ρ = 300 km, and the radiating heights h = 10 m, h = 20 m, h = 30 m, respectively. It is seen that the lowest observable height restriction for the first hop sky wave is weakened as the increase of radial distance.

In Fig. 5, the electromagnetic field $\left|E_r^{he}(r,\theta)\right|$ are computed and plotted at the radiating height z_s = 5 km, radial distance ρ = 100 km due to unit horizontal electric dipole excitation, at the operating frequency f = 200 KHz and f = 300 KHz, respectively. It is shown that the maximum values and minimum values of the first two hop sky waves occur periodically because of the interferences caused by the enhanced angular distances.



(a) f = 200 KHz; z_s = 5 km

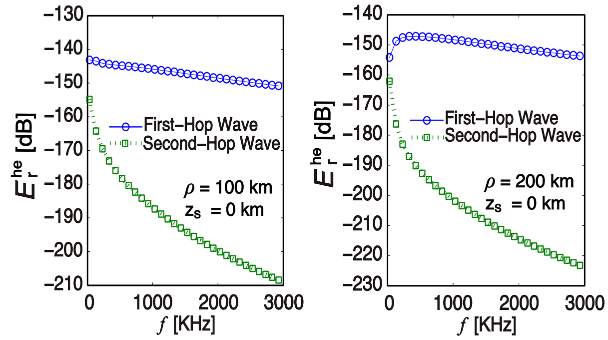


(b) f = 300 KHz; z_s = 5 km

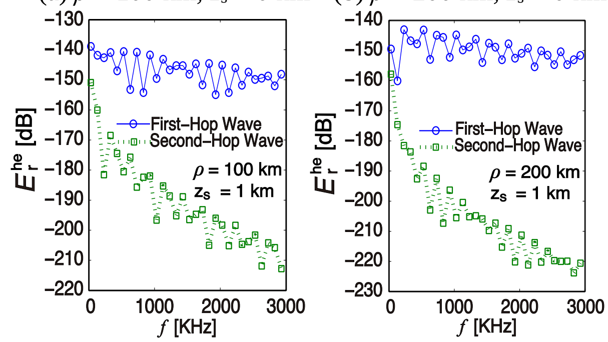
Fig. 5. Amplitudes of the electric field $\left|E_r^{he}(r,\theta)\right|$ in dB versus the propagating distances ρ due to unit horizontal electric dipole excitation; a = 6370 km, ε_r = 80, σ = 5 S·m^{-1}, and z_s = 5 km; f = 200 KHz, 300 KHz, respectively.

C. Frequency-domain response

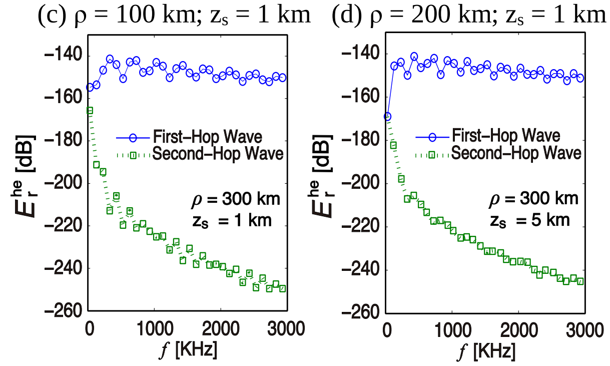
In Fig. 6, the electromagnetic components are evaluated by the variance of operating frequencies, with the radiating heights z_s = 0 km, 1 km, 5 km, and the distances ρ = 100 km, 200 km, 300 km, respectively. The equivalent height of the ionosphere assumed to be h = 90 km for summer day-time, electron density is N = 10^9 m^{-3}, electron collision frequency is $\nu = 10^7$ s^{-1}, the ground which is characterized by the relative dielectric constant ε_r = 80 and electrical conductivity σ = 5 S·m^{-1}. It demonstrates that the radiation efficiency is improved by the enhancement of the radiating height. In the meanwhile, when the radiating source is located above the ground, the curves appear oscillatory decaying by the variance of frequencies.



(a) ρ = 100 km; z_s = 0 km (b) ρ = 200 km; z_s = 0 km



(c) ρ = 100 km; z_s = 1 km (d) ρ = 200 km; z_s = 1 km



(e) ρ = 300 km; z_s = 1 km (f) ρ = 300 km; z_s = 5 km

Fig. 6. The electric field of sky waves due to horizontal unit electric dipole excitation versus the frequency; a = 6370 km, ε_r = 80, σ = 5 S·m^{-1}, and z_r = 0 km; ρ = 100 km, 200 km, 300 km, z_s = 0 km, 1 km, 5 km, respectively.

D. Time-domain response

Assuming that all parameters are same with those in Fig. 6, the characteristics of the transient electromagnetic field generated by a horizontal electric dipole are similar to those of vertical electric dipole case. The transient electric component $\left|E_r^{he}(r,t)\right|$ consisting of ground wave and two sky waves is computed by the inverse Fourier transformation excited by unit horizontal electric dipole and plotted in Figs. 7-8. This demonstrates the fact that excitation efficiency of a vertical dipole is much higher than that of a horizontal electric dipole for ground wave component, whereas with the increase of propagation distance, the sky wave part of the horizontal electric dipole radiation performance is better than vertical electric dipole. Therefore, the radiating source is better described as a superposition of a vertical electric dipole representing for the ground wave and a horizontal electric dipole representing for the sky waves.

In Fig. 9, the ground wave and first two sky wave pulses are computed respectively. It appears that the ground pulse and sky waves arrive at distance ρ = 80 km and ρ = 150 km with different time-delays. Specifically, the time delay for each wave stay longer at the farther propagating distance.

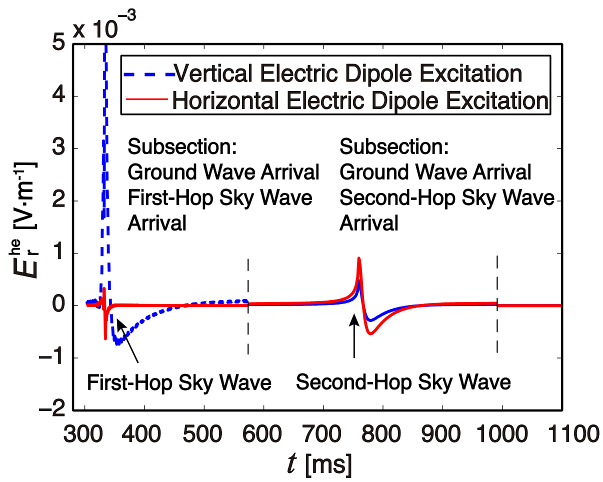


Fig. 7. The transient electric field $\left|E_r^{he}(r,t)\right|$ in V· m^{-1} due to a horizontal unit electric dipole; a = 6370 km, ε_r = 80, σ =5 S · m^{-1}, z_s =0 km, z_r = 0 km, ρ = 100 km, f = 200 KHz.

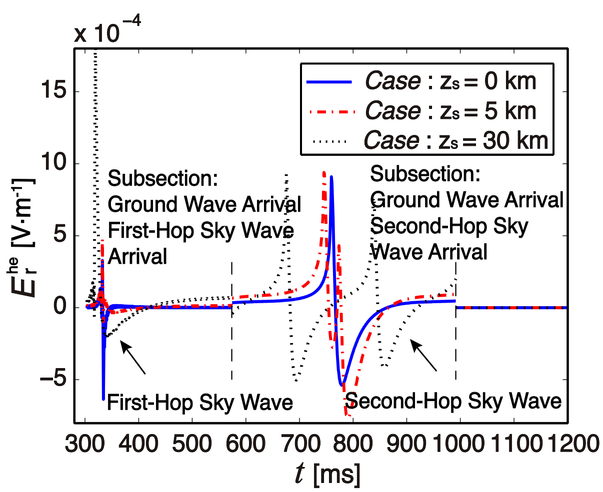


Fig. 8. The transient electric field $\left|E_r^{he}(r,t)\right|$ in V·m^{-1} due to a horizontal unit electric dipole; a = 6370 km, ε_r = 80, σ =5 S·m^{-1}, z_s =0 km, 5 km, 30 km, respectively, z_r = 0 km, ρ = 100 km, f = 200 KHz.

Table 1: Restriction on observing heights for first-hop sky wave

Distance (ρ/km)	HED Source (z_s/m)	Limit Observable Height at Receiving Point (z_r/m)
100	10	8.2777
	20	17.4862
	30	26.8464
200	10	6.7228
	20	15.1516
	30	23.8845
300	10	5.3333
	20	12.9931
	30	21.1103

**The limited observable heights are listed for the first-hop sky waves; for the higher modes of sky waves, the influence of revised angular distances are negligible compared to the first-hop wave. They are not listed.*

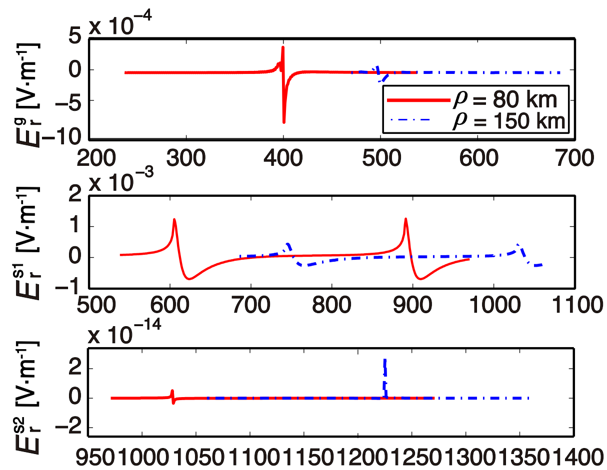


Fig. 9. The transient electric field $\left|E_r^{he}(r,t)\right|$ in V · m^{-1} due to horizontal unit electric dipole excitation; a = 6370 km, ε_r = 80, σ = 5 S · m^{-1}, and z_r = z_s = 0 km; ρ = 80 km, 150 km, respectively.

IV. CONCLUSIONS

The matter of LF sky wave propagation has been investigated by analytical approximation. In the present study, it is provided that: (i) Explicit formulas derived for the evaluations of LF ground wave and sky waves due to a horizontal electric dipole excitation; (ii) When both the observation point and the radiating source are located on the ground, it is demonstrated of high performance of radiation efficiency for ground wave by vertical dipole excitation over a horizontal electric one, while it is opposite for sky waves. (iii) The influence of LF radiation mechanism appears in time-domain by different time-delays for the pulses. (iv) When the observation point and the radiating source are not located on the ground, the LF waves in variant of propagating paths are affected by the change of height factors and angular distances with restricted observable receiving heights for sky waves.

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