Single-Snapshot Time-Domain Direction of Arrival Estimation under Bayesian Group-Sparse Hypothesis and Vector Sensor Antennas

Marco Muzi[1,2], Nicola Tedeschi[1], Luca Scorrano[3], Vincenzo Ferrara[1], and Fabrizio Frezza[1]

[1]Department of Information Engineering, Electronics, and Telecommunications,
"La Sapienza", University of Rome, Via Eudossiana 18, 00184, Rome, Italy.
marco.muzi@uniroma1.it, nicola.tedeschi@uniroma1.it, fabrizio.frezza@uniroma1.it

[2]Engineering departmental faculty, Campus Bio-Medico University of Rome,
Via Alvaro del Portillo 21, 00128, Rome, Italy.

[3]Elettronica Group, Via Tiburtina Valeria Km 13.700, 00131, Rome, Italy.
Luca.Scorrano@elt.it

Abstract **─** In this work, an optimal single-snapshot, time domain, group-sparse optimal Bayesian DOA estimation method is proposed and tested on a vector sensors antenna system. Exploiting the group-sparse property of the DOA and the Bayesian formulation of the estimation problem, we provide a fast and accurate DOA estimation algorithm. The proposed estimation method can be used for different steering matrix formulations since the optimal standardization matrix is computed directly from the knowledge of the steering matrix and noise covariance matrix. Thanks to this, the algorithm does not requires any kind of calibration or human supervision to operate correctly. In the following, we propose the theoretical basis and details about the estimation algorithm and a possible implementation based on FISTA followed by the results of our computer simulations test.

Index Terms **─** Bayesian optimization, DOA estimation, group-sparsity norm, single snapshot signal, vector sensor antennas.

I. INTRODUCTION

Classic Direction of Arrival (DOA) estimation methods, like ESPRIT [1] and MUSIC [2], based on the signal and noise subspaces subdivision, ensure good performances in the cases of long-snapshots data scenarios. However, the performance of such algorithms degenerates into the presence of short-snapshot signals; as well as when the sources are correlated, or in the presence of low SNR.

An interesting alternative to the subspace methods is the Minimum Norm or the Bayesian approaches, especially after the recent development of the compressive sensing theory [3-6]. Indeed, it is possible to explain the unknown parameter probability distributions as the parameter set that maximizes their a-posteriori probability.

The application of the compressive sensing to electromagnetic problems is reported for example in [7-9], where the problem of the DOA identification of a certain number of incoming waves is studied, and a possible resolution method based on single and long snapshot noisy electromagnetic field measurement is proposed.

Most literature on DOA estimation algorithms, takes into account antenna array configurations for the ease of the steering matrix computation. Usually, the array antennas are sensitive only to one polarization, further simplifying the mathematical model.

Despite the analytical model simplification, the antenna array needs a certain space for the installation.

Thanks to the vector sensor antennas, it is possible to obtain an unambiguous DOA estimation through the simultaneous measurement of the electric and magnetic fields, univocally defining the Poynting vector direction.

As it is known, the field measured by a vector sensor antenna depends on the polarization of the impinging wave. Generally, we must take into account the polarization angle or, at least, the horizontal and vertical components of the wave polarization vector. A way to overcome this problem is to consider the horizontal and the vertical polarizations as different cases, allowing the system to be sensitive only to one of them at a time.

The use of a vector sensor antenna allows measuring simultaneously all the electromagnetic field components permitting a DOA estimation regardless the actual field polarization.

In this work, the adoption of a time domain, single snapshot, optimal group-sparse Bayesian algorithm for the DOA estimation problem is proposed, and its performances are reported.

Submitted On: November 30, 2017
Accepted On: July 27, 2018

The choice of the group-sparse solution hypothesis, allows us to simplify the construction of the steering matrix, thanks to the superposition effect of the components of the polarization vector. Indeed, the group sparsity on the problem solution (i.e., the elevation and azimuth angles of the DOA impinging waves) means that the sparsity is imposed on the number of the impinging waves and not on their singular components, as in the case of a simply sparse solution. The main advantage of the group sparsity hypothesis, with respect to the simple sparsity hypothesis, consists in the mitigation of spurious elements in the estimated solution.

II. METHODS

The first step in the DOA estimation is the steering matrix definition. We suppose that the impinging wave is homogeneous and plane, without restraint about the polarization.

We define the steering matrix associated with an ideal vector sensor antenna, with all the elements co-located, able to measure all the six components of the electric and magnetic field according to [10]:

$$A(\theta,\varphi,\gamma,\eta)=\left[\dots,a_k\left(\theta_k,\varphi_k,\gamma_{k,l},\eta_{k,l}\right),\dots\right]=$$
$$\left[\dots,\begin{matrix}\cos(\theta_k)\cos(\varphi_k) & -\sin(\varphi_k)\\ \cos(\theta_k)\sin(\varphi_k) & \cos(\varphi_k)\\ -\sin(\theta_k) & 0\\ -\sin(\varphi_k) & -\cos(\theta_k)\cos(\varphi_k)\\ \cos(\varphi_k) & -\cos(\theta_k)\sin(\varphi_k)\\ 0 & \sin(\theta_k)\end{matrix},\dots\right]\begin{bmatrix}\sin\left(\gamma_{k,l}\right)e^{j\eta_{k,l}}\\ \cos\left(\gamma_{k,l}\right)\end{bmatrix}, \quad (1)$$

where: $a_k\left(\theta_k,\varphi_k,\gamma_{k,l},\eta_{k,l}\right)$ is the k–th steering vector representing the Green function associated to the k–th incoming wave with: elevation angle $\theta_k\in[0,2\pi]$; azimuth angle $\varphi_k\in[0,\pi]$; auxiliary polarization angle $\gamma_{k,l}$ and polarization phase difference $\eta_{k,l}$. In our case, we assume that $\eta_{k,l}$ is equal to zero and $\gamma_{k,l}$ is alternately equal to zero or $\pi/2$ for each k-value: in this way we obtain a group of two steering vectors defining the φ - and θ – wave polarization components for each DOA, obtaining a steering matrix with a number of columns twice the number of DOA taken into account for its construction:

$$A\left(\theta,\varphi,0,\left[0,\frac{\pi}{2}\right]\right)=\left[\dots,a_k\left(\theta_k,\varphi_k,0,\left[0,\frac{\pi}{2}\right]\right),\dots\right]=$$
$$=\left[\dots,\begin{matrix}\cos(\theta_k)\cos(\varphi_k) & -\sin(\varphi_k)\\ \cos(\theta_k)\sin(\varphi_k) & \cos(\varphi_k)\\ -\sin(\theta_k) & 0\\ -\sin(\varphi_k) & -\cos(\theta_k)\cos(\varphi_k)\\ \cos(\varphi_k) & -\cos(\theta_k)\sin(\varphi_k)\\ 0 & \sin(\theta_k)\end{matrix},\dots\right]. \quad (2)$$

The steering matrix obtained from Eq. (2) is purely real, and each column element defines the gain of the system as a function of the wave polarization components and DOA.

Once defined the steering matrix, we can write the measurement model as follows:

$$S(t)=A\left(\theta_k,\varphi_k,0,\left[0,\frac{\pi}{2}\right]\right)X(t)+n(t), \quad (3)$$

where: $S(t)$ is the signal recorded by the electromagnetic sensors; $X(t)$ is the time-varying signal of the incoming electromagnetic wave; $n(t)$ is a zero-mean Gaussian additive noise with covariance matrix Ψ_n: $N(0,\Psi_n)$. For real signals, we can make use directly of Eq. 3 with the steering matrix defined in Eq. 2.

A. Optimal Bayesian group-sparse DOA estimation method

Vector sensor antennas allow unambiguous DOA estimation. In this work, we propose an approach for the DOA estimation different from the classic ones based on MUSIC or on the direct computation of the Poynting vector from the measured fields. In fact, unlike MUSIC, we use only a single-snapshot for the estimation and, in place of the direct Poynting vector, we take into account all the field components, mitigating the contribution of noise spikes on the single field components.

As proposed in [11] it is possible to solve efficiently a severely ill-posed linear system (i.e., with a number of unknowns many order of magnitude greater than the number of equations) under the hypothesis of the existence of a sparse solution.

As usual in the resolution of ill-posed ill-conditioned minimization problems, we must resolve a so-called, fat and short matrix. In this kind of problems, the choice of a proper pre-conditioner is of fundamental importance, since it can ensure the correct estimation of the sparse solution.

In this work, we propose an optimal Bayesian group-sparse method, able to provide an unbiased DOA estimator derived from the Bayesian formulation of the DOA estimation problem for the group-sparse regularized problem.

The unknown parameters vector X is supposed to be sparse, thanks to the limited number of expected DOA to estimate. The simple sparsity condition, however, can lead to a wrong solution especially if the wave does not presents a dominant polarization component. It is possible to overcome this issue through the group sparse hypothesis. Since the chosen model associates a group of parameters at each DOA, we consider all the elements of the group as a unique entity (e.g., through the L_2^2 norm evaluated over the group or a similar metric) and then we impose that only a small amount of these entities present non-zero metric obtaining a group sparse condition. This change allows a better identification of the polarization components of the impinging wave and, finally of the DOA.

Recalling Eq. (3), we can state that after the elision of the dependencies:

$$S\text{-}AX=n \rightarrow S\text{-}AX\sim N(0,\Psi_n). \quad (4)$$

Then we can formulate the following likelihood function for the signal revealed by the sensors from the generic impinging EM field as a function of its DOA components:

$$p(S|X)\propto\exp(-0.5[AX-S]^T[\Psi^N]^{-1}[AX-S]), \quad (5)$$

we assume that the impinging field can be decomposed into a finite and restrained number of elementary plane waves (i.e., the actual DOA number is of the order of few unities, eventually just equal to one), each one characterized by their polarization components. Then we can define the following likelihood function for the polarization component magnitudes:

$$p(X|\lambda)\propto\exp\left(-\sum_{k=1:N}\lambda_{X_k}\left[X_k{}^T\Psi^{X_k}X_k\right]^{0.5}\right)= \\ =\exp\left(-\lambda_X[X^T\Psi^X X]^{0.5}\right), \quad (6)$$

where we denote with X_k the k-th steering vector associated to the k-th DOA; the matrix Ψ^X is, for construction, a block diagonal matrix with the task to select, and eventually weight, only the steering vectors associated to the k-th DOA; and λ is a diagonal matrix with entries described by the exponential probability distribution:

$$p(\lambda_X)=\exp\left(-\beta\sum_{k=1:N}\lambda_{X_k}\right), \quad (7)$$

where the parameter β is such that the mean value is near zero and then the realization of $p(X|\lambda)$ can be considered sparse.

Finally, we define the probability that an impinging plane wave from a certain DOA can produce the measured signal combining Eqs. 5-7 thanks to the Bayesian chain rule:

$$p(X|S)\propto p(S|X)p(X|\lambda)p(\lambda)= \\ \exp\left(-0.5[AX-S]^T[\Psi^N]^{-1}[AX-S]-\lambda_X(X^T\Psi^X X)^{0.5}-\beta\lambda_X\right). \quad (8)$$

We impose the change of variable: $Z=[\Psi^X]^{0.5}X$, and then Eq. 8 becomes:

$$p(Z|S)\propto\exp(-0.5\left[A[\Psi^X]^{-0.5}Z-S\right]^T[\Psi^N]^{-1}\left[A[\Psi^X]^{-0.5}Z-S\right]- \\ -\lambda_X(Z^TZ)^{0.5}-\beta\lambda_X). \quad (9)$$

Since the exponent argument is composed by the sum of positive defined matrices, we can assert that the optimal DOA weights array $\hat{Z}$ is such to maximize the a posteriori probability density expressed in Eq. 8; in other words:

$$\hat{Z}=\min_Z F(Z)=\min_Z(0.5[A[\Psi^X]^{-0.5}Z-S]^T *. \\ *[\Psi^N]^{-1}[A[\Psi^X]^{-0.5}Z-S]+\lambda_X(Z^TZ)^{0.5}+\beta\lambda_X) \quad (10)$$

The minimum point $\hat{Z}$, thanks to the convexity of $F(Z)$, must satisfy the nullification of the first-derivative of the argument of Eq. 10:

$$\left.\frac{\partial F(Z)}{\partial Z}\right|_{Z=\hat{Z}}=0.5\left[A\left[\Psi^X\right]^{-0.5}\hat{Z}-S\right]^T\left[\Psi^N\right]^{-1}A\left[\Psi^X\right]^{-0.5}+ \\ +0.5\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}\hat{Z}^T=0. \quad (11)$$

Rearranging the Eq. 11 terms we obtain the following expression for $\hat{Z}$ value:

$$\hat{Z}=\left[[\Psi^X]^{-0.5}A^T[\Psi^N]^{-1}A[\Psi^X]^{-0.5}+\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}I\right]^{-1}* \\ *A^T[\Psi^N]^{-1}[\Psi^X]^{-0.5}S. \quad (12)$$

Now we can derive the expression of the matrix Ψ^X. Note that, although Eq. 12 is implicit, the quantity $\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}$ is the inverse of the L_2 norm of the unknown solution multiplied by the unknown parameter λ_X; since both multiplicands are unknowns, we can take into account them as a single unknown parameter.

The optimal definition of the standardization matrix Ψ^X is an active branch of the inverse-problems research, and the formulation is related to the form of the minimum problem to be solved. The role of the standardization matrix consists in the minimization of the correlation between the steering matrix columns in order to mitigate the DOA estimation error. The structure of the optimal standardization matrix is strictly related to the steering matrix singular values spectrum and to the restrains terms imposed to the minimum problem solution. Since the steering matrix is related to the actual antenna system, it is important to provide an optimal standardization matrix for any given steering matrix, ensuring the optimality of the computed solution independently of the particular form of the steering matrix.

In this work, we provide the optimal formulation of Ψ^X, ensuring the maximal independence between the DOAs (i.e., the covariance matrix must be proportional to the identity matrix). In particular, we impose that the estimated solution covariance matrix must be diagonal:

$$E\{\hat{Z}\hat{Z}^T\}=\left(\left[\hat{\Psi}^X\right]^{-0.5}A^T[\Psi^N]^{-1}A\left[\hat{\Psi}^X\right]^{-0.5}+\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}I\right)^{-1}* \\ *\left[\hat{\Psi}^X\right]^{-0.5}A^T[\Psi^N]^{-1}E\{SS^T\}[\Psi^N]^{-1}A\left[\hat{\Psi}^X\right]^{-0.5}* \\ *\left(\left[\hat{\Psi}^X\right]^{-0.5}A^T[\Psi^N]^{-1}A\left[\hat{\Psi}^X\right]^{-0.5}+\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}I\right)^{-1}\propto I. \quad (13)$$

Since: $E\{SS^T\}=\Psi^N$, from Eq. 13, with some algebra, we obtain:

$$\left(1-2\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}\right)I\propto A^T[\Psi^N]^{-1}A\left[\hat{\Psi}^X\right]^{-1}+ \\ +\lambda_x^2\left(\hat{Z}^T\hat{Z}\right)^{-1}(A^T[\Psi^N]^{-1}A)^{-1}\left[\hat{\Psi}^X\right]. \quad (14)$$

Since, for invertible matrices, the relation: $AB^{-1}=BA^{-1}$ holds, from Eq. 14 we obtain:

$$\frac{1-2\lambda_X\left(\hat{Z}^T\hat{Z}\right)^{-0.5}}{1+\lambda_x^2\left(\hat{Z}^T\hat{Z}\right)^{-1}}I\propto A^T[\Psi^N]^{-1}A\left[\hat{\Psi}^X\right]^{-1}. \quad (15)$$

The left-hand side term of Eq. 15 is a diagonal matrix, then, since $\hat{\Psi}^X$ is a diagonal block matrix, we can assert that:

$$\left[\hat{\Psi}_k^X\right]=\mathrm{diag}(a_k{}^T[\Psi^N]^{-1}*a_k). \quad (16)$$

The estimated solution covariance matrix $\hat{\Psi}^X$, in the form presented in Eq. 16, ensures the maximal independence between the estimated elements.

B. Numerical results

The simplest way to obtain a group-sparse solution is a proper numerical code able to solve the problem in Eq. 9. Indeed, the minimization problem in Eq. (9) is solved by FISTA [11], with a modified shrinkage operator implementation for the group-sparse property imposition.

We use a hard threshold shrinkage instead of the soft one proposed in the original work in order to obtain a gain in terms of computation time. In our experience, this choice does not affect the algorithm performances in terms of estimation precision.

The group-sparse shrinkage operator is obtained by the suppression of all the solution elements that present a group L_2 norm lower than the user-defined threshold value.

The estimation performances, in term of RMS estimation error, are evaluated for different SNR levels: 30 dB, 20 dB, 10 dB, 5 dB and 0 dB. For each SNR value, the algorithm is tested for different steering matrices, with the spatial resolution equal to 1, 2 and 4 degrees, in order to evaluate the influence of the resolution on the final estimation result. The same simulations are been executed with and without the optimal standardization.

The effects of the optimal standardization are evaluated by the introduction of a uniformly distributed scalar gain (defined in the interval 0.5-1) in the steering vectors, in order to simulate a generic anisotropic antenna system gain pattern, and then evaluate the capability of the algorithm to recover the antenna system isotropy.

The algorithm is implemented in MATLAB on a PC with a CPU Intel i7 @ 3.07GHz, RAM 24 GB, and Windows 10 OS.

The synthetic data are generated making use of Eq. 3; varying the DOA angles (θ,φ) in the interval $[0,\pi] \times [0,2\pi]$ in order to cover the field of view of the sensor antenna. The different polarization angles are obtained by the weighted sum of the group steering vectors correspondent to the k-th DOA couple angle: i.e., for each DOA, the algorithm is tested for eight different polarization angles.

The average execution time, estimated over two and half million algorithm runs (i.e., five SNR values, for eight different polarization angles, and 180 for 360 different DOA directions), is reported in Table 1 as a function of the steering matrix spatial resolution.

The run-time do not takes into account the computation of the standardization matrix and of the steering matrix, since they depend only on a priori known parameters like the noise covariance matrix and the sensor antenna configuration and gain pattern, and then they can be pre-computed and stored once the antenna system is defined.

Table 1: Algorithm run-time as a function of the steering matrix resolution

Steering Matrix Resolution	**Average Run-Time**
1 degree	0.050 s
2 degrees	0.028 s
4 degrees	0.015 s

III. DISCUSSION

As shown in Table 2, the RMS estimation errors degrade, as expected, with the SNR, and the estimation can be considered poor for SNR values lower than 5 dB SNR. It is important to note that the SNR, here, is defined on the single time snapshot, and then without any information about the time course of the signal; then the SNR is to be intended as the residual SNR from the filtering step.

It is interesting to note that the steering matrix spatial resolution does not affect the estimation error when greater than 10 dB; this result can be used to choose the steering matrix resolution in function of the expected SNR level, with a significant computation time saving.

As expected, the adoption of the optimal standardization matrix (Table 2, normal entries) ensures an improvement of the estimation accuracy. In fact, when the standardized steering matrix is adopted, the RMS estimation error is significantly lower; this is evident in the higher SNR level cases, where the estimation error is equal, in the worst cases, to half of the steering vectors spatial resolution. In general, the estimation accuracy is considered “good” until the threshold of 10 dB SNR, where the RMS estimation error is less than five degrees in elevation and azimuth angles, regardless the spatial resolution.

IV. CONCLUSIONS

In this work, an optimal single snapshot, time domain, group-sparse Bayesian DOA estimation method is proposed and tested on a vector sensor antenna system. As reported in the discussion section, it is possible to obtain an accurate DOA estimation also in the presences of imperfections in the steering matrix definition and with a single, noisy, time signal snapshot. The algorithm can be extended to sensors array configuration, or more complex sensor antennas elements [12-14], just with a proper definition of the steering matrix.

Table 2: RMS error percentile distributions (in degree) for optimally (bold) and non-optimally (plain) standardized steering matrix for 1, 2 and 4 degrees steering matrix resolution

SNR (Res.)	**Low Limit**	**25-Perc.**	**Median**	**75-Perc.**	**Up Limit**	**Low Limit**	**25-Perc.**	**Median**	**75-Perc.**	**Up Limit**
	Elevation RMSE					**Azimuth RMSE**				
30 dB (1°)	**0**/0	**0.1**/2.2	**0.2**/3.5	**0.2**/4.7	**0.5**/8.5	**0**/0	**0.1**/3.5	**0.2**/5.7	**0.2**/10	**0.5**/21
30 dB (2°)	**0**/0	**0**/3	**0.5**/4.5	**1**/6.5	**1**/11	**0**/0	**0**/5	**0**/8	**1**/16	**1**/32
30 dB (4°)	**0**/0	**0**/4	**1**/6	**1**/8	**2**/14	**0**/0	**1**/7	**1**/11	**2**/24	**3**/49
20 dB (1°)	**0**/0	**0.1**/2.2	**0.2**/3.5	**0.2**/4.7	**0.5**/8.5	**0**/0	**0.1**/4	**0.2**/6	**0.2**/9	**0.5**/20
20 dB (2°)	**0**/0	**0**/3	**0.2**/4	**0.5**/7	**1**/10	**0**/0	**0**/5	**0**/7	**1**/17	**2.2**/32
20 dB (4°)	**0**/0	**0**/4	**1**/6	**1.2**/8	**2**/14	**0**/0	**1**/7	**1**/11	**2**/25	**3.5**/49
10 dB (1°)	**0**/0	**1.1**/2.7	**1.8**/4	**2.5**/5.3	**4.5**/9	**0**/0	**1.8**/4	**2.3**/5.5	**3.5**/10	**4.8**/22
10 dB (2°)	**0**/0	**1.2**/3	**1.7**/5	**2.5**/8	**4**/11	**0**/0	**2**/6	**2.7**/8	**3.5**/20	**5**/35
10 dB (4°)	**0**/0	**1.5**/6	**2**/8	**2.7**/12	**4.5**/18	**0**/0	**2**/7	**2.8**/11	**4.5**/25	**5**/49
5 dB (1°)	**0**/0	**3.8**/5	**6.2**/7	**8.5**/9	**15**/16	**0**/0	**6.3**/6	**9**/8	**11**/11	**20**/23
5 dB (2°)	**0**/0	**4**/5.5	**6.2**/8	**8.5**/10	**15**/18	**0**/0	**6**/9	**9**/14	**12**/24	**20**/47
5 dB (4°)	**0**/0	**4**/6	**6**/9	**8**/12	**15**/19	**0**/0	**6**/11	**9**/17	**12**/37	**21**/77
0 dB (1°)	**0**/0	**19**/19	**27**/28	**38**/37	**68**/69	**0**/0	**43**/41	**61**/63	**87**/30	**154**/157
0 dB (2°)	**0**/0	**18**/19	**27**/28	**38**/39	**68**/70	**0**/0	**43**/44	**62**/60	**88**/90	**155**/158
0 dB (4°)	**0**/0	**17**/18	**27**/28	**38**/38	**68**/70	**0**/0	**42**/43	**63**/64	**88**/90	**154**/155

REFERENCES

[1] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 7, pp. 984-995, 1989.

[2] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, 1986.

[3] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, 1289-1306, 2006.

[4] D. L. Donoho and Y. Tsaig, "Extensions of compressed sensing," *Signal Processing*, vol. 86, no. 3, pp. 533-548, 2006.

[5] S. S.Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM Review*, vol. 43, no. 1, pp. 129-159, 2001.

[6] E. J. Candes and T. Tao, "Near-optimal signal recovery from random projections: Universal encoding strategies," *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 5406-5425, 2006.

[7] W. Zhu and B. X. Chen, "Novel methods of DOA estimation based on compressed sensing," *Multidimensional Systems and Signal Processing*, vol. 26, no. 1, pp. 113-123, 2015.

[8] M. Carlin, P. Rocca, G. Oliveri, F. Viani, and A. Massa, "Directions-of-arrival estimation through Bayesian compressive sensing strategies," *IEEE Transactions on Antennas and Propagation*, vol. 61, no. 7, pp. 3828-3838, 2013.

[9] A. Massa, P. Rocca, and G. Oliveri, "Compressive sensing in electromagnetics - A review," *IEEE Antennas and Propagation Magazine*, vol. 57, no. 1, pp. 224-238, 2015.

[10] A. Nehorai and E. Paldi, "Vector-sensor array processing for electromagnetic source localization," *IEEE Transactions on Signal Processing*, vol. 42, no. 2, pp. 376-398, 1994.

[11] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," *SIAM Journal on Imaging Sciences*, vol. 2, no. 1, pp. 183-202, 2009.

[12] M. Parise, "Exact EM field excited by a short horizontal wire antenna lying on a conducting soil," *AEU - International Journal of Electronics and Communications*, vol. 70, pp. 676-680, 2016.

[13] M. Parise, "Transverse magnetic field of infinite line source placed on ground surface," *Electronics Letters*, vol. 51, pp. 1478-1480, 2015.

[14] M. Parise, "An exact series representation for the EM field from a circular loop antenna on a lossy half-space," *IEEE Antennas and Wireless Propagation Letters*, vol. 13, pp. 23-26, 2014.

Marco Muzi received his B.S. in Biomedical Engineering in 2009. He received his Ph.D. in Applied Physics in 2014 with a dissertation on MEG Inverse Problems. He currently works on low frequency dielectric behavior and characterization of biological cell culture, optimization problems and metamaterials design at "La Sapienza" university of Rome.

Nicola Tedeschi received the Master degree in Electronic Engineering, from "La Sapienza" University of Rome in 2009. He received the Ph.D. in Electromagnetics from the same University in 2013. His research interests concern microwave amplifiers, electromagnetic scattering by objects near to interfaces, propagation of inhomogeneous waves in dissipative media, and the characterization of dispersive properties of natural and artificial materials.

Luca Scorrano (M'12) received the B.S. degree (summa cum laude) in Electrical Engineer from University Roma Tre, Rome, Italy in 2005 and the M.S. (summa cum laude) in Electrical Engineer from University Roma Tre, Rome, Italy in 2007. He earned the Ph.D. in Applied Electromagnetics in 2011. From 2015 he is a System Engineer in the Research & Innovation Department at Elettronica S.p.A., Rome, Italy.

His research interests are in the field of broadband antennas and arrays, Frequency Selective Surfaces (FSS), Artificial Magnetic Conductors (AMC), composite material, meta-materials, electromagnetic theory, photonics and plasmonics. He currently serves as peer reviewer for many IET journals in the field of electromagnetics and optics and for IEEE APS Letters. He is co-author of a book, author of more than 40 international publications, and 6 patent applications.

Vincenzo Ferrara joined the Department DIET, Sapienza University of Rome, as Associate Professor of Electronics since 2001, and he is scientific responsible of Electronics for the Environment Lab of the DIET. The scientific research concerns different topics, focusing electronic systems for the environment, such as technologies for planning and sustainable development, satellite navigation, WSN, energy harvesting and so on.

Fabrizio Frezza was born in Rome, Italy, on October 31, 1960. He received the "Laurea" (degree) "cum laude" in Electronic Engineering in 1986 and the Doctorate degree in Applied Electromagnetics and Electrophysical Sciences in 1991, both from "La Sapienza" University of Rome. In 1986, he joined the Department of Electronics of the same University, where he has been Full Professor of Electromagnetic Fields since 2005. His research activity has concerned guiding structures, antennas and resonators for microwaves and millimeter waves, numerical methods, scattering, optical propagation, plasma heating, anisotropic media, artificial materials and metamaterials, cultural-heritage applications. Frezza is a Member of Sigma Xi and Senior Member of IEEE and OSA.