Stochastic Analysis of Multi-conductor Cables with Uncertain Boundary Conditions

Gang Zhang, Jinjun Bai, Lixin Wang, and Xiyuan Peng

School of Electrical Engineering and Automation
Harbin Institute of Technology, Harbin, 150001, China
zhang_hit@hit.edu.cn, 13B906024@hit.edu.cn, wlx@hit.edu.cn, pxy@hit.edu.cn

Abstract **─** This paper provides two novel Stochastic Galerkin Method strategies to undertake stochastic analysis of the crosstalk in the multi-conductor cables with uncertain boundary conditions. Two different uncertain boundary conditions, stochastic lumped source and stochastic lumped load, are considered into stochastic Transmission Line Model. With the help of the Feature Selective Validation, it is verified that the proposed strategies is accurate by comparing with the reference results provided by Monte Carlo Method. At last, advantage of the proposed strategies in computational efficiency is presented.

Index Terms **─** EMC simulation, stochastic Galerkin method, uncertain boundary conditions, uncertainty analysis.

I. INTRODUCTION

Recent years, in order to reasonably accomplish simulation recurrence of actual situation, Electromagnetic Compatibility (EMC) community is facing a growing demand for developing uncertainty analysis into simulations. Among these new-type simulations, input parameters can be modeled by random variables, due to the lack of knowledge or the existence of tolerance factor [1, 2].

Solving stochastic Transmission Line Model is a typical uncertainty analysis problem in EMC simulation. For example, in calculating crosstalk of the wires in a cable bundle, the height of the wires might be uncertain in the complicated actual situation. In this case, the crosstalk voltages at terminations will be no longer deterministic. Obtaining these uncertain outputs is what uncertainty analysis does.

Many uncertainty analysis methods are presented to deal with this typical problem. Monte Carlo Method (MCM) is a widely used uncertainty analysis method, which owns high accuracy [1-3]. It is easy to realize as there is no need to change solver during uncertainty analysis. However, low computational efficiency makes MCM uncompetitive in EMC simulation, especially in some complicated problems.

Stochastic Galerkin Method (SGM) is another accurate uncertainty analysis method that provides high computational efficiency [4, 5]. Discussion of the crosstalk problem can be seen in literatures [6-8]. It shows that SGM is as accurate as MCM, and much more effective than MCM. Nevertheless, the original solver must be changed during uncertainty analysis in SGM. This character limits the application of SGM. If the solver of the problem is complex, realizing of SGM will become difficult. In existing literatures of SGM, only the uncertainty in telegraph equations is considered. Thus, further studies about applying SGM are still necessary to solve more complex problems.

In this paper, in order to improve the applicable scope of SGM in solving complex EMC problems, uncertain boundary conditions will be considered in stochastic Transmission Line Model. Novel SGM strategies will be introduced in detail towards two different uncertain boundary conditions, stochastic lumped source and stochastic lumped load. By using Feature Selective Validation [9-12], the strength of the SGM strategies will be testified.

The structure of the paper is as follows. Section II employs a brief description of Stochastic Galerkin Method; solving of stochastic Transmission Line Model with uncertain boundary conditions is presented in Section III; algorithm validation is presented in Section IV; Section V provides a summary of this paper.

II. SGM OVERVIEW

In traditional EMC simulation models, all input parameters are supposed certain. However, in actual situation, some parameters might be unknown as the lack of knowledge, or may change arbitrarily like the cables in a moving car, or may be random as the existence of manufacturing tolerance. In this case, some input parameters of the simulation must be uncertain in order to improve the reliability of simulation results. In such simulation, the output parameters which we are interested in will be influenced by the uncertainty inputs. Uncertainty analysis methods can provide these uncertain outputs.

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SGM is an effective uncertainty analysis method which is rooted in the generalized Polynomial Chaos (gPC) theory, and it has been successfully applied in many fields [4, 5].

Modeling the uncertain inputs is the primary task of SGM. And random events θ , which are caused by the uncertain input parameters, can be modeled by random variable space $\xi(\theta)$. The random variable space $\xi(\theta)$ is made up by several random variables, shown in (1). Moreover, each random variable has its own distribution:

$$\xi(\theta)=\left\{\xi_1(\theta),\ \xi_2(\theta),\ \cdots,\ \xi_n(\theta)\right\}. \tag{1}$$

Independence of the random variables in the random variable space is an essential requirement in the application of SGM. Karhunen-Loeve expansion can provide technical support for such independence [5].

Based on the gPC theory, the outputs of uncertainty analysis can be unfolded by polynomial form shown in (2):

$$V(\xi)=v_0\varphi_0(\xi)+v_1\varphi_1(\xi)+\cdots+v_P\varphi_P(\xi), \tag{2}$$

where $V(\xi)$ is the output result which is influenced by the random variable space ξ , and $\varphi_i(\xi)$ stands for the Chaos polynomial of the random variables which is given by the Askey rule. Where $P+1$ is total number of the polynomial, and v_i is the polynomial coefficient, which needs to be solved.

Table 1 presents the Askey rule. The form of polynomial is determined by distribution of the random variables. The Askey rule can guarantee best convergence of the polynomial expansion according to the gPC theory [5].

Table 1: Askey rule

Random Variables	Wiener-Askey Chaos	Support
Gaussian	Hermite-chaos	$(-\infty,+\infty)$
Gamma	Laguerre-chaos	$[0,+\infty)$
Beta	Jacobi-chaos	$[0,\ 1]$
Uniform	Legendre-chaos	$[-1,\ 1]$

Suppose that there are two random variables ξ_1 and ξ_2 in a random variable space $\xi=\{\xi_1,\ \xi_2\}$. ξ_1 is in Gaussian distribution, and ξ_2 is in Uniform distribution. First three polynomials of the Hermite-chaos are $\varphi_0(\xi_1)=1$, $\varphi_1(\xi_1)=\xi_1$ and $\varphi_2(\xi_1)=\xi_1^2-1$. First three polynomials of the Legendre-chaos are $\varphi_0(\xi_2)=1$, $\varphi_1(\xi_2)=\xi_2$ and $\varphi_2(\xi_2)=3\times\xi_2^2-1$. The polynomial expansion of ξ is in form of tensor product, as Table 2 shown.

Table 2: Polynomial for the case with two random variables

The Number of Polynomial *P*	The Order *d*	Polynomial
0	0	1
1	1	ξ_1
2	1	$1.732\times\xi_2$
3	2	$0.707\times(\xi_1^2-1)$
4	2	$1.732\times\xi_1\times\xi_2$
5	2	$1.118\times(3\times\xi_2^2-1)$

If the order of the polynomial is *d* and the number of the random variables is *n*, the number of polynomials *P*+1 will be calculated by following relationship:

$$(P+1)=\frac{(n+d)!}{n!d!}. \tag{3}$$

The polynomials given by the Askey rule are orthogonal to each other, which implies that,

$$\left\langle\phi_i,\ \phi_j\right\rangle=\left\langle\phi_i^2\right\rangle\delta_{ij}. \tag{4}$$

Where δ_{ij} is Kronecker function which leads to the relation:

$$\delta_{ij}=\begin{cases}1 & (i=j)\\ 0 & (i\neq j)\end{cases}. \tag{5}$$

Inner product computation $\langle .\ ,\ .\rangle$ is defined as:

$$\left\langle\phi_i,\ \phi_j\right\rangle=\int\phi_i(\xi)\phi_j(\xi)w(\xi)d\xi, \tag{6}$$

where $w(\xi)$ is weight function, and it can be obtained by the joint probability density of the random variables because the random variables are independent. In the same way, the supports of the integration are the joint supports of every random variable.

III. SOLUTION OF STOCHASTIC TRANSMISSION LINE MODEL

Transmission line equations include two types of equations, and they are telegraph equations and boundary conditions equations. Previous research only focuses on uncertain analysis of telegraph equations, shown in [6-8]. In contract, little attention is gained about boundary conditions equations. Therefore, this study aims to undertake uncertainty analysis in boundary conditions equations.

Boundary conditions equations are given as (7) and (8):

$$V(0)=\mathrm{V}_s-Z_sI(0), \tag{7}$$

$$V(L)=\mathrm{V}_l+Z_lI(L), \tag{8}$$

where L is the length of the transmission line. $I(0)$ and $V(0)$ stand for the current value and the voltage value in source end. $I(L)$ and $V(L)$ are the current value and the voltage value in load end. V_s is the voltage source in the source end, and V_l presents the voltage source in load end. Z_s and Z_l present the loads in source end and load end respectively.

Two different uncertain boundary conditions are considered. One is stochastic lumped source, and the other is stochastic lumped load.

A. Stochastic lumped source

In the first model, uncertainty in lumped source V_s is considered, and equation (7) is translated into:

$$V(0,\xi) = \mathrm{V}_s(\xi) - Z_s I(0,\xi), \tag{9}$$

where $\mathrm{V}_s(\xi)$ is the uncertain input which is inference by the random event ξ. $V(0,\xi)$ and $I(0,\xi)$ are outputs of the calculation.

According to the gPC theory, the output parameters can be expanded by the polynomial of the random variables like (2), so they become:

$$\begin{cases} V(0,\xi) = v_0\varphi_0(\xi) + v_1\varphi_1(\xi) + v_2\varphi_2(\xi) \\ I(0,\xi) = i_0\varphi_0(\xi) + i_1\varphi_1(\xi) + i_2\varphi_2(\xi) \end{cases}. \tag{10}$$

Substituting (10) into (9), (9) can be expanded as:

$$\begin{aligned} &v_0\varphi_0(\xi) + v_1\varphi_1(\xi) + v_2\varphi_2(\xi) = \\ &\mathrm{V}_s(\xi) - Z_s(i_0\varphi_0(\xi) + i_1\varphi_1(\xi) + i_2\varphi_2(\xi)). \end{aligned} \tag{11}$$

Using Galerkin progress, inner product computation with $\varphi_0(\xi)$ is taken on both sides of (11). And (11) is rewritten as:

$$\begin{aligned} &v_0\langle\varphi_0(\xi),\varphi_0(\xi)\rangle + v_1\langle\varphi_1(\xi),\varphi_0(\xi)\rangle + \\ &v_2\langle\varphi_2(\xi),\varphi_0(\xi)\rangle = \langle\mathrm{V}_s(\xi),\varphi_0(\xi)\rangle - \\ &Z_s(i_0\langle\varphi_0(\xi),\varphi_0(\xi)\rangle + i_1\langle\varphi_1(\xi),\varphi_0(\xi)\rangle + \\ &i_2\langle\varphi_2(\xi),\varphi_0(\xi)\rangle). \end{aligned} \tag{12}$$

Considering the properties given in (4), (12) can be rearranged to obtain:

$$v_0 = \langle\mathrm{V}_s(\xi),\varphi_0(\xi)\rangle - Z_s i_0, \tag{13}$$

where $\langle\mathrm{V}_s(\xi),\varphi_0(\xi)\rangle$ is integral calculation, so its result is a constant.

Inner product computation with $\varphi_1(\xi)$ and $\varphi_2(\xi)$ is done in the similar way like (12), Equation (14) is got:

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \langle\mathrm{V}_s(\xi),\varphi_0(\xi)\rangle \\ \langle\mathrm{V}_s(\xi),\varphi_1(\xi)\rangle \\ \langle\mathrm{V}_s(\xi),\varphi_2(\xi)\rangle \end{bmatrix} - Z_s \begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix}. \tag{14}$$

Compared with the original equation in (7), the random variable ξ is disappeared, and (14) only includes three certain equations. Randomness is transferred into the Polynomial chaos in (10).

Traditional EMC simulation method can be used to solve the equations in (14), and coefficients like v_0 and i_0 can be calculated. After substituting these coefficients into the Polynomial chaos in (10), uncertainty analysis results of the SGM are presented. If the random variables in (10) are sampled, the statistical property which we need will be obtained, such as the expectation, the standard deviation, the worst case values, the probability density curves, and so on.

B. Stochastic lumped load

In the second model, stochastic lumped load is considered into boundary conditions equation (7). The equation will become:

$$V(0,\xi) = \mathrm{V}_s - Z_s(\xi) I(0,\xi). \tag{15}$$

Compared with the equation (9), the lumped source V_s is a certain input, but the lumped load is turned into an uncertain value, $Z_s(\xi)$.

The output parameters $V(0,\xi)$ and $I(0,\xi)$ can also be expanded by substituting (10), equation (15) is unfolded as:

$$\begin{aligned} &v_0\varphi_0(\xi) + v_1\varphi_1(\xi) + v_2\varphi_2(\xi) = \\ &\mathrm{V}_s - Z_s(\xi)(i_0\varphi_0(\xi) + i_1\varphi_1(\xi) + i_2\varphi_2(\xi)). \end{aligned} \tag{16}$$

Taking inner product computation on both sides with $\varphi_0(\xi)$, (16) is rewritten as:

$$\begin{aligned} &v_0\langle\varphi_0(\xi),\varphi_0(\xi)\rangle + v_1\langle\varphi_1(\xi),\varphi_0(\xi)\rangle + \\ &v_2\langle\varphi_2(\xi),\varphi_0(\xi)\rangle = \langle\mathrm{V}_s,\varphi_0(\xi)\rangle - \\ &(i_0\langle Z_s(\xi)\varphi_0(\xi),\varphi_0(\xi)\rangle + i_1\langle Z_s(\xi)\varphi_1(\xi),\varphi_0(\xi)\rangle + \\ &i_2\langle Z_s(\xi)\varphi_2(\xi),\varphi_0(\xi)\rangle). \end{aligned} \tag{17}$$

Using the inner product properties given in (4), (17) should be:

$$\begin{aligned} &v_0 = \mathrm{V}_s - (i_0\langle Z_s(\xi)\varphi_0(\xi),\varphi_0(\xi)\rangle + \\ &i_1\langle Z_s(\xi)\varphi_1(\xi),\varphi_0(\xi)\rangle + i_2\langle Z_s(\xi)\varphi_2(\xi),\varphi_0(\xi)\rangle). \end{aligned} \tag{18}$$

It is worth noting that the first Chaos polynomial $\varphi_0(\xi)$ is equal to 1. Thus, the calculating progress like (19) can be obtained:

$$\langle\mathrm{V}_s,\varphi_0(\xi)\rangle = \langle\mathrm{V}_s\,\varphi_0(\xi),\varphi_0(\xi)\rangle = \mathrm{V}_s. \tag{19}$$

In the similar way, (20) and (21) can be shown as the relations:

$$\langle\mathrm{V}_s,\varphi_1(\xi)\rangle = \langle\mathrm{V}_s\,\varphi_0(\xi),\varphi_1(\xi)\rangle = 0, \tag{20}$$

$$\langle\mathrm{V}_s,\varphi_2(\xi)\rangle = \langle\mathrm{V}_s\,\varphi_0(\xi),\varphi_2(\xi)\rangle = 0. \tag{21}$$

Taking inner product computation with $\varphi_1(\xi)$ and $\varphi_2(\xi)$, the Galerkin Process results can be rearranged to get:

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \mathrm{V}_s \\ 0 \\ 0 \end{bmatrix} - T \begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix}, \tag{22}$$

$$T = \begin{bmatrix} \langle Z_s(\xi)\varphi_0,\varphi_0\rangle & \langle Z_s(\xi)\varphi_1,\varphi_0\rangle & \langle Z_s(\xi)\varphi_2,\varphi_0\rangle \\ \langle Z_s(\xi)\varphi_0,\varphi_1\rangle & \langle Z_s(\xi)\varphi_1,\varphi_1\rangle & \langle Z_s(\xi)\varphi_2,\varphi_1\rangle \\ \langle Z_s(\xi)\varphi_0,\varphi_2\rangle & \langle Z_s(\xi)\varphi_1,\varphi_2\rangle & \langle Z_s(\xi)\varphi_2,\varphi_2\rangle \end{bmatrix}, \tag{23}$$

where T is only an intermediate variable. The random variable ξ in the Chaos polynomial $\varphi_1(\xi)$ is ignored to simplify the expression.

Unlike (14), equations (22) and (23) provide an augmented certain equation. Traditional EMC simulation method can also be used to calculate the coefficients in (10), like v_0 and i_0. Final uncertainty analysis results can also be got by sampling equation (10).

IV. ALGORITHM VALIDATION

In this section, algorithm validation of the proposed strategies is presented. The validation model is got by improving a published model mentioned in literature [6]. Only geometrical uncertainty has been considered in [6]. In our model, uncertain boundary conditions will also be considered at the same time.

The validation model is to calculate crosstalk between two coupled lines, and it is shown in Fig. 1.

The radius of the radiating conductor and the disturbed conductor are both 0.1mm. The horizontal distance between two conductors is 0.05m. The height of the disturbed conductor is 0.035m, and the height of the radiating conductor H_g obeys Gaussian distribution $N(0.04, 0.005)$m. All the loads are supposed 50Ω, except for Z_{load} under identified. The amplitude of the excitation source U_{source} is another undefined value. The radiating conductor and the disturbed conductor are surrounded by vacuum. The relative dielectric constant and the relative magnetic permeability of the vacuum are both 1.

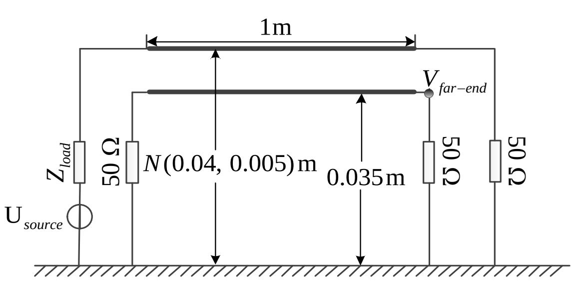

Fig. 1. Basic Stochastic Transmission Line Model.

A random variable can be used to express the uncertainty in the height of the radiating conductor, and it shows in:

$$H_g = 0.04 + 0.005 \times \xi_1, \tag{24}$$

where, ξ_1 is standard normal distribution.

A. Validation of the first strategy

The strategy about the stochastic lumped source is validated at first. Uncertainty in lumped source is considered as:

$$U_{source} = 1.1 + 0.1 \times \xi_2, \tag{25}$$

where ξ_2 is the Uniform distribution in $[-1, 1]$.

On the contrary, the lumped load is a certain value shown as:

$$Z_{load} = 50. \tag{26}$$

In this case, the random variable space is $\{\xi_1, \xi_2\}$, thus Chaos polynomials should be chosen as Table 2. Using the process given in Section III, the probability density curve of crosstalk voltage value $V_{far-end}$ at the far end of the disturbed conductor can be calculated. Figure 2 shows the results of the crosstalk voltage value at single frequency point 20 MHz and 5 MHz.

Results calculated by MCM can be regarded as reference data, and 20000 times of sampling are used. The sampling times judgment is based on the method given in [13], in order to make sure that MCM is convergence.

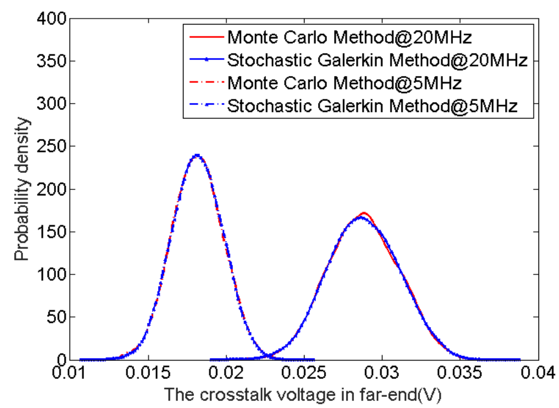

Fig. 2. Results at single frequency point.

Figure 2 indicates that the results given by the first SGM strategy and MCM are nearly the same at these two frequency points.

Figure 3 shows the expectation results of $V_{far-end}$ with frequency from 1MHz to 100 MHz. Meanwhile, Fig. 4 presents the 'worst case' information.

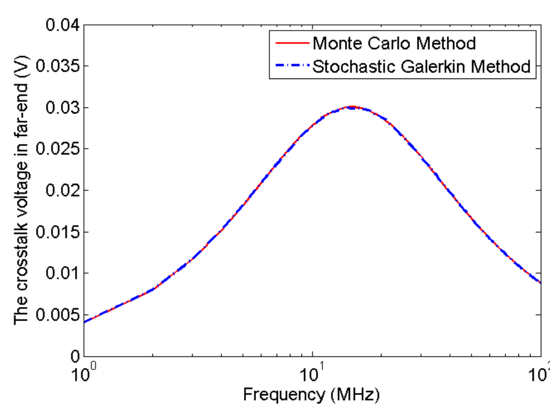

Fig. 3. Expectation results of crosstalk voltage value from 1MHz to 100 MHz.

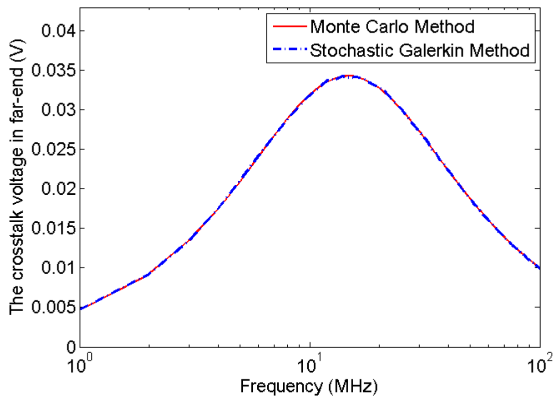

Fig. 4. Worst case values of crosstalk voltage value from 1MHz to 100 MHz.

By using FSV, the Total-GDM of the results between two methods in Fig. 3 is 0.036. It indicates that the results in the first strategy are an 'Excellent' match with the results in MCM. The Total-GDM of the results according to Fig. 4 is 0.043, and it is also an 'Excellent' match. The details about FSV can be found in [9, 10]. In Fig. 5, all the relative errors between SGM and MCM in every frequency point are less than 1%, and it means that SGM and MCM are in the same accuracy level.

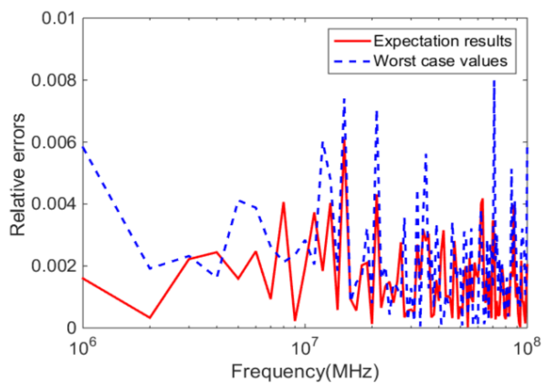

Fig. 5. Relative errors between SGM and MCM in the first strategy.

According to all Total-GDM results above, it is testified that the first SGM strategy in considering the stochastic lumped source is as accurate as MCM.

B. Validation of the second strategy

Uncertainty in the lumped load is considered in this part, it shows in:

$$Z_{load} = 55+5\times\xi_3, \tag{27}$$

where ξ_3 is also the Uniform distribution in $[-1, 1]$.

The lumped source is a certain value this time, and it should be:

$$U_{source} = 1. \tag{28}$$

The random variable space turns to be $\{\xi_1, \xi_3\}$, the Chaos polynomials also can be provided by Table 2. In this case, the random variable ξ_2 in the table is changed to be ξ_3.

By using the process provided in Section III, the uncertain crosstalk results can be obtained. Fig. 6 also gives the PDF results at single frequency point 20 MHz and 5 MHz. It shows that the MCM results are quite similar to the results given by the second SGM strategy.

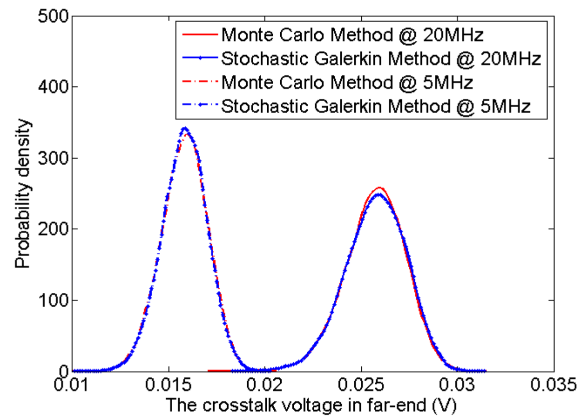

Fig. 6. Results at single frequency point.

The expectation results and Worst case values at the whole frequency from 1MHz to 100 MHz are presented in Fig. 7 and Fig. 8 respectively.

In the same way, by using FSV, the Total-GDM of the results between two methods in Fig. 7 is 0.014, and it is 0.020 in Fig. 8. Both of them indicate the 'Excellent' match. Furthermore, the relative errors given in Fig. 9 can provide the same conclusion.

In a word, it is demonstrated that the accuracy of the second SGM strategy is also in the same level with MCM in considering the stochastic lumped load.

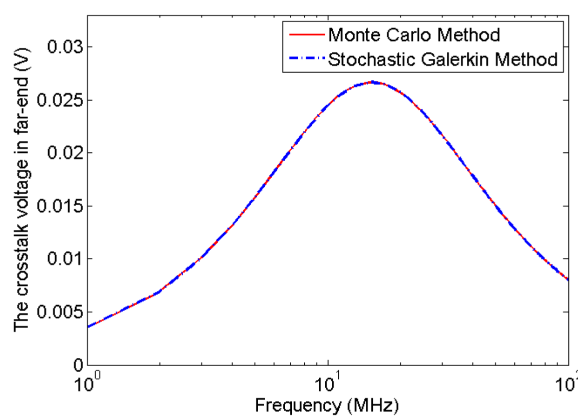

Fig. 7. Expectation results of crosstalk voltage value from 1MHz to 100 MHz.

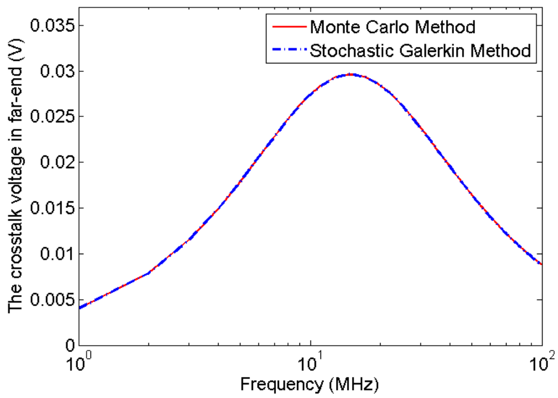

Fig. 8. Worst case values of crosstalk voltage value from 1MHz to 100 MHz.

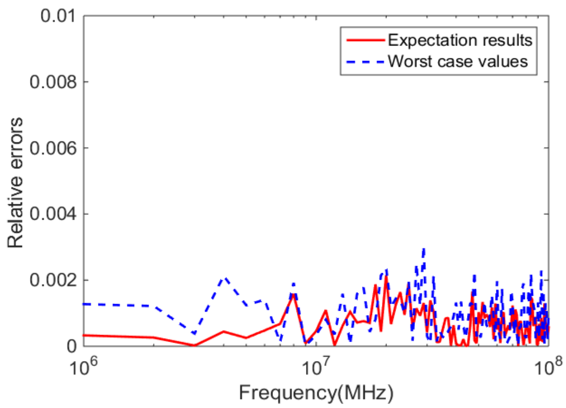

Fig. 9. Relative errors between SGM and MCM in the second strategy.

C. Discussion of computational efficiency

The simulation time comparison between SGM and MCM in two strategies is shown in Table 3. The total time includes pre-processing time, calculating time and post processing time.

For pre-processing time, some inner product computations must be calculated firstly in using SGM. On the contrary, nothing needs to be done in MCM. In calculating time, several certain equations like (14) or an augmented equation like (23) should be solved in SGM, thus the calculated quantity of the SGM is equal to several times of the original equation like (7). However, 20000 times of the original equation like (7) need to be calculated in MCM. As to the post processing time, statistical properties of the results can be obtained. The SGM samples the random variables before statistical properties calculation, so it needs a little more time than MCM.

Therefore, the difference in computational efficiency between SGM and MCM is decided by the calculating time. If the single simulation time in solving equation (7) is long, the difference will be more obvious.

Table 3: The simulation time comparison between the SGM and the MCM

	Pre-Processing	Calculating	Post Processing	Total
MCM(1)	0s	373.83s	0.15s	372.98s
SGM(1)	10.25s	0.26s	0.81s	11.32s
MCM(2)	0s	355.29s	0.21s	355.5s
SGM(2)	10.84s	0.25s	0.97s	12.06s

In short, the computational efficiency of SGM is much higher than MCM.

V. CONCLUSION

In this paper, considering uncertain boundary conditions, two novel strategies based on Stochastic Galerkin Method are presented to solve the stochastic transmission line equations. By using Feature Selective Validation, the proposed strategies are demonstrated as accurate as Monte Carlo Method. Furthermore, the computation efficiency of the strategies is proved much higher than that of Monte Carlo Method. In a word, the proposed strategies improve the competitiveness of the Stochastic Galerkin Method in solving complex multi-conductor cables problems.

At last, the usage of uncertainty analysis results is discussed. Expectation results are the most likely values, so it is useful in EMC prediction field. Standard deviation information plays an important role in sensitivity analysis and robustness analysis of EMC simulation. Worst case values are a special focus in EMC field, and it is an important value in EMC optimization design.

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REFERENCES

[1] S. Sun, G. Liu, J. L. Drewniak, et al., "Hand-assembled cable bundle modeling for crosstalk and common-mode radiation prediction," *IEEE Transactions on Electromagnetic Compatibility*, vol. 49, no. 3, pp. 708-718, 2007.

[2] D. Bellan and S. A. Pignari, "Efficient estimation of crosstalk statistics in random wire bundles with lacing cords," *IEEE Transactions on Electromagnetic Compatibility*, vol. 53, no. 1, pp. 209-218, 2011.

[3] G. Spadacini and S. A. Pignari, "Numerical assessment of radiated susceptibility of twisted-wire pairs with random nonuniform twisting," *IEEE Transactions on Electromagnetic Compatibility*, vol. 55, no. 5, pp. 956-964, 2013.

[4] D. Xiu and G. E. Karniadakis, "The Wiener Askey polynomial chaos for stochastic differential equations," *SIAM Journal on Scientific Computing*, vol. 24, no. 2, pp. 619-644, 2002.

[5] D. Xiu, *Numerical Methods for Stochastic Computations*. Princeton University Press, pp. 1-23, 2010.

[6] I. S. Stievano, P. Manfredi, and F. G. Canavero, "Stochastic analysis of multiconductor cables and interconnects," *IEEE Transactions on Electromagnetic Compatibility*, vol. 53, no. 2, pp. 501-507, 2011.

[7] P. Manfredi and F. G. Canavero, "Numerical calculation of polynomial chaos coefficients for stochastic per-unit-length parameters of circular conductors," *IEEE Transactions on Magnetics*, vol. 50, no. 3, pp. 74-82, 2014.

[8] P. Manfredi and F. Canavero, "Statistical analysis of multiconductor cables and interconnects with internal variability and field coupling," *42nd European Microwave Conference*, pp. 348-351, 2012.

[9] IEEE Standard for Validation of Computational Electromagnetics Computer Modeling and Simulations. IEEE STD 15971-2008, pp. 1-41, 2008.

[10] IEEE Recommended Practice for Validation of Computational Electromagnetics Computer Modeling and Simulations. IEEE STD 15972-2010, pp. 1-124, 2011.

[11] A. Duffy, A. Orlandi, and G. Zhang, "Review of the feature selective validation method (FSV). Part I: Theory," *IEEE Transactions on Electromagnetic Compatibility*, vol. 6, no. 4, pp. 814-821, 2018.

[12] A. Orlandi, A. P. Duffy, and G. Zhang, "Review of the feature selective validation method. Part II: Performance analysis and research fronts," *IEEE Transactions on Electromagnetic Compatibility*, vol. 6, no. 4, pp. 1029-1035, 2018.

[13] J. Bai, L. Wang, and D. Wang, "Validity evaluation of the uncertain EMC simulation results," *IEEE Transactions on Electromagnetic Compatibility*, vol. 59, no. 3, pp. 797-804, 2017.