Multi-Beamformer with Adjustable Gain: Projection Approach

Jie Chen [1] and Yingzeng Yin [2]

[1] School of Electronic Engineering
Xi'an Aeronautical University, Xi'an, 710077, China
chenbinglin88888@163.com

[2] National Key Laboratory of Antenna and Microwave Technology
School of Electronic Engineering, Xidian University, Xi'an, 710071, China
yyzeng@mail.xidian.edu.cn

Abstract **─** In this paper, a new multi-beam forming approach is presented. First, we divide the steering vectors into two parts. The first part is the beam vector space. The second part is the side lobe vector space. Given that the inner product of two orthogonal vectors is zero, to minimize the gains of the side lobes, the excitation vector of the antenna array elements has to be placed in the orthogonal projection matrix of the side lobe vector space. Then, we obtain a vector by linearly combining the beam vectors and orthogonally projecting the combined vector into the orthogonal projection matrix of the side lobe vector space, and this projection vector is just the solution of the excitation vector of the antenna array elements. Since the combined vector is the linear combination of the beam vectors, we can control the beam gain by adjusting the linear combination coefficients. This new method can be easily used to form multi-beams and adjust each beam gain. The results of simulations show, that the new method acts effectively and efficiently.

Index Terms **—** Antenna array, array manifold, beam forming, linear space, orthogonal projection.

I. INTRODUCTION

Beam forming technologies have been well studied in past decades. Many articles have been published in this domain. The well-known analytical function approaches, such as Taylor and Chebyshev beam forming, were the earliest method developed to form beams. These methods generally investigated radiation beam forming. Microwave beam forming (MBF) is another developing approach. Several techniques of MBF have been developed to accomplish adaptive beam forming (ABF) [1, 2].

In recent years, along with the development of the digital processing and microelectronic technologies, digital beam forming has become a hot topic for researchers. A beam forming approach used in wide band multiple-input multiple-output (MIMO) systems was discussed in reference [3]. Reference [4] adopted the compressed sensing method to form beams. Under the constraint of *l*1-norm minimization, article [5] developed a new beam forming method. In reference [6], to adapt to real time beam forming, real weight adaptive processing based on a direct data domain least squares approach was presented. The optimization of an arbitrary side lobe attenuation level was proposed in reference [7]. Article [8] put forward a multiple beam forming approach. Reference [9] developed a beam forming means for a phase-configurable antenna array. In reference [10], a differential evolution genetic algorithm beam forming approach was presented. In reference [11], many digital processing methods were discussed. In reference [12], beam forming was viewed as a space filtering issue. Phased array beam steering through serial control of the phase shifters was presented in another article [13]. Reference [14] studied the phase and pattern characteristics of a sub-wavelength broadband reflectarray unit element based on triple concentric circular-rings.

In this paper, a new multi-beam forming approach is presented. First, we divide the steering vectors into two parts. The first part is the beam vector space. The second part is the side lobe vector space. Given that the inner product of two orthogonal vectors is zero, to minimize the gains of the side lobes, the excitation vector of the antenna array elements has to be settled on the orthogonal projection matrix of the side lobe vector space. Then, we obtain a vector by linearly combining the beam vectors and orthogonally projecting the combined vector into the orthogonal projection matrix of the side lobe vector space, and this projection vector is just the solution of the excitation vector of the antenna array elements. Since the combined vector is obtained from the linear combination of the beam vectors, we can control the beam gain by adjusting the linear combination coefficients. Compared with

Submitted On: March 1, 2017
Accepted On: August 11, 2018

conventional methods, this new method can be easily used to form multi-beams and adjust each beam gain. The results of the simulations show, that the new method acts effectively and efficiently.

The remainder of this paper is organized as follows: Section II presents the beam forming paradigm; Section III presents the new projection beam forming approach; Section IV shows several simulations of the new approach; and Section V draws a conclusion.

II. BEAM FORMING PARADIGM

We aim to investigate the beam forming of a uniform linear antenna array. This array has N isotropic antenna elements, which are arranged along a line with spacing d. The far field is considered, and the narrow band signal centered at a wave length of λ is transmitted by each antenna element. The element arrangement numbered from 1 to N is shown in figure 1. The angle between the transmitting signal and the axis of the array denoted as θ in figure 1 is the signal transmitting angle.

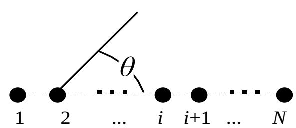

Fig. 1. Arrangement of the elements in the uniform linear array.

The beam pattern formed by the array in Fig. 1 in the far field can be written as:

$$f(\theta)=\sum_{i=1}^{N} I_i e^{j2\pi(i-1)d\cos\theta/\lambda}. \quad (1)$$

In equation (1), I_i is the current excitation of the i^{th} antenna element. We define $\mathbf{a}(\theta)=[1,e^{j2\pi d\cos\theta/\lambda},...,e^{j2\pi d(N-1)\cos\theta/\lambda}]^{\mathrm{T}}$ as the steering vector and denote the antenna element excitation vector as $\mathbf{W}=[I_1,I_2,...,I_i,...,I_N]^{\mathrm{T}}$, where the superscript T denotes the transpose operation. Hence, equation (1) can be rewritten as:

$$f(\theta)=\mathbf{a}^{\mathrm{T}}(\theta)\mathbf{W}. \quad (2)$$

Because the pattern formed is the periodic function of the signal transmitting angle θ, we set θ in a cycle from 0° to 180°. For convenience, we denote θ as discrete values of θ_1, θ_2, ..., θ_k, ..., θ_K in sequence. Let the expected beam vector be:

$$\mathbf{P}=[P(\theta_1),P(\theta_2),...,P(\theta_k),...,P(\theta_K)]^{\mathrm{T}}. \quad (3)$$

Then, the beam forming issue is transformed to design the array element excitation vector $\mathbf{W}$ to make the following equation valid:

$$\begin{aligned}\mathbf{P}&=abs(f(\theta))\\&=abs([\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),...,\mathbf{a}(\theta_k),...,\mathbf{a}(\theta_K)]^{\mathrm{T}}\mathbf{W}),\end{aligned} \quad (4)$$

where abs denotes the absolution value operation.

Let $\mathbf{A}=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),...,\mathbf{a}(\theta_k),...,\mathbf{a}(\theta_K)]$, where $\mathbf{A}$ is called the array manifold matrix. Then, equation (4) can be rewritten as:

$$\mathbf{P}=abs(\mathbf{A}^{\mathrm{T}}\mathbf{W}). \quad (5)$$

The least square solution of equation (5) is:

$$\mathbf{W}_{\mathrm{LS}}=((\mathbf{A}^{\mathrm{T}})^{\mathrm{H}}\mathbf{A}^{\mathrm{T}})^{-1}(\mathbf{A}^{\mathrm{T}})^{\mathrm{H}}\mathbf{P}. \quad (6)$$

III. BEAM FORMING OF THE PROJECTION APPROACH

If the beam points to the direction of θ_n, as well as in other directions, there is no beam. The beam forming process can be mathematically expressed as:

$$\mathbf{a}(\theta_n)^{\mathrm{T}}\mathbf{W}\neq 0, \quad (7)$$

$$\mathbf{a}(\theta_m)^{\mathrm{T}}\mathbf{W}=0 \quad \forall m\neq n, \quad (8)$$

where both m and n are integer variables. Let,

$$\mathbf{B}=\mathbf{a}(\theta_n), \quad (9)$$

and

$$\mathbf{Z}=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),...,\mathbf{a}(\theta_{n-1}),\mathbf{a}(\theta_{n+1})...,\mathbf{a}(\theta_K)], \quad (10)$$

where matrix $\mathbf{B}$ is called the beam matrix, and matrix $\mathbf{Z}$ is named as the null matrix.

Obviously, both matrix $\mathbf{Z}$ and $\mathbf{B}$ are a division of matrix $\mathbf{A}$ and they can be combined into $\mathbf{A}$.

Then, equation (7) and (8) can be rewritten as:

$$\mathbf{B}^{\mathrm{T}}\mathbf{W}\neq 0, \quad (11)$$

$$\mathbf{Z}^{\mathrm{T}}\mathbf{W}=\mathbf{0}. \quad (12)$$

Equation (12) means that vector $\mathbf{W}^*$ is perpendicular to the column vector space of matrix $\mathbf{Z}$ with the superscript * indicating the conjugate operation. Hence, vector $\mathbf{W}^*$ must locate in the orthogonal projection space of the column vector space of matrix $\mathbf{Z}$, which can be mathematically written as:

$$\mathbf{W}^*=(\mathbf{I}-\mathbf{Z}(\mathbf{Z}^{\mathrm{H}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{H}})\mathbf{Y}, \quad (13)$$

where $\mathbf{I}$ denotes the unit matrix, the superscript H indicates the conjugate transpose operation, and $\mathbf{Y}$ is an arbitrary vector.

Let,

$$\mathbf{Y}=\mathbf{B}\mathbf{X}, \quad (14)$$

where $\mathbf{X}$ is an arbitrary vector, and its length equals the column vector number of matrix $\mathbf{B}$. Equation (14) means that $\mathbf{Y}$ is a linear combination of the column vectors of matrix $\mathbf{B}$. Substituting equation (14) into (13), we can obtain:

$$\mathbf{W}^*=(\mathbf{I}-\mathbf{Z}(\mathbf{Z}^{\mathrm{H}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{H}})\mathbf{B}\mathbf{X}. \quad (15)$$

It is easy to verify that $\mathbf{W}$ in equation (15) satisfies equation (11) and (12).

It is necessary to point out that if the expected pattern has multiple beams that direct to several different directions, matrix $\mathbf{B}$ has to include the steering vectors of these directions, and on the other hand, matrix $\mathbf{Z}$ has to exclude the steering vectors of these directions.

To further improve the flexibility, in equation (15), we can add a small perturbation to the diagonal elements of the unit matrix **I**, which can be mathematically expressed as:

$$\mathbf{W}^* = (s\mathbf{I} - \mathbf{Z}(\mathbf{Z}^H\mathbf{Z})^{-1}\mathbf{Z}^H)\mathbf{B}\mathbf{X}. \qquad (16)$$

In equation (16), $s = ue^{j\varphi}$ where u is a real number and φ is an arbitrary angle.

IV. SIMULATIONS OF THE NEW APPROACH

In this section, several examples are given to demonstrate the new approach and its performance.

We aim to investigate the beam forming of a uniform linear antenna array. It has N isotropic antenna elements, which are arranged along a line with spacing d. Let $d=\lambda/2$.The far field is considered and the narrow band signal centered at the wave length of λ is transmitted by each antenna element. The antenna array element arrangement is shown in Fig. 1.

Let the discrete values of θ, θ_1, θ_2, ..., θ_k, ..., θ_K, be equal to 0°, 1°, …, 179°, 180° respectively, which is in accord with the real practice and is convenient for digital processing. The pattern can be formed through following algorithm steps: i) Determine the beam directions and then use their steering vectors to make up matrix **B**; ii) Determine the non-beam directions and then use their steering vectors to make up matrix **Z**; iii) Obtain vector $\mathbf{W}_{LS}$ and **W** respectively through equation (6) and (16); iv) Obtain the formed pattern using equation (5).

In the first example, let N=12, the expected beams direct to θ_{40}=40° and θ_{90}=90°, let **X**=[1+j157/4π, 1+j157/4π]^T, s=1+j1000π, and the pattern generated by equation (16) is shown in Fig. 2. It can be seen in Fig. 2 that two beams pointing to the directions of θ_{40}=40° and θ_{90}=90° have the same gain. Then, we let the pattern generated by equation (16) be the vector **P** in equation (6). Thus, the weight vector $\mathbf{W}_{LS}$ can be obtained from equation (6). The pattern created by $\mathbf{W}_{LS}$ is also shown in Fig. 2. In the legend of Fig. 2, the new method indicates the pattern generated by equation (16), and LS refers to the pattern created by $\mathbf{W}_{LS}$. Figure 2 shows that the new method has a better performance than the least square method.

In the second example, still let N=12, the expected beams direct to θ_{40}=40° and θ_{90}=90°, let **X**=[1000+j157/4π, 500+j157/4π]^T, s=1+j1000π, and the pattern generated by equation (16) is shown in Fig. 3. It can be learned easily from Fig. 3 that the beam pointing to the direction of θ_{90}=90° has an approximately 6 dB attenuation compared with the beam directing to θ_{40}=40°. Then, we let the pattern generated by equation (16) be the vector **P** in equation (6). Thus, the weight vector $\mathbf{W}_{LS}$ can be obtained from equation (6). The pattern created by $\mathbf{W}_{LS}$ is also shown in Fig. 3. In the legend of Fig. 3, the new method indicates the pattern generated by equation (16), and LS refers to the pattern created by $\mathbf{W}_{LS}$. Figure 3 shows that the new method has a better performance than the least square method.

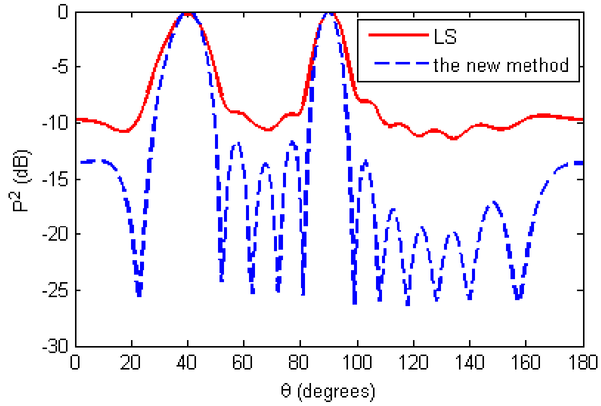

Fig. 2. The outcome of the first example (N=12).

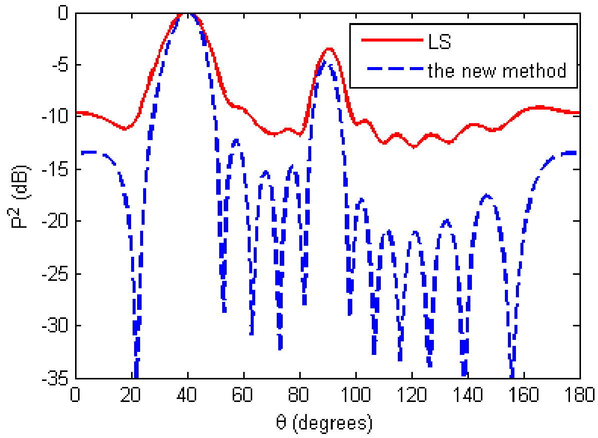

Fig. 3. The outcome of the second example (N=12).

In the third example, let N=24, the expected beams direct to θ_{40}=40°, θ_{90}=90° and θ_{120}=120°, let **X**=[1+j157/4π, 1+j157/4π, 1+j157/4π]^T, s=1+j1000π, and the pattern generated by equation (16) is shown in Fig. 4. It can be learned from Fig. 4 that three beams pointing to the directions of θ_{40}=40°, θ_{90}=90° and θ_{120}=120° have the same gain. Then, we let the pattern generated by equation (16) be the vector **P** in equation (6). Thus, the weight vector $\mathbf{W}_{LS}$ can be obtained from equation (6). The pattern created by $\mathbf{W}_{LS}$ is also shown in Fig. 4. In the legend of Fig. 4, the new method indicates the pattern generated by equation (16), and LS refers to the pattern created by $\mathbf{W}_{LS}$. Figure 4 shows that the new method has a better performance than the least square method.

In the fourth example, still let N=24, the expected beams direct to θ_{40}=40°, θ_{90}=90° and θ_{120}=120°, let **X**=[500+j157/4π, 1000+j157/4π, 500+j157/4π]^T, s=1+j1000π, and the pattern generated by equation (16) is shown in Fig. 5. It can be seen easily from Fig. 5 that the beams pointing to the directions of θ_{40}=40° and θ_{120}=120° have an approximately 6 dB attenuation compared with the beam directing to θ_{90}=90°. Then, we let the pattern generated by equation (16) be the vector **P** in equation (6). Thus, the weight vector $\mathbf{W}_{LS}$ can be obtained from equation (6). The pattern created by $\mathbf{W}_{LS}$ is also shown in Fig. 5. In the legend of Fig. 5, the new method indicates the pattern generated by equation (16), and LS refers to the pattern created by $\mathbf{W}_{LS}$. Figure 5 shows that the new method has a better performance than the least square method.

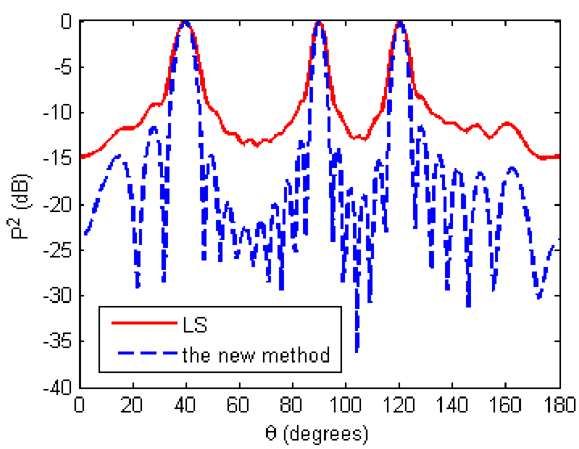

Fig. 4. The outcome of the third example (*N*=24).

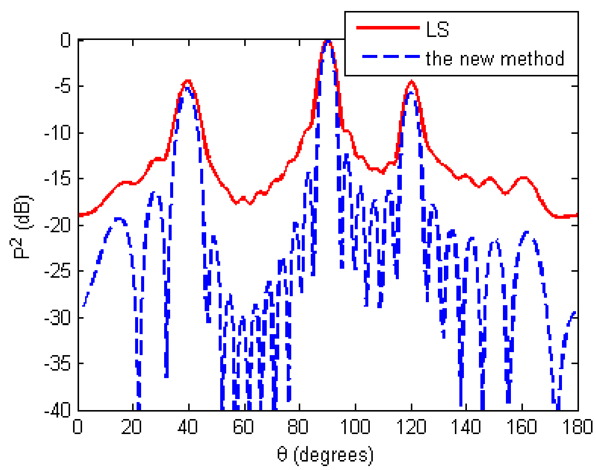

Fig. 5. The outcome of the fourth example (*N*=24).

According to our simulations, the beam gain is mainly determined by the image part of *s* and the real part ratio of the elements of **X**. To easily adjust the beam gain, the real parts of the elements of **X** should be a number of several hundreds, and the image parts of the elements of **X** have little effect on the beam gain. Additionally, the image part of *s* is no less than 3π, while the real part of *s* is equal to 1. To estimate the computational complexity of the new method, we compare equation (16) with equation (6). Since both **B** and **Z** are a part of matrix **A**, the computational complexity of equation (16) is similar to that using equation (6) to form the pattern. We simulate the new approach using the MATLAB software platform on an HP notebook PC with a core i5-5200U CPU and a 4G memory. All simulations in this paper take approximately less than one second to obtain the final outcomes.

V. CONCLUSION

For an antenna array whose manifold matrix has been determined, we divide the matrix into two parts. The first part is the beam vector space. The second part is the side lobe vector space. We obtain a vector by linearly combining the beam vectors and orthogonally projecting the combined vector into the orthogonal projection matrix of the side lobe vector space, and this projection vector is just the solution of the excitation vector of the antenna array elements. Since the combined vector is obtained from the linear combination of the beam vectors, we can control the beam gain by adjusting the linear combination coefficients. This new method can be easily used to form multi-beams and adjust each beam gain. The results of the simulations show, that the new method acts effectively and efficiently.

ACKNOWLEDGEMENT

This work has been supported in part by the Chinese Government under the National Natural Science Foundation No. 61501340, and Education Department of Shanxi province under the Special Science and Technology Foundation No. 17JK0397.

REFERENCES

[1] T. Ohira, "Adaptive array antenna beamforming architectures as viewed by a microwave circuit designer," *Proc. Asia–Pacific Microw. Conf.*, Sydney, Australia, pp. 828-833, Dec. 2000.

[2] S. Denno and T. Ohiro, "Modified constant modulus algorithm for digital signal processing adaptive antennas with microwave analog beam-forming," *IEEE Trans. Antennas Propag.*, vol. 50, no. 6, pp. 850-857, June 2002.

[3] H. He, P. Stoica, and J. Li, "Wideband MIMO systems: Signal design for transmit beampattern synthesis," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 618-628, Feb. 2011.

[4] G. Oliveri and A. Massa, "Bayesian compressive sampling for pattern synthesis with maximally sparse non-uniform linear arrays," *IEEE Transactions on Antennas Propagation*, vol. 59, no. 2, pp. 467-481, Feb. 2011.

[5] H. Chen, Q. Wang, and R. Fan, "Beampattern

synthesis using reweighted *l*1-norm minimization and array orientation diversity," *Radioengineering*, vol. 22, no. 2, pp. 602-609, June 2013.
[6] W. Choi, T. K. Sarkar, H. Wang, and E. L. Mokole, "Adaptive processing using real weights based on a direct data domain least squares approach," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 1, pp. 182-191, Jan. 2006.
[7] A. Morabito and P. Rocca, "Optimal synthesis of sum and difference patterns with arbitrary side-lobes subject to common excitations constraints," *IEEE Antennas Wireless Propag. Lett.*, vol. 9, pp. 623-626, July 2010.
[8] T. K. Sarkar, S. Bhumkar, N. Yilmazer, et al., "Non-statistical multi-beamformer," *Journal of Signal and Information Processing*, vol. 3, no. 1, pp. 26-29, Jan. 2012.
[9] A. F. Morabito, A. Massa, P. Rocca, and T. Isernia, "An effective approach to the synthesis of phase-only reconfigurable linear arrays," *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 3622-3631, Aug. 2012.
[10] X. Li, W. T. Li, X. W. Shi, et al., "Modified differential evolution strategy for antenna array pattern synthesis," *Progr. Electromagn. Res.*, vol. 137, no. 2, pp. 371-388, Feb. 2013.
[11] X. D. Zhang, *Modern Signal Processing*. Tsinghua University Press, Beijing, 2012.
[12] T. K. Sarkar, J. Koh, R. Adve, et al., "A pragmatic approach to adaptive antennas," *IEEE Acoust., Speech, and Signal Processing Mag.*, vol. 42, no. 2, pp. 39-55, Feb. 2000.
[13] R. L. Haupt, "Phased array beam steering through serial control of the phase shifters," *Appl. Comput. Electrom. Society Journal*, vol. 32, no. 12, pp. 1140-1143, Dec. 2017.
[14] J. Nourinia, C. Ghobadi, B. Mohammadi, and F. Alizadeh, "Study of phase and patterns characteristics of a sub-wavelength broadband reflectarray unit element based on triple concentric circular-rings," *Appl. Comput. Electrom. Society Journal*, vol. 33, no. 6, pp. 714-718, June 2018.

Jie Chen is now an Associate Professor of School of Electronic Engineering, Xi'an Aeronautical University, No.259 Xi'an West Second Ring Road, Xi'an, 710077, China. He received his bachelor and master degree in Electronic Science and Technology major from School of Electronic and Information, Xi'an Jiaotong University in 1997 and 2004 respectively. In 2013, he achieves doctor degree in Electromagnetic Theory and Microwave Technique major from School of Electronic Engineering, Xidian University. His research interests are in the areas of smart antenna, antenna array, microwave circuit design, wireless communication and RFID technique.

Yingzeng Yin is now a Professor of Xidian University. He works in National Key Laboratory of Antenna and Microwave Technology, School of Electronic Engineering, Xidian University, Xi'an, 710071, China. His research activities are in the fields of microwave circuit design, antenna design, smart antenna design and RFID technique.