LCP Plane Wave Scattering by a Chiral Elliptic Cylinder Embedded in Infinite Chiral Medium

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Abstract **─** An analytic solution is presented to the scattering of a left circularly polarized (LCP) plane wave from a chiral elliptic cylinder placed in another infinite chiral medium, using the method of separation of variables. The incident, scattered, as well as the transmitted electromagnetic fields are expressed using appropriate angular and radial Mathieu functions and expansion coefficients. The unknown scattered and transmitted field expansion coefficients are subsequently determined by imposing proper boundary conditions at the surface of the elliptic cylinder. Numerical results are presented graphically as normalized scattering widths for elliptic cylinders of different sizes and chiral materials, to show the effects of these on the scattering widths.

Index Terms **─** Chiral-chiral material, elliptic cylinder, LCP and RCP, Mathieu functions, scattering cross section.

I. INTRODUCTION

It is well known that a reciprocal and isotropic chiral medium is characterized by different phase velocities for right- and left-circularly polarized (RCP and LCP) waves. In a lossless isotropic chiral medium, a linearly polarized wave undergoes a rotation of its polarization while it propagates. Numerous developments linked to chiral media overall are described in [1-3], and some analytical and numerical solutions to scattering from various types of chiral objects are given in [4-11].

The elliptic cylinder is a geometry that has been extensively analyzed in literature due to its ability to create cylindrical cross sections of different shapes by changing the axial ratio of the ellipse. Furthermore, since the elliptic cylindrical coordinate system is one of the coordinate systems in which the wave equation is separable, solutions to problems involving elliptic cylinders can be obtained in exact form. The solution of chiral cylinder immersed in unbounded chiral media will be the first step in providing an analytic solution to the complex problem of chiral elliptic cylinder coated with another layer of chiral media. Also, the analytic solution may be used as a benchmark for validating solutions to similar scattering problems using approximate or numerical methods.

In this paper, we present a solution to the problem of the scattering of a left-circularly polarized (LCP) plane wave from a chiral elliptic cylinder of arbitrary axial ratio placed in a distinct infinite chiral medium, while in [11] the chiral cylinder was embedded in free space. The solution of the chiral cylinder immersed in unbounded free space involves co and cross polarized fields with free space wave numbers where in the case of unbounded chiral media the solution involves both co and cross polarized fields with left and right circularly polarized wave numbers regardless if the incident field is left or right circularly polarized field. It is required to compute Mathieu functions with left and right circulation wave numbers inside and outside the elliptic cylinder. Both chiral media in this problem are isotropic. The obtained solution will therefore provide more parameters to control the normalized bistatic scattering width when compared to that in reference [11].

II. FORMULATION

Consider a LCP plane wave that is propagating in an infinite isotropic chiral medium, being incident on an infinitely long elliptic cylinder at an angle φ_i with respect to the minus x-axis of a Cartesian coordinate system located at the center of a cross section of the cylinder with its z-axis along the axis of the cylinder, which is made up of a different chiral material and of major axis length 2*a* and minor axis length 2*b*, as shown in Fig. 1.

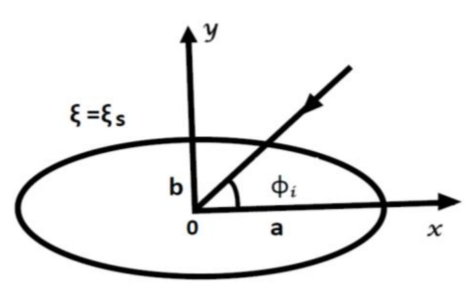

Fig. 1. Geometry of the scattering problem.

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The incident electric field for LCP can be expanded in terms of elliptical vector wave functions as:

$$\mathbf{E}^{i} = \sum_{m=0,1}^{\infty} A_{\substack{em\\om}}[\mathbf{N}^{(1)}_{\substack{em\\om}}(c_{L1},\mathbf{r}) - \mathbf{M}^{(1)}_{\substack{em\\om}}(c_{L1},\mathbf{r})], \quad (1)$$

in which,

$$A_{\substack{em\\om}} = E_0 j^m \frac{\sqrt{8\pi}}{N_{\substack{em\\om}}(c_{L1})} S_{\substack{em\\om}}(c_{L1},\cos\phi_i), \quad (2)$$

where,

$$N_{\substack{em\\om}}(c_{L1}) = \int_0^{2\pi} [S_{\substack{em\\om}}(c_{L1},\cos v)]^2 dv, \quad (3)$$

with $c_{L1} = k_{L1}F$, $S_{qn}(c_\alpha, \cos v)$ for q=e,o being the angular Mathieu function of order n and arguments c_α and $\cos v$, and F being the semi-focal length of the cylinder. $\Xi^{(i)}_{qm}(c_\alpha, \mathbf{r})$ for q=e,o, and $\mathbf{\Xi} = \mathbf{M}, \mathbf{N}$ are defined in [13-14] in terms of angular and radial Mathieu functions, with $\mathbf{r}$ designating the elliptic coordinate dyad (ξ,η). The summation over m in (1) starts from 0 for even (e) functions and from 1 for odd (o) functions, and is the same for the other field expansions given below too. The wavenumber of the left circularly polarized wave is given by [12],

$$k_{L1} = \frac{k_0\sqrt{\mu_{r1}\varepsilon_{r1}}}{1 + k_0\gamma_1\sqrt{\mu_{r1}\varepsilon_{r1}}}, \quad (4)$$

where k_0 is the wavenumber in free space, γ_1 is the chirality parameter of the external chiral medium, and μ_{r1} and ε_{r1} are the relative permeability and relative permittivity of the external chiral medium.

The incident magnetic field may be expressed in terms of elliptical vector wave functions as:

$$\mathbf{H}^{i} = \frac{j}{Z_1}\sum_{m=0,1}^{\infty} A_{\substack{em\\om}}[\mathbf{M}^{(1)}_{\substack{em\\om}}(c_{L1},\mathbf{r}) - \mathbf{N}^{(1)}_{\substack{em\\om}}(c_{L1},\mathbf{r})], \quad (5)$$

where $Z_1 = Z_0\sqrt{\mu_{r1}/\varepsilon_{r1}}$, with Z_0 denoting the free space wave impedance.

As the elliptic cylinder consists of an isotropic chiral material and is surrounded by another infinite isotropic chiral medium, the scattered and transmitted electromagnetic fields will have both co-polar and cross-polar components. The scattered fields can be written as:

$$\mathbf{E}^{s}_{\mathbf{cp}} = \sum_{m=0,1}^{\infty} B_{\substack{em\\om}}[\mathbf{N}^{(4)}_{\substack{em\\om}}(c_{L1},\mathbf{r}) - \mathbf{M}^{(4)}_{\substack{em\\om}}(c_{L1},\mathbf{r})], \quad (6)$$

$$\mathbf{E}^{s}_{\mathbf{xp}} = \sum_{m=0,1}^{\infty} C_{\substack{em\\om}}[\mathbf{N}^{(4)}_{\substack{em\\om}}(c_{R1},\mathbf{r}) + \mathbf{M}^{(4)}_{\substack{em\\om}}(c_{R1},\mathbf{r})], \quad (7)$$

$$\mathbf{H}^{s}_{\mathbf{cp}} = \frac{j}{Z_1}\sum_{m=0,1}^{\infty} B_{\substack{em\\om}}[\mathbf{M}^{(4)}_{\substack{em\\om}}(c_{L1},\mathbf{r}) - \mathbf{N}^{(4)}_{\substack{em\\om}}(c_{L1},\mathbf{r})], \quad (8)$$

$$\mathbf{H}^{s}_{\mathbf{xp}} = \frac{j}{Z_1}\sum_{m=0,1}^{\infty} C_{\substack{em\\om}}[\mathbf{N}^{(4)}_{\substack{em\\om}}(c_{R1},\mathbf{r}) + \mathbf{M}^{(4)}_{\substack{em\\om}}(c_{R1},\mathbf{r})], \quad (9)$$

where B_{qm} and C_{qm} for q=e,o are the unknown field expansion coefficients, $c_{R1} = k_{R1}F$, with the wave-number k_{R1} for the RCP wave given by [12],

$$k_{R1} = \frac{k_0\sqrt{\mu_{r1}\varepsilon_{r1}}}{1 - k_0\gamma_1\sqrt{\mu_{r1}\varepsilon_{r1}}}. \quad (10)$$

The fields transmitted into the chiral elliptic cylinder may also be expressed as:

$$\mathbf{E}^{t}_{\mathbf{cp}} = \sum_{m=0,1}^{\infty} D_{\substack{em\\om}}[\mathbf{N}^{(1)}_{\substack{em\\om}}(c_{L2},\mathbf{r}) - \mathbf{M}^{(1)}_{\substack{em\\om}}(c_{L2},\mathbf{r})], \quad (11)$$

$$\mathbf{E}^{t}_{\mathbf{xp}} = \sum_{m=0,1}^{\infty} G_{\substack{em\\om}}[\mathbf{N}^{(1)}_{\substack{em\\om}}(c_{R2},\mathbf{r}) + \mathbf{M}^{(1)}_{\substack{em\\om}}(c_{R2},\mathbf{r})], \quad (12)$$

$$\mathbf{H}^{t}_{\mathbf{cp}} = \frac{j}{Z_2}\sum_{m=0,1}^{\infty} D_{\substack{em\\om}}[\mathbf{M}^{(1)}_{\substack{em\\om}}(c_{L2},\mathbf{r}) - \mathbf{N}^{(1)}_{\substack{em\\om}}(c_{L2},\mathbf{r})], \quad (13)$$

$$\mathbf{H}^{t}_{\mathbf{xp}} = \frac{j}{Z_2}\sum_{m=0,1}^{\infty} G_{\substack{em\\om}}[\mathbf{N}^{(1)}_{\substack{em\\om}}(c_{R2},\mathbf{r}) + \mathbf{M}^{(1)}_{\substack{em\\om}}(c_{R2},\mathbf{r})], \quad (14)$$

where D_{qm} and G_{qm} for q=e,o are the unknown field expansion coefficients, $c_{R2} = k_{R2}F$ and $c_{L2} = k_{L2}F$, with expressions for k_{R2} and k_{L2} obtained from (4) and (10), respectively, by changing the subscript 1 in these equations to 2.

The unknown expansion coefficients can be obtained by imposing the boundary conditions corresponding to the continuity of the tangential field components at the surface $\xi = \xi_s$ of the chiral elliptic cylinder [15], which may be expressed mathematically as:

$$(\mathbf{E}^{i} + \mathbf{E}^{s}) \times \hat{\xi}\big|_{\xi=\xi_s} = \mathbf{E}^{t} \times \hat{\xi}\big|_{\xi=\xi_s}, \quad (15)$$

$$(\mathbf{H}^{i} + \mathbf{H}^{s}) \times \hat{\xi}\big|_{\xi=\xi_s} = \mathbf{H}^{t} \times \hat{\xi}\big|_{\xi=\xi_s}, \quad (16)$$

where $\hat{\boldsymbol{\xi}}$ is the outward unit normal to the surface of the elliptic cylinder.

Substituting the above developed expressions into the fields of (15) and (16), and applying the orthogonal property of the angular Mathieu functions, yield:

$$\begin{aligned}&[A_{qn}R^{(1)}_{qn}(c_{L1},\xi_s) + B_{qn}R^{(4)}_{qn}(c_{L1},\xi_s)]N_{qn}(c_{L1})\\ &+\sum_m C_{qm}R^{(4)}_{qm}(c_{R1},\xi_s)M_{qmn}(c_{L1},c_{R1})\\ &=\sum_m D_{qm}R^{(1)}_{qm}(c_{R2},\xi_s)M_{qmn}(c_{R2},c_{L1})\\ &+\sum_m G_{qm}R^{(1)}_{qm}(c_{L2},\xi_s)M_{qmn}(c_{L2},c_{L1})\end{aligned}, \quad (17)$$

$$\begin{aligned}&[A_{qn}R^{(1)'}_{qn}(c_{L1},\xi_s) + B_{qn}R^{(4)'}_{qn}(c_{L1},\xi_s)]N_{qn}(c_{L1})\\ &-\frac{k_{L1}}{k_{R1}}\sum_m C_{qm}R^{(4)'}_{qm}(c_{R1},\xi_s)M_{qmn}(c_{L1},c_{R1})\\ &=\frac{k_{L1}}{k_{R2}}\sum_m D_{qm}R^{(1)'}_{qm}(c_{R2},\xi_s)M_{qmn}(c_{R2},c_{L1})\\ &-\frac{k_{L1}}{k_{L2}}\sum_m G_{qm}R^{(1)'}_{qm}(c_{L2},\xi_s)M_{qmn}(c_{L2},c_{L1})\end{aligned}, \quad (18)$$

$$\begin{aligned}&[A_{qn}R_{qn}^{(1)}(c_{L1},\xi_s)+B_{qn}R_{qn}^{(4)}(c_{L1},\xi_s)]N_{qn}(c_{L1})\\&-\sum_m C_{qm}R_{qm}^{(4)}(c_{R1},\xi_s)M_{qmn}(c_{L1},c_{R1})\\&=\frac{Z_1}{Z_2}\sum_m D_{qm}\,R_{qm}^{(1)}(c_{R2},\xi_s)M_{qmn}(c_{R2},c_{L1})\\&-\frac{Z_1}{Z_2}\sum_m G_{qm}\,R_{qm}^{(1)}(c_{L2},\xi_s)M_{qmn}(c_{L2},c_{L1})\end{aligned}\quad, \tag{19}$$

$$\begin{aligned}&[A_{qn}R_{qn}^{(1)'}(c_{L1},\xi_s)+B_{qn}R_{qn}^{(4)'}(c_{L1},\xi_s)]N_{qn}(c_{L1})\\&+\frac{k_{L1}}{k_{R1}}\sum_m C_{qm}R_{qm}^{(4)'}(c_{R1},\xi_s)M_{qmn}(c_{L1},c_{R1})\\&=\frac{k_{L1}Z_1}{k_{R2}Z_2}\sum_m D_{qm}\,R_{qm}^{(1)'}(c_{R2},\xi_s)M_{qmn}(c_{R2},c_{L1})\\&+\frac{k_{L1}Z_1}{k_{L2}Z_2}\sum_m G_{qm}\,R_{qm}^{(1)'}(c_{L2},\xi_s)M_{qmn}(c_{L2},c_{L1})\end{aligned}\quad, \tag{20}$$

for q=e,o where $R_{qn}^{(i)}(c_\alpha,\xi_s)$ is the radial Mathieu function of order n and kind (i) of arguments c_α and ξ_s, and $M_{qmn}(c_\alpha,c_\sigma)$ is given by:

$$M_{qmn}(c_\alpha,c_\sigma)=\int_0^{2\pi}S_{qm}(c_\alpha,\cos v)S_{qn}(c_\sigma,\cos v)dv\,. \tag{21}$$

The system of equations (17) to (20) may be written in matrix form as:

$$\begin{bmatrix}Q_{11}&Q_{12}&Q_{13}&Q_{14}\\Q_{21}&Q_{22}&Q_{23}&Q_{24}\\Q_{31}&Q_{32}&Q_{33}&Q_{34}\\Q_{41}&Q_{42}&Q_{43}&Q_{44}\end{bmatrix}\begin{bmatrix}B_q\\C_q\\D_q\\G_q\end{bmatrix}=\begin{bmatrix}V_1\\V_2\\V_3\\V_4\end{bmatrix}, \tag{22}$$

where the elements of the submatrices are defined in Appendix A. We solve for unknown expansion coefficients B_q, C_q, D_q, G_q from equation (22) by using matrix inversion technique.

Using asymptotic expressions of the radial Mathieu functions of the fourth kind and their first derivatives, we can write expressions for the normalized bistatic echo width of the right- and left-polarized waves can then be written as:

$$\frac{\sigma_L}{\lambda_L}=\left|\Omega_{cp}(\phi)\right|^2,\quad \frac{\sigma_R}{\lambda_R}=\left|\Omega_{xp}(\phi)\right|^2, \tag{23}$$

where,

$$\Omega_{cp}(\phi)=\sum_{q=e,o}\sum_{m=0,1}^{\infty}j^m B_{qm}\,S_{qm}(c_{L1},\cos\phi)\,, \tag{24}$$

$$\Omega_{xp}(\phi)=\sum_{q=e,o}\sum_{m=0,1}^{\infty}j^m C_{qm}\,S_{qm}(c_{R1},\cos\phi)\,, \tag{25}$$

where B_{qm}, C_{qm} can be obtained by invoking equation (22).

III. NUMERICAL RESULTS

Since the summations are infinite in extent, to obtain numerical results these summations have to be truncated to include only the first N terms, where N is an integer proportional to the electrical size and the constitutive parameters of the composite object. The results given in this paper have been checked for convergence, and obtained by considering only the first 10 terms (i.e., N = 10) of the infinite series associated with each even and odd function.

Numerical results are presented as normalized echo pattern widths for isotropic chiral elliptic cylinders of different axial ratios, embedded in another infinite isotropic chiral medium of different relative permittivities and chirality parameters. First, we select the parameters $\varepsilon_{r1}=1.0$, $\mu_{r1}=1.0$, $k_0\gamma_1=0.0$ for the exterior region while $k_0a=0.16\pi$, $\varepsilon_{r2}=4.0$, $\mu_{r2}=2.0$, $k_0\gamma_2=0.15$, and axial ratio a/b = 1.001 for the cylinder and $\varphi_i=180^{\circ}$. To validate the analysis and the calculated results, we computed the normalized echo pattern widths for the above chiral cylinder when it is excited by a plane wave that is transverse magnetically (TM) polarized in the axial z-direction. The results shown in Fig. 2 are in good agreement with those in [5] (circles) for an analogous chiral circular cylinder in free space, verifying the accuracy of the analysis and calculated results.

Figure 3 displays the normalized left- and right-polarized echo-width patterns for a chiral elliptic cylinder of axial ratio 2, with same the parameters as in Fig. 2 when it is embedded in an exterior chiral medium having the parameters $\varepsilon_{r1}=1.0$, $\mu_{r1}=1.0$, $k_0\gamma_1=0.15$ while excited by a LCP plane wave incident with $\varphi_i=180^{\circ}$. In this plot, the dominant left-polarized echo-width magnitude decreases gradually as the scattering angle increases from 0° to 180° while the corresponding right-polarized has an almost constant value at all scattering angles of -16 dB.

Figure 4 shows the left- and right-polarized echo-width patterns for the chiral elliptic cylinder in Fig. 2, when it is placed in a chiral medium which is similar to that in Fig. 3, but with $\varepsilon_{r1}=3.0$. When the results are compared with the ones in Fig. 3, we see that as the scattering angle increases from 0° to 180°, the reduction of the left-polarized echo-width magnitude is much higher. Also the right-polarized echo-width magnitude is much lower for all scattering angles.

Figure 5 shows is similar to Fig. 4, but with $\varepsilon_{r1}=3.0$ and $k_0\gamma_1=0.1$. We observe that as the scattering angle increases from 0° to 180°, the reduction of the left-polarized echo-width magnitude is much higher than in Figs. 3 and 4. Also the right-polarized echo-width magnitude is becoming closer to the left-polarized echo-width as the scattering angles increase. Figures 6 and 7 are similar to Fig. 5 but with $\varphi_i=45^0$ and $k_0\gamma_1=0.2$.

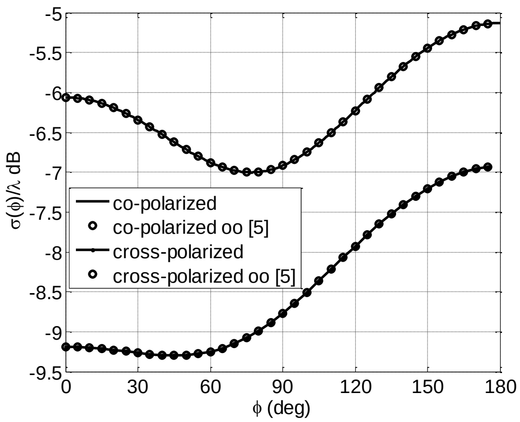

Fig. 2. Normalized co-polar and cross polar bistatic scattering widths against the scattering angle, for a chiral elliptic cylinder of axial ratio *a*/*b* =1.001, with $k_0 a = 0.16\pi$, $\varepsilon_{r2} = 4.0$, $\mu_{r2} = 2.0$, $k_0\gamma_2 = 0.15$, and located in free space, when it is excited by a TM polarized plane wave incident at $\varphi_i = 180^{\mathrm{o}}$. Circles [5].

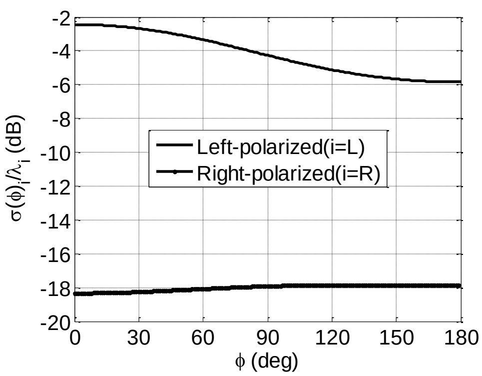

Fig. 3. Normalized left- and right-polarized bistatic scattering widths against the scattering angle for a chiral elliptic cylinder of axial ratio 2.0 and having the same parameters as those in Fig. 2, with $\varepsilon_{r1} = 1.0$, $\mu_{r1} = 1.0$, $k_0\gamma_1 = 0.15$ and $\varphi_i = 180^{\mathrm{o}}$.

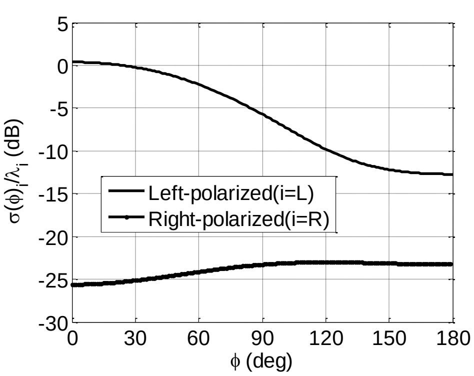

Fig. 4. Normalized left- and right-polarized bistatic scattering widths against the scattering angle for the chiral elliptic cylinder as in Fig. 3 while $\varepsilon_{r1} = 3.0$.

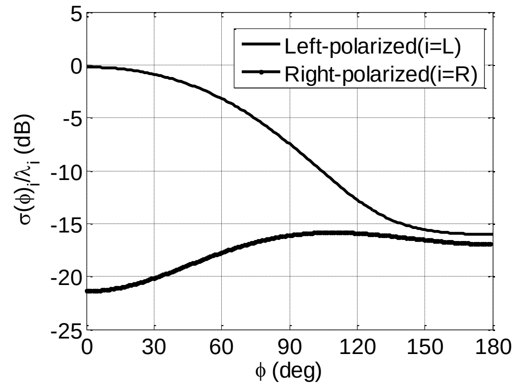

Fig. 5. Normalized left- and right-polarized bistatic scattering widths against the scattering angle for the chiral elliptic cylinder as in Fig. 4 while $\varepsilon_{r1} = 5.0$ and $k_0\gamma_1 = 0.1$.

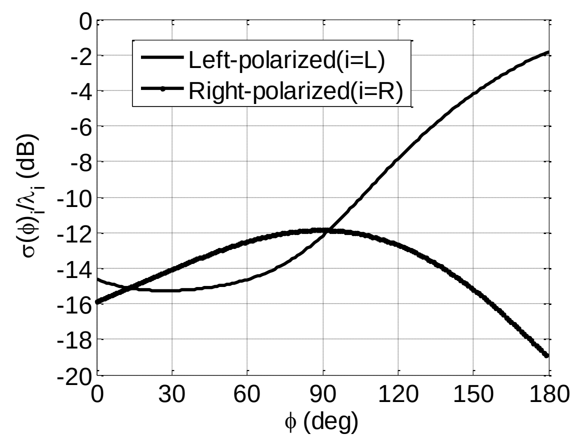

Fig. 6. Normalized left- and right-polarized bistatic scattering widths against the scattering angle for the chiral elliptic cylinder as in Fig. 5 while $\varphi_i = 45^0$.

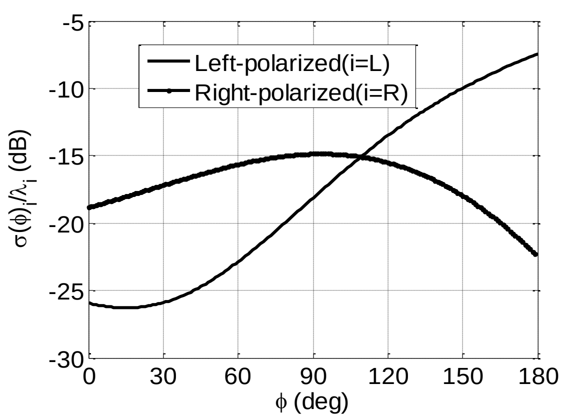

Fig. 7. Normalized left- and right-polarized bistatic scattering widths against the scattering angle for the chiral elliptic cylinder as in Fig. 5 while $k_0\gamma_1 = 0.2$.

IV. CONCLUSIONS

An analytic solution to the problem of scattering of an LCP plane wave by a chiral elliptic cylinder embedded in another infinite chiral medium is presented using the method of the separation of variables. Results have been presented as normalized bistatic for co and cross-polarized echo-width patterns for chiral elliptic cylinders of different axial ratios and chiral materials, to show the effects of these on scattering. It is seen that the presence of two different chiral materials could significantly influence the co and cross-polarized pattern widths and can be used to control the radar cross section of targets or antenna radiation pattern [8]. Finally, the solution presented in this paper is for LCP incident field while in [15] was for RCP.

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Appendix A

$$Q_{11nn} = R_{qn}^{(4)}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A1}$$

$$Q_{12nm} = R_{qm}^{(4)}(c_{R1}, \xi_s) M_{q\,mn}(c_{L1}, c_{R1}), \tag{A2}$$

$$Q_{13nm} = -R_{qm}^{(1)}(c_{R2}, \xi_s) M_{q\,mn}(c_{L1}, c_{R2}), \tag{A3}$$

$$Q_{14nm} = -R_{qm}^{(1)}(c_{L2}, \xi_s) M_{q\,mn}(c_{L2}, c_{L1}), \tag{A4}$$

$$Q_{21nn} = R_{qn}^{(4)'}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A5}$$

$$Q_{22nm} = -\frac{k_{L1}}{k_{R1}} R_{qm}^{(4)'}(c_{R1}, \xi_s) M_{q\,mn}(c_{L1}, c_{R1}), \tag{A6}$$

$$Q_{23nm} = -\frac{k_{L1}}{k_{R2}} R_{qm}^{(1)'}(c_{R2}, \xi_s) M_{q\,mn}(c_{L1}, c_{R2}), \tag{A7}$$

$$Q_{24nm} = \frac{k_{L1}}{k_{L2}} R_{qm}^{(1)'}(c_{L2}, \xi_s) M_{q\,mn}(c_{L2}, c_{L1}), \tag{A8}$$

$$Q_{31nn} = R_{qn}^{(4)}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A9}$$

$$Q_{32nm} = -R_{qm}^{(4)}(c_{R1}, \xi_s) M_{q\,mn}(c_{L1}, c_{R1}), \tag{A10}$$

$$Q_{33nm} = -\frac{Z_1}{Z_2} R_{qm}^{(1)}(c_{R2}, \xi_s) M_{q\,mn}(c_{L1}, c_{R2}), \tag{A11}$$

$$Q_{34nm} = \frac{Z_1}{Z_2} R_{qm}^{(1)}(c_{L2}, \xi_s) M_{q\,mn}(c_{L2}, c_{L1}), \tag{A12}$$

$$Q_{41nn} = R_{qn}^{(4)'}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A13}$$

$$Q_{42nm} = \frac{k_{L1}}{k_{R1}} R_{qm}^{(4)'}(c_{R1}, \xi_s) M_{q\,mn}(c_{L1}, c_{R1}), \tag{A14}$$

$$Q_{43nm} = -\frac{k_{L1}Z_1}{k_{R2}Z_2} R_{qm}^{(1)'}(c_{R2}, \xi_s) M_{q\,mn}(c_{L1}, c_{R2}), \tag{A15}$$

$$Q_{44nm} = -\frac{k_{L1}Z_1}{k_{L2}Z_2} R_{qm}^{(1)'}(c_{L2}, \xi_s) M_{q\,mn}(c_{L2}, c_{L1}), \tag{A16}$$

$$V_{1n} = -A_{qn} R_{qn}^{(1)}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A17}$$

$$V_{1n} = -A_{qn} R_{qn}^{(1)'}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A18}$$

$$V_2 = -A_{qn} R_{qn}^{(1)}(c_{L1}, \xi_s) N_{qn}(c_{L1}), \tag{A19}$$

$$V_3 = V_1 , \tag{A20}$$

$$V_4 = V_2 . \tag{A21}$$

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