

# An Analytical Approach for a Miniaturized Unequal Wilkinson Power Divider with Filtering Response

Y. Torabi and G. Dadashzadeh

Department of Electrical Engineering  
University of Shahed, Tehran, 3319118651, Iran  
y.torabi@shahed.ac.ir, gdadashzadeh@shahed.ac.ir

**Abstract** — In this paper, a closed-form design method for a novel compact unequal filtering Wilkinson power divider, WPD, based on symmetric and asymmetric right/left-handed (CRLH) coupled-line circuit structures is proposed and investigated. The structure is composed of an input asymmetric CRLH coupled-line (A-CRLH) section along with two output symmetric CRLH coupled-line band-pass filter transformers to provide unequal filtering performance. The analytical design equations along with ideal closed-form expressions for scattering parameters are derived for the proposed structure by using in- and anti-phase ( $c/\pi$ -mode) decomposition method. To theoretically verify the proposed design approach, two filtering unequal WPDs with different high-power dividing ratios of 1:10 and 1:15 are designed and simulated. Finally, as a typical example, a practical filtering WPD with a power-splitting ratio of 1:15 operating at 1.4 GHz, is implemented with more than 55.5% size reduction as compared to conventional one. Good agreement between calculated, simulated and measured results verifies the effectiveness of the proposed power divider for miniaturized, large power dividing ratio and filtering applications.

**Index Terms** — CRLH, filtering, scattering parameters, unequal power divider.

## I. INTRODUCTION

Various Wilkinson power dividers (WPD) with simultaneously filtering and unequal response are important components for microwave subsystems. As it is well known, for industrial systems and other applications like phased-array systems and beam-forming networks, compact unequal power dividers with arbitrary power division ratios and filtering response are required as key components. To date, the filtering performance of WPDs has been motivation of many research works and different techniques have been used in practice to provide unequal power dividing function with favorable selectivity [1]-[4], however, all the proposed topologies are primarily limited to lower power division ratios and/or non-exact analytical design

equations. Recently, the authors introduced in [5], that asymmetric composite right/left handed (A-CRLH) coupled-line structure, realized by loading host coupled transmission line medium with series capacitors and shunt inductors to the ground, has the capability to be flexibly designed for any desired modal characteristic impedances and frequency dispersion without manufacturing difficulty as opposed to the conventional coupled lines. By using this fact, a novel unequal WPD with high power splitting ratio is proposed, however, it is not able to effectively suppress the undesired frequency signals out of its operating bandwidth [5].

This paper documented here and in an earlier work published by the authors as the conference paper [6], is the first report on the application of symmetric and asymmetric CRLH coupled lines for the designing of a novel filtering WPD with the ability to simultaneously provide sharp filtering response with high power-dividing ratio as 1:15. The current study, presents a substantially expanded analysis of the briefly introduced earlier work [6], with analytic and measurement verifications. Particularly, synthesis equations along with ideal closed-form expressions for scattering parameters, which allow the designer to conveniently design and simulate the proposed structure, are derived. Furthermore, to demonstrate the proposed analytical design approach, two examples including unequal filtering WPDs with 1:10 and 1:15 power splitting ratios supported by S-plots obtained from analytical closed-form equations are presented. Finally, a prototype filtering 1:15 unequal WPD is implemented and tested for validation. The measured results verify the application of the proposed structure, experimentally.

## II. THEORY AND DESIGN

Figure 1 (a) shows the outline of the proposed WPD. It comprises an A-CRLH coupled-line section as a quarter-wavelength transformer between input and output ports to provide unequal power division ratio and two output symmetric CRLH coupled-line band-pass filter transformers to fulfill the matching condition for the system impedance of  $50 \Omega$  as well as to obtain the

favorite selectivity in each transmission path. Typical A-CRLH coupled line as depicted in Fig. 1 (b), consists of two different CRLH transmission lines which can be assembled from  $N$  unit cells with right-handed (RH) and left-handed (LH) sections [5]. The LH section is based on lumped loading series capacitors ( $C_{La}$  and  $C_{Lb}$ ) and shunt inductors ( $L_{La}$  and  $L_{Lb}$ ) and the RH section is realized with conventional microstrip coupled lines as seen in Fig. 1 (b). Since each A-CRLH coupled-line section is implemented by periodically chaining  $N$  number of unit cells with electrical length ' $p$ ' much smaller than the wavelength, it can be regarded effectively homogeneous and characterized by the general coupled-mode differential equations [7]. To derive closed-form design equations, first of all, the power dividing ratio and filtering response analyses are investigated using coupled-mode formulation in sections A and B, respectively, then, the generalized scattering parameters are obtained in a closed-form in section C.

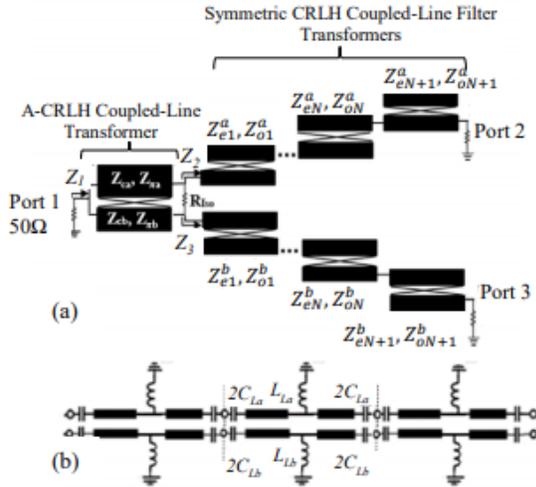


Fig. 1. (a) Schematic representation of a proposed unequal filtering WPD based on symmetric and asymmetric CRLH coupled lines, and (b) a typical structure of the asymmetric CRLH coupled lines.

### A. Power-dividing ratio analysis

Based on in- and anti-phase ( $c/\pi$ -mode) decomposition method, the equivalent c-mode characteristic impedances of the A-CRLH coupled-line section ( $Z_{ca}$  and  $Z_{cb}$ ) and isolation resistor ( $R_{Iso}$ ) can be obtained as following to satisfy the demanding power dividing ratio,  $k^2$ , ( $P_2/P_3=k^2$ ), with perfect input and output matching and isolation between output ports:

$$\begin{aligned} Z_{ca} &= \sqrt{\frac{1+k^2}{k^2}} Z_1 Z_2, \\ Z_{cb} &= \sqrt{(1+k^2)} k^2 Z_1 Z_2 = k^2 Z_{ca}, \\ R_{Iso} &= (1+k^2) Z_2. \end{aligned} \quad (1)$$

Here,  $Z_1$  and  $Z_2$  are the input and output impedances seen from ports 1 and 2, respectively, as indicated in Fig. 1 (a). From equation (1), the values of  $Z_{ca}$  and  $Z_{cb}$  can be estimated based on desired power-dividing ratio of  $k^2$ . The demanded modal characteristic impedances,  $Z_{ca}$  and  $Z_{cb}$ , can be straightforwardly tuned for wide-ranging values by proper choosing the loading element values of A-CRLH coupled-line section,  $C_{La/b}$  and  $L_{La/b}$ , as well as coupling capacitance value,  $C_m$ , without manufacturing complexity based on following equations [5]:

$$C_{La/b} = \frac{Np(Z_{\pi a/b} + Z_{ca/b})}{\omega\pi Z_{\pi a/b} Z_{ca/b} + \omega^2 L_R Np(Z_{\pi a/b} + Z_{ca/b})}, \quad (2)$$

$$L_{La/b} = \frac{4NpZ_{\pi a/b} Z_{ca/b}}{\omega\pi(Z_{\pi a/b} + Z_{ca/b}) + 4Np\omega^2 C_R Z_{\pi a/b} Z_{ca/b}}, \quad (3)$$

$$C_m = \frac{\pi}{4Nk\omega p} \left( \frac{1}{Z_{\pi a}} - \frac{1}{Z_{ca}} \right) = \frac{k\pi}{4N\omega p} \left( \frac{1}{Z_{\pi b}} - \frac{1}{Z_{cb}} \right). \quad (4)$$

Here, parameters  $L_R$ ,  $C_R$  and  $C_m$  are per unit cell self-inductance, self-capacitance, and mutual capacitance, respectively that model the edge-coupled interconnecting transmission line segments. For estimating loading element values, RH-parameters including  $C_R$  and  $L_R$  as two degrees of freedom can be assumed arbitrary, resulting in the favorite TL characteristic impedances for intersecting transmission lines. This avoids the fabrication limitation to realize high power division ratios in conventional coupled lines. With selected values of  $C_R$  and  $L_R$  and determined values of  $N$  and  $p$ , the loading element values can be easily calculated from equations (2) and (3).

### B. Filtering response analysis

As well as unequal performance, the proposed WPD supports the filtering response by using two symmetric CRLH coupled-line filter transformers near the output ports as shown in Fig. 1 (a). The structure of CRLH coupled-line band-pass filter employed in the design of output filter transformers is shown in Fig. 2, wherein symmetric CRLH coupled-line sections with equivalent even- and odd-mode characteristic impedances of  $Z_{ei}$  and  $Z_{oi}$ ,  $i = 1, \dots, N+1$ , and electrical length of  $\theta$  have been used in a cascaded form to develop a narrowband band-pass filter. Its equivalent circuit is also represented in Fig. 2, where each CRLH coupled-line section is roughly modeled using two CRLH transmission line sections with characteristic impedance of  $Z_0$  and the electrical length of  $\theta = -\pi/2$ , added on both ends of an inverter  $J_i$  ( $i=1, \dots, N+1$ ). By equating their propagation constants and image impedances, the following equation is found for even- and odd-mode characteristic impedances of each CRLH coupled-line section:

$$Z_{ei/oi} = Z_0(1 \pm J_i Z_0 \pm (J_i Z_0)^2) \quad i=1, \dots, N+1. \quad (5)$$

The equation (5) indicates that the even-mode and odd-mode characteristic impedances of a CRLH coupled-line section can be described by the system impedance

of  $Z_0$  and the admittance inverter of  $J_i$ , which has the same depiction of those of conventional coupled-line filters [7]. Therefore, the design equations of a typical couple-line filter can be assumed to design the CRLH coupled-line filter transformer. The values of admittance inverters for a band-pass filter are given by [7]:

$$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{2g_1}}, Z_0 J_i = \frac{\pi\Delta}{2\sqrt{g_{n-1}g_n}}, Z_0 J_{N+1} = \sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}. \quad (6)$$

Here,  $g_0, g_1, \dots, g_{n+1}$  are the elements of an equal-ripple low-pass prototype with a normalized cutoff  $\Omega_c=1$  and  $\Delta$  is the fractional bandwidth of the band-pass filter. Consequently, the transmission matrix of the CRLH coupled-line filter can be written for even-order and odd-order,  $N$ , as following:

$$T_{N=odd} = \begin{bmatrix} T_0 & 0 \\ 0 & T_1 \end{bmatrix}, T_{N=Even} = \begin{bmatrix} 0 & T_2 \\ T_3 & 0 \end{bmatrix},$$

$$T_0 = (-1)^{N+1/2} \left( \frac{\prod_{n=1}^{N+1/2} J_{2n-1}}{N+1/2} \right), T_1 = (-1)^{N+1/2} \left( \frac{\prod_{n=1}^{N+1/2} J_{2n}}{N+1/2} \right), \quad (7)$$

$$T_2 = (-1)^{N/2} j Z_0^2 \left( \frac{\prod_{n=1}^{N/2+1} J_{2n-1}}{N/2} \right), T_3 = (-1)^{N/2} \frac{j}{Z_0^2} \left( \frac{\prod_{n=1}^{N/2+1} J_{2n-1}}{N/2+1} \right).$$

Since in the proposed power dividing circuitry, such CRLH coupled-line filter is employed instead of the conventional output transformers in the ideal WPD, hence, its transmission matrixes should be approximately equal to transmission matrix of ideal impedance transformer to satisfy matching condition which can be determined as following:

$$T_{Ideal} = \begin{bmatrix} 0 & j(-1)^n Z_{0j} \\ j(-1)^n / Z_{0j} & 0 \end{bmatrix}, \quad (8)$$

where,  $Z_{0j}$ ,  $j = 2, 3$ , is the characteristic impedance of the conventional transmission line functioning as an impedance transformer at output 2 or 3:

$$Z_{02} = \sqrt{50Z_2} = 50\sqrt{\frac{1}{k}}, \quad Z_{03} = \sqrt{50Z_3} = 50\sqrt{k}. \quad (9)$$

Equating (7) and (8) implies that the even-order CRLH coupled-line filter with system impedance of  $Z_0$  equal to  $Z_{0j}$ ,  $j = 2, 3$ , should be employed for satisfying matching conditions and improving the selectivity in each transmission path. The design guideline for the proposed WPD can be briefly summarized as:

**Step1)** Calculate  $Z_{Ca}$  and  $Z_{Cb}$  of A-CRLH coupled-line transformer based on equation (1) for preferred power dividing ratio ( $k^2$ ). **Step2)** Calculate  $Z_{e/oi}^a$  and  $Z_{e/oi}^b$  ( $i = 1, \dots, N + 1$ ) of output CRLH coupled-line filter transformers based on equation (5) for desired selectivity in each transmission path and satisfying

matching condition. **Step3)** Find the corresponding mutual capacitance value,  $C_m$ , from equation (4) and Set suitable values for the strip width,  $w$ , and separation,  $s$ , using the approximate coupled-line formula [8] and appropriate circuit simulator for required  $C_m$ . **Step4)** Calculate the values of RH parameters,  $C_R$  and  $L_R$ , for determined values of  $w$  and  $s$ . **Step5)** Determine the capacitance and inductance values of chip or distributed loading element values  $L_L$  and  $C_L$  from equations (2) and (3).

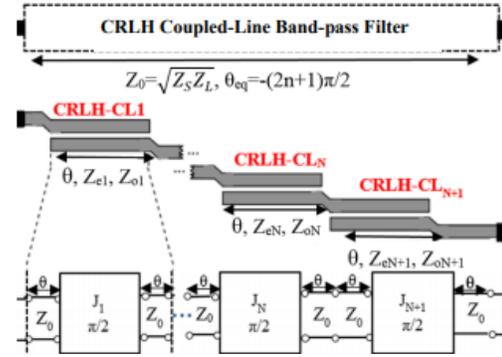


Fig. 2. CRLH coupled-line band-pass filter with its equivalent circuit.

### C. Analytical closed form scattering parameters for the proposed WPD

As the common circuit simulators like Agilent Advanced Design System (ADS) do not have electrical parameter-based transmission line models for the asymmetric coupled lines without physical dimensions, hence, these circuit simulation tools, cannot give the accurate and lossless scattering parameters for the proposed WPD. In order to have convenient accurate analysis, it is necessary to study the theoretical scattering parameters of the presented WPD. In this section, the closed-form mathematical expressions for scattering parameters are presented. Consequently, once the design parameters are defined based on theoretical analysis in sections A and B, the scattering parameters of the power divider can be conveniently obtained using the ideal-closed form equations. Due to the asymmetric nature of the proposed WPD, in- and anti-phase ( $c/\pi$ -mode) analysis is adopted to analyze the proposed structure. After using  $c/\pi$ -mode analysis, Fig. 1 can be simplified to two half circuits as depicted in Figs. 3 (a) and 3 (b). Here,  $\theta_c = \beta_c N p$  and  $\theta_\pi = \beta_\pi N p$  represent the equivalent  $c$ - and  $\pi$ -mode electrical lengths of the A-CRLH coupled-line section consisting of  $N$  unit-cells of  $p$ -length.  $Z_{Ca/b}$  and  $Z_{\pi a/b}$ , are the equivalent  $c$ - and  $\pi$ -mode characteristic impedances of the asymmetric CRLH coupled-line section calculated from equation (1). Moreover, for easy analysis, each output filter transformer composed of the cascade connection of  $N+1$

CRLH coupled-line sections is represented by its transmission matrix,  $[A^{a/b}, B^{a/b}, C^{a/b}, D^{a/b}]$ , which is equal to the product of the  $[A, B, C, D]$  matrices of individual two-port CRLH coupled-line sections,  $[A_k^{a/b}, B_k^{a/b}, C_k^{a/b}, D_k^{a/b}]$  as:

$$\begin{bmatrix} A_{eq}^{a/b} & B_{eq}^{a/b} \\ C_{eq}^{a/b} & D_{eq}^{a/b} \end{bmatrix} = \prod_{k=1}^{N+1} \begin{bmatrix} A_k^{a/b} & B_k^{a/b} \\ C_k^{a/b} & D_k^{a/b} \end{bmatrix}. \quad (10)$$

The  $[A, B, C, D]$  matrix for each two-port CRLH coupled-line section, which is approximately modeled by using two transmission lines added on both ends of an inverter,  $J_i^{a/b}$ , is thus given by equation (11):

$$\begin{bmatrix} A_k^{a/b} & B_k^{a/b} \\ C_k^{a/b} & D_k^{a/b} \end{bmatrix} = \begin{bmatrix} \cos \theta & jZ_0^{a/b} \sin \theta \\ j \frac{\sin \theta}{Z_0^{a/b}} & \cos \theta \end{bmatrix} \times \begin{bmatrix} 0 & -j/J_k^{a/b} \\ -jJ_k^{a/b} & 0 \end{bmatrix} \begin{bmatrix} \cos \theta & jZ_0^{a/b} \sin \theta \\ j \frac{\sin \theta}{Z_0^{a/b}} & \cos \theta \end{bmatrix}. \quad (11)$$

Where,  $Z_0^{a/b}$  and  $\theta$  are the characteristic impedance and electrical length of the added CRLH transmission lines. In the abovementioned expressions, ‘a’ and ‘b’ subscripts refer to the output transformers of path ‘a’ and path ‘b’, respectively. To the c-mode equivalent circuit, Fig. 3 (a), the ABCD-matrix between two ports (Port 1 and Port 2) can be obtained as:

$$\begin{bmatrix} A_c^{a/b} & B_c^{a/b} \\ C_c^{a/b} & D_c^{a/b} \end{bmatrix} = \begin{bmatrix} \cos(\theta_c) & jZ_{ca/b} \sin(\theta_c) \\ j \frac{\sin(\theta_c)}{Z_{ca/b}} & \cos(\theta_c) \end{bmatrix} \begin{bmatrix} A_{eq}^{a/b} & B_{eq}^{a/b} \\ C_{eq}^{a/b} & D_{eq}^{a/b} \end{bmatrix}. \quad (12)$$

According to the transformation between ABCD matrix and scattering parameters, the closed-form c-mode scattering parameters can be calculated as equation (13). The final input scattering parameter,  $S_{11}$ , and transmission parameters,  $S_{12}$  and  $S_{13}$ , of Fig. 1 can be derived as:

$$S_{11} = \frac{S_{11,c}^a + S_{11,c}^b}{2}, \quad S_{21} = \frac{kS_{21,c}^a}{\sqrt{1+k^2}}, \quad S_{31} = \frac{S_{21,c}^b}{\sqrt{1+k^2}}. \quad (14)$$

Considering the  $\pi$ -mode sub-circuits in Fig. 3 (b),

the output scattering parameters of  $\pi$ -mode can be also obtained as equation (15), where  $Z_{in}^{a/b}$  is the input impedance looking into the equivalent half-circuit at the output port as indicated in Fig. 3 (b). To calculate this impedance, first, the  $\pi$ -mode ABCD-matrix from port 2 to port 1 is obtained as equation (16), shown in the following, then as the two-port sub-network is terminated with a short circuit at port 1 in  $\pi$ -mode excitation,  $Z_{in}^{a/b}$  can be derived based on ABCD parameters as equation (17):

$$S_{22,\pi}^{a/b} = \frac{Z_{in}^{a/b} - 50}{Z_{in}^{a/b} + 50}, \quad (15)$$

$$\begin{bmatrix} A_{\pi}^{a/b} & B_{\pi}^{a/b} \\ C_{\pi}^{a/b} & D_{\pi}^{a/b} \end{bmatrix} = \begin{bmatrix} A_{eq}^{a/b} & B_{eq}^{a/b} \\ C_{eq}^{a/b} & D_{eq}^{a/b} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_{Iso}^{a/b}} & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_{\pi}) & jZ_{\pi a/b} \sin(\theta_{\pi}) \\ j \frac{\sin(\theta_{\pi})}{Z_{\pi a/b}} & \cos(\theta_{\pi}) \end{bmatrix}, \quad (16)$$

$$Z_{in}^{a/b} = \frac{B_{\pi}^{a/b}}{D_{\pi}^{a/b}} = \frac{j(A_{eq}^{a/b} + \frac{B_{eq}^{a/b}}{R_{Iso}^{a/b}})Z_{\pi a/b} \sin(\theta_{\pi}) + B_{eq}^{a/b} \cos(\theta_{\pi})}{j(C_{eq}^{a/b} + \frac{D_{eq}^{a/b}}{R_{Iso}^{a/b}})Z_{\pi a/b} \sin(\theta_{\pi}) + D_{eq}^{a/b} \cos(\theta_{\pi})}, \quad (17)$$

The final output scattering parameters,  $S_{22}$  and  $S_{33}$ , and the isolation parameter,  $S_{23}$ , can be expressed by:

$$S_{22} = \frac{k^2 S_{22,c}^a + S_{22,\pi}^a}{1+k^2}, \quad S_{33} = \frac{S_{22,c}^b + k^2 S_{22,\pi}^b}{1+k^2}, \quad S_{23} = \frac{k(S_{22,c}^b - S_{22,\pi}^b)}{1+k^2}. \quad (18)$$

To demonstrate the proposed design method, 2 examples including WPDs with 1:10 and 1:15 power dividing ratios and fractional bandwidth of 15% comprising an A-CRLH coupled-line section with two 2<sup>th</sup> order symmetric CRLH coupled-line filters are designed. The corresponding S-plots obtained from the analytical closed-form equations, (13)-(18), for RO4003 substrate with dielectric constant of 3.55 and thickness of 0.813 mm are presented in Fig. 4. It is seen that the proposed structure can function as an unequal power divider with high-selectivity band-pass responses.

$$\begin{bmatrix} S_{11,c}^{a/b} & S_{12,c}^{a/b} \\ S_{21,c}^{a/b} & S_{22,c}^{a/b} \end{bmatrix} = \frac{\begin{bmatrix} [50(A_{eq}^{a/b} - Z_1^{a/b} C_{eq}^{a/b}) + B_{eq}^{a/b} - Z_1^{a/b} D_{eq}^{a/b}] \cos(\theta_c) + \\ j[50(C_{eq}^{a/b} Z_{ca/b} - Z_1^{a/b} \frac{A_{eq}^{a/b}}{Z_{ca/b}}) + D_{eq}^{a/b} Z_{ca/b} - \frac{Z_1^{a/b} B_{eq}^{a/b}}{Z_{ca/b}}] \sin(\theta_c) \\ 2\sqrt{50Z_1^{a/b}} \\ [-50(A_{eq}^{a/b} + Z_1^{a/b} C_{eq}^{a/b}) + B_{eq}^{a/b} + Z_1^{a/b} D_{eq}^{a/b}] \cos(\theta_c) + \\ j[-50(C_{eq}^{a/b} Z_{ca/b} + Z_1^{a/b} \frac{A_{eq}^{a/b}}{Z_{ca/b}}) + D_{eq}^{a/b} Z_{ca/b} + \frac{Z_1^{a/b} B_{eq}^{a/b}}{Z_{ca/b}}] \sin(\theta_c) \\ 2\sqrt{50Z_1^{a/b}} \\ [50(A_{eq}^{a/b} + Z_1^{a/b} C_{eq}^{a/b}) + B_{eq}^{a/b} + Z_1^{a/b} D_{eq}^{a/b}] \cos(\theta_c) + j[50(C_{eq}^{a/b} Z_{ca/b} + Z_1^{a/b} \frac{A_{eq}^{a/b}}{Z_{ca/b}}) + D_{eq}^{a/b} Z_{ca/b} + \frac{Z_1^{a/b} B_{eq}^{a/b}}{Z_{ca/b}}] \sin(\theta_c) \end{bmatrix}}{\begin{bmatrix} [50(A_{eq}^{a/b} - Z_1^{a/b} C_{eq}^{a/b}) + B_{eq}^{a/b} - Z_1^{a/b} D_{eq}^{a/b}] \cos(\theta_c) + \\ j[50(C_{eq}^{a/b} Z_{ca/b} - Z_1^{a/b} \frac{A_{eq}^{a/b}}{Z_{ca/b}}) + D_{eq}^{a/b} Z_{ca/b} - \frac{Z_1^{a/b} B_{eq}^{a/b}}{Z_{ca/b}}] \sin(\theta_c) \\ 2\sqrt{50Z_1^{a/b}} \\ [-50(A_{eq}^{a/b} + Z_1^{a/b} C_{eq}^{a/b}) + B_{eq}^{a/b} + Z_1^{a/b} D_{eq}^{a/b}] \cos(\theta_c) + \\ j[-50(C_{eq}^{a/b} Z_{ca/b} + Z_1^{a/b} \frac{A_{eq}^{a/b}}{Z_{ca/b}}) + D_{eq}^{a/b} Z_{ca/b} + \frac{Z_1^{a/b} B_{eq}^{a/b}}{Z_{ca/b}}] \sin(\theta_c) \\ 2\sqrt{50Z_1^{a/b}} \\ [50(A_{eq}^{a/b} + Z_1^{a/b} C_{eq}^{a/b}) + B_{eq}^{a/b} + Z_1^{a/b} D_{eq}^{a/b}] \cos(\theta_c) + j[50(C_{eq}^{a/b} Z_{ca/b} + Z_1^{a/b} \frac{A_{eq}^{a/b}}{Z_{ca/b}}) + D_{eq}^{a/b} Z_{ca/b} + \frac{Z_1^{a/b} B_{eq}^{a/b}}{Z_{ca/b}}] \sin(\theta_c) \end{bmatrix}}. \quad (13)$$

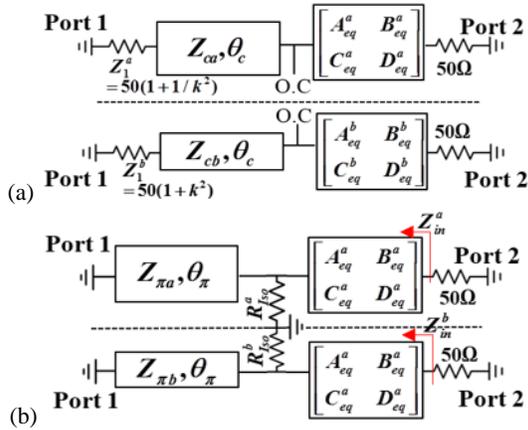


Fig. 3. Equivalent half-circuits of the proposed WPD: (a) c-mode half-circuit and (b)  $\pi$ -mode half-circuit.

### III. CIRCUIT IMPLEMENTATION

To experimentally verify the proposed design method, a prototype filtering unequal WPD with high power dividing ratio of 1:15 and fractional bandwidth of 15% is fabricated for operating at 1.4 GHz as shown in Fig. 5. It is implemented on R04003 substrate with dielectric constant of 3.55 and thickness of 0.813mm. It consists of an A-CRLH coupled-line transformer comprises conventional coupled lines loaded by surface mount technology (SMT) components with inductance and capacitance values derived for preferred power dividing ratio of  $k^2$  and two output CRLH coupled-line impedance transformers composed of three symmetric CRLH coupled lines formed of coupled lines loaded by distributed elements including shorted stub inductors and series interdigital capacitors (IDCs) with inductance and capacitance values derived for the desired selectivity based on aforementioned design principle. The geometrical parameters of IDCs and shorted stub inductors meeting the required electrical LC parameters can be determined with the help of established approximate formulas, [8], as a starting guess for design and then applying a suitable optimization algorithm in a full-wave software like high frequency structure simulator (HFSS) for accurate realizing the desired characteristics. The designed values of lumped loading elements with geometrical dimensions are as following:  $C_{La} = 3.2$  pF,  $C_{Lb} = 0.3$  pF,  $L_{La} = 3.2$  nH,  $L_{Lb} = 16$  nH,  $w'_{1a} = w'_{3a} = 0.3$  mm,  $w'_{2a} = 0.4$  mm,  $w'_{1b} = w'_{3b} = 0.8$  mm,  $w'_{2b} = 0.9$  mm,  $s'_{1a} = s'_{3a} = 0.2$  mm,  $s'_{2a} = 0.4$  mm,  $s'_{1b} = s'_{3b} = 0.9$  mm,  $s'_{2b} = 0.4$  mm,  $s_{1a} = s_{3a} = 0.1$  mm,  $s_{2a} = 0.6$  mm,  $s_{1b} = s_{3b} = 0.2$  mm,  $s_{2b} = 0.4$  mm,  $l_{1a}^{IDC} = l_{2a}^{IDC} = l_{3a}^{IDC} = 8.7$  mm,  $l_{1b}^{IDC} = l_{3b}^{IDC} = 8.7$  mm,  $l_{2b}^{IDC} = 7.7$  mm,  $l_{1a}^{stub} = l_{3a}^{stub} = 13$  mm,  $l_{2a}^{stub} = 12.5$  mm,  $l_{3b}^{stub} = l_{1b}^{stub} = 9.4$  mm,  $l_{2b}^{stub} = 8.9$  mm.

To avoid confusion, the geometrical values are

denoted by their corresponding CRLH coupled-line section number, i.e., the dimensions that correspond to CRLH coupled-line section represented by  $CL_{2a}$  are denoted by  $w'_{2a}$ ,  $s'_{2a}$ ,  $s_{2a}$ ,  $l_{2a}^{IDC}$ ,  $l_{2a}^{stub}$  and the same for other sections.

The measurement results along with full-wave and analytical theory simulations using high frequency structure simulator (HFSS) and MATLAB are depicted in Fig. 6. As shown, there is a good agreement between HFSS, theory and measurement data and the desired power division ratio with relatively sharp band-pass responses is achieved. The measured insertion loss of  $S_{21}$  and  $S_{31}$ , are about -1.3 and -13.1 dB, respectively. The measured center frequency is 1.4 GHz, with the fractional bandwidth of 15.5%. The in-band return loss,  $|S_{22}|$ ,  $|S_{33}|$  and  $|S_{11}|$ , are better than 30 dB. The isolation,  $|S_{23}|$ , is greater than 35 dB over the whole band. The measured in-band phase difference between output ports is between  $-2.5^\circ$  and  $2.8^\circ$ . The performance comparison of the proposed power divider with other compact unequal WPDs in terms of power splitting ratio, filtering response, relative occupied area and isolation between output ports is summarized in Table 1. As the results show, the proposed WPD has the most compact size and the highest power division ratio, since the circuit size is reduced to 44.5% of the conventional one and the power splitting ratio is enhanced to 1:15. Additionally, despite power ratio, the proposed design has a filtering function with a steep skirt selectivity and high isolation in a wide frequency range.

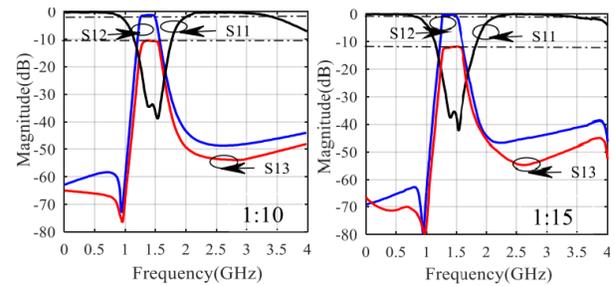


Fig. 4. Theoretical results of WPDs with (left) 1:10 and (right) 1:15 power dividing ratios.

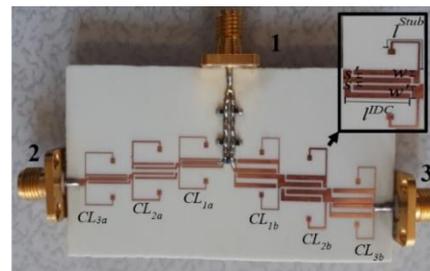
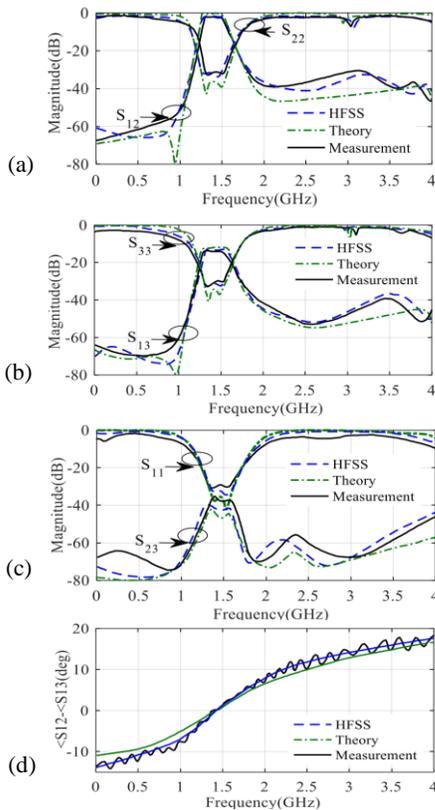


Fig. 5. Photograph of the proposed filtering 1:15 WPD fabricated on RO4003.

Table 1: Performance comparison with the previous works

Ref.	Filtering Response	Dividing Ratio	Relative Area (%)	Isolation (in-band)
[1]	Yes	1:3/2	200	>23 dB
[3]	Yes	1:8	40	N/A
[4]	Yes	1:2	200	>20 dB
[5]	No	1:15	41.3	>25 dB
[9]	No	1:6	100	>20 dB
[10]	No	1:4	100	>20 dB
[11]	No	1:10	100	>20 dB
<b>This work</b>	Yes	1.15	44.5	>35 dB

Fig. 6. Measured and simulated results of the filtering power divider: (a)  $S_{12}$  and  $S_{22}$ , (b)  $S_{13}$ , and  $S_{33}$ , (c)  $S_{23}$ , and  $S_{11}$ , and (d) phase difference of output ports.

#### IV. CONCLUSION

This paper presents a new compact filtering unequal WPD with high power-splitting ratio application using symmetric and asymmetric CRLH coupled lines. The analytical design equations along with ideal closed-form expressions for scattering parameters are derived for the proposed structure by using in- and anti-phase ( $c/\pi$ -mode) decomposition method. To verify the theoretical design procedure, an unequal compact WPD with a high-power division ratio of 1:15 is designed, simulated and fabricated at 1.4 GHz. The measurement results along

with theoretical and simulation data validate that the proposed WPD not only exhibits a sharp frequency selectivity but also a large power splitting ratio with desired performance.

#### REFERENCES

- [1] P. H. Deng and L. C. Dai, "Unequal Wilkinson power dividers with favorable selectivity and high-isolation using coupled-line filter transformers," *IEEE Transactions on Microwave Theory and Techniques*, vol. 60, no. 6, pp. 1520-1529, June 2012.
- [2] L. Gao and X. Y. Zhang, "Novel 2:1 Wilkinson power divider integrated with bandpass filter," *Microwave and Optical Technology Letters*, vol. 55, no. 3, pp. 646-648, 2013.
- [3] M. Danaeian, A. R. Moznebi, K. Afrooz, and H. Hakimi, "Miniaturised equal/unequal SIW power divider with bandpass response loaded by CSRRs," *Electronics Letters*, vol. 52, no. 22, pp. 1864-1866, 2016.
- [4] Z. Zhuang, Y. Wu, L. Jiao, W. Wang, and Y. Liu, "Wideband balanced-to-unbalanced filtering unequal power divider with wide stopband and isolation," *Electronics Letters*, vol. 53, no. 13, pp. 892-894, 2017.
- [5] Y. Torabi and G. Dadashzadeh, "A compact 1:15 unequal Wilkinson power divider using A-CRLH coupled lines," *Microwave and Optical Technology Letters*, vol. 60, no. 1, pp. 117-122, 2017.
- [6] Y. Torabi and G. Dadashzadeh, "Compact filtering unequal Wilkinson power divider with 1:15 power dividing ratio using composite right/left handed-coupled lines," *International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 2017.
- [7] R. Mongia, I. Bahl, and P. Bhartia, *RF and Microwave Coupled-Line Circuits*. Norwood, MA: Artech House, 1999.
- [8] C. Caloz and T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications: the Engineering Approach*. Hoboken, N.J: John Wiley & Sons, 2006.
- [9] C.-P. Chang, C.-C. Su, S.-H. Hung, Y.-H. Wang, and J.-H. Chen, "A 6:1 unequal Wilkinson power divider with EBG CPW," *Progress In Electromagnetics Research Letters*, vol. 8, pp. 151-159, 2009.
- [10] M. Moradian and H. Oraizi, "Application of grooved substrates for design of unequal Wilkinson power dividers," *Electronics Letters*, vol. 44, no. 1, June 2008.
- [11] B. Li, X. Wu, and W. Wu, "A 10:1 unequal Wilkinson power divider using coupled lines with two shorts," *IEEE Microwave Wireless Component Letter*, vol. 19, no. 12, pp. 789-791, Dec. 2009.



**Yalda Torabi** was born in Zanjan, Iran in 1987. She received her B.Sc. degree in Electrical Engineering from Tabriz University, Tabriz, Iran in 2009 and the M.Sc. degree from the Iran University of Science and Technology (IUST), Tehran, Iran in 2012 and is currently working towards the Ph.D. degree in Electrical Engineering at Shahed University, Tehran, Iran.



**Gholamreza Dadashzadeh** was born in Urmia, Iran, in 1964. He received the B.Sc. degree in Communication Engineering from Shiraz University, Shiraz, Iran in 1992 and M.Sc. and Ph.D. degree in Communication Engineering from Tarbiat Modarres University (TMU), Tehran, Iran, in 1996 and 2002, respectively. He is currently a Professor in the Department of Electrical Engineering at Shahed University, Tehran, Iran.