Asymmetric Band Structure Calculations Using the Plane Wave Expansion Method with Time-Modulated Permittivity

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***Abstract*—In this work we show how the plane wave expansion method for calculating the band structure of materials with periodic electric permittivity may be extended to calculate the band structure of periodic materials that also possess a sinusoidal time-modulation. The numerical technique is applied to a structure possessing a synthetic linear momentum which causes unidirectional bandgaps to appear in the band structure. Such devices could be of use for tunable magnet-free isolators in integrated photonics platforms.**

I. INTRODUCTION

Electromagnetic devices that allow only one-way wave propagation have many practical uses including in optical isolators [1], antennas [2] and duplexed communication transceivers [3]. Any static linear electromagnetic device is time-reversible and therefore cannot be used for unidirectional wave propagation. However, incorporating a synthetic momentum into the system via directional time-modulation can break time-reversal symmetry and induce one-way electromagnetic wave propagation [4]. Most traditional computational techniques assume static material properties. However, some recent reports have detailed methods for incorporating a sinusoidal time-modulated permittivity into computational electromagnetic tools [5]. In this work we extend the plane wave expansion method for calculating the photonic band structure of static periodic structures to materials that have harmonically time-modulated electric permittivities. With this method we show how a directional spatio-temporal modulation results in an asymmetric (in wavevector) bandstructure that produces unidirectional wave propagation in a range of frequencies.

II. METHOD

This work is concerned with electric permittivities of the form $\varepsilon(z,t) = \varepsilon_a(z) + \varepsilon_b(z)\cos[2\pi\Omega t + \phi]$ where $\varepsilon_a(z+\Lambda) = \varepsilon_a(z)$ and $\varepsilon_b(z+\Lambda) = \varepsilon_b(z)$ are both periodic in Λ. For one-dimensional material variation, the Maxwell equations become:

$$\frac{\partial E_x(z,t)}{\partial z} = -\mu_0 \frac{\partial H_y(z,t)}{\partial t}, \tag{1a}$$

$$-\frac{\partial H_y(z,t)}{\partial z} = -\varepsilon_0 \frac{\partial}{\partial t}[\varepsilon(z,t)E_x(z,t)]. \tag{1b}$$

If the time derivative in (1b) is expanded, one obtains:

$$-\frac{\partial H_y(z,t)}{\partial z} = -\varepsilon_0\{\varepsilon_a(z)\frac{\partial E_x(z,t)}{\partial t} + \varepsilon_b(z)\cos[\Omega t+\phi]\frac{\partial E_x(z,t)}{\partial t} - \varepsilon_b(z)\Omega\sin[\Omega t+\phi]E_x(z,t)\}.$$

The spatial periodicity motivates the use of a Bloch form, and the time-modulation motivates inclusion of harmonics $\omega + n\Omega$ for integer n. Our ansatz for solution of Eqs. 1 is:

$$E_x(z,t) = \sum_{G,n} E_x(G,n)e^{i(k+G)z}e^{-i(\omega+n\Omega)t}, \tag{2a}$$

$$H_y(z,t) = \sum_{G,n} H_y(G,n)e^{i(k+G)z}e^{-i(\omega+n\Omega)t}, \tag{2b}$$

where $G = m2\pi/\Lambda$ for m integer. Combining Eqs. 2 with Eqs. 1 produces:

$$\mu_0(\omega+n\Omega)H_y(G,n) - GE_x(G,n) = kE_x(G,n), \tag{3a}$$

$$-GH_y(G,n) + \varepsilon_0\sum_{G'}\{\varepsilon_a(G-G')(\omega_0+n\Omega)E_x(G',n) + [e^{i\phi}E_x(G',n+1) + e^{-i\phi}E_x(G',n-1)] = kH_y(G,n), \tag{3b}$$

which is an eigenvalue equation with eigenvalue k and eigenvector made up of the components $E_x(G,n)$ and $H_y(G,n)$. To obtain the bandstructure for these dynamic geometries, a range of frequency values is chosen, and for each frequency, the corresponding eigenvalue k is obtained.

III. SYNTHETIC MOMENTUM

To illustrate the physical properties of time-modulated periodic structures, consider the geometry shown in Fig. 1 (a). The structure consists of a repeating unit cell consisting of three layers each modulated by the same frequency but with relative phase offsets of $2\pi/3$. The phase sequence of 0, $2\pi/3$, $4\pi/3$ from left to right produces a non-zero overlap

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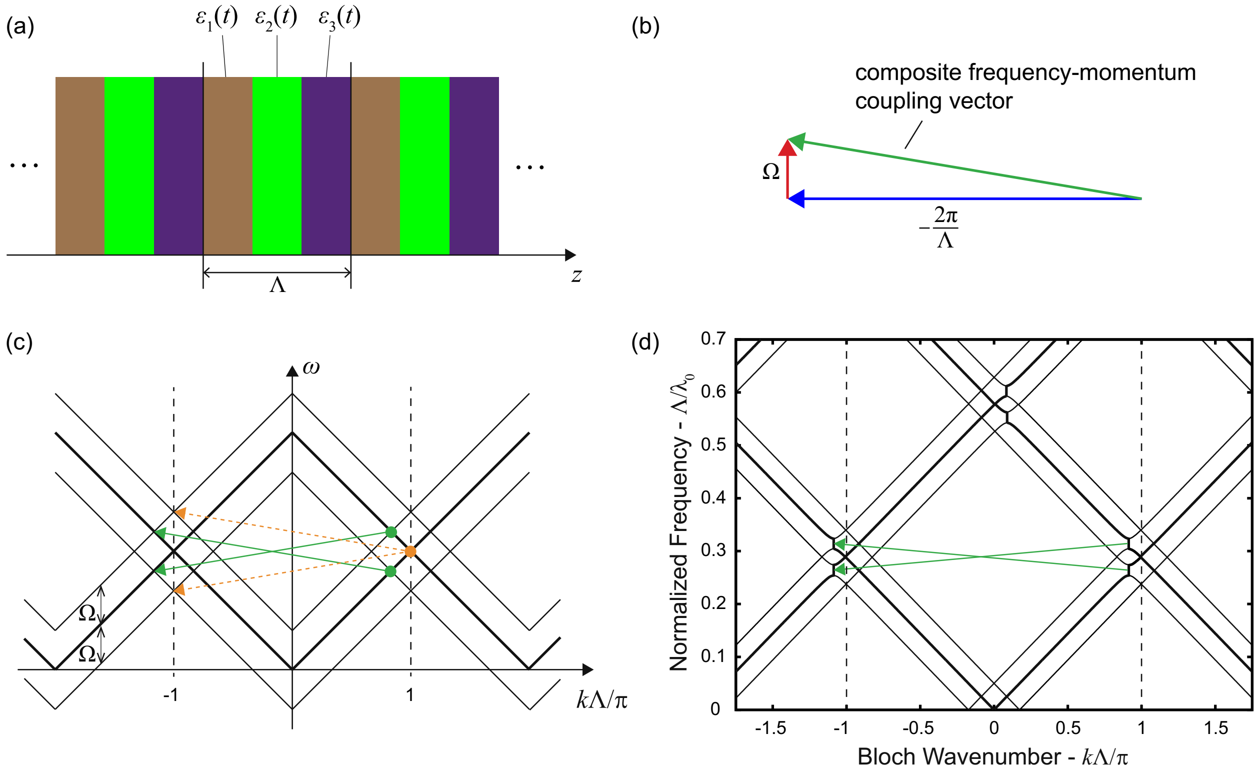

Fig. 1. (a) One-dimensional periodic structure with time-modulated permittivity. $\varepsilon_1(t) = 3.0 + 0.5\cos(\Omega t)$, $\varepsilon_2(t) = 3.0 + 0.5\cos(\Omega t + 2\pi/3)$ and $\varepsilon_3(t) = 3.0 + 0.5\cos(\Omega t + 4\pi/3)$. (b) Mode coupling vector in the spatio-temporally modulated system in (a). (c) The mode-coupling scheme in the empty lattice. The bold line is the fundamental band ($n = 0$) whereas the thin lines correspond to harmonics which is the fundamental band shifted vertically by $\pm n\Omega$ (only $n = 1$ is depicted). (d) The bandstructure of the dynamic geometry shown in (a) using the proposed numerical method.

with the continuous travelling wave modulation of the form $\cos(\Omega t + \frac{2\pi}{\Lambda}z)$ which produces a synthetic momentum pointing toward $-z$. Fig. 1 (b) shows the net momentum-frequency coupling vector. In a static periodic system, the coupling vector would be bidirectional, horizontal and of length $2\pi/\Lambda$. In the time-modulated system, it is unidirectional and tilted up by the amount Ω with concomitant length.

Fig. 1 (c) shows how the mode coupling works in this system. The empty lattice (a lattice with periodicity Λ and time-modulation frequency Ω but $\varepsilon_b \to 0$) is shown. Because of the directionality of the spatio-temporal modulation, the coupling vectors point only from right to left. The green arrows show couplings that result in photonic bandgaps. The green vectors couple band crossings that involve the fundamental band, and, therefore, bandgaps are expected to open at the green dots. The orange vectors show couplings between band crossings, but these bands belong to different order harmonics, and, therefore, no coupling occurs.

Fig. 1 (d) shows the band structure obtained using the proposed method. Indeed one sees bandgaps in the first Brillouin zone corresponding to the points highlighted in Fig. 1 (c). Specifically, there is a bandgap near $k\Lambda/\pi \lesssim 1.0$ but no bandgap near $k\Lambda/\pi \gtrsim -1.0$. This means that incident waves propagating along $+k$ at a frequency in the bandgap will be reflected; whereas, incident waves propagating along $-k$ at a frequency in the same frequency range will be transmitted as if there were no periodic perturbation to the electric permittivity at all. Due to the broken time-reversal symmetry, the band structure is clearly asymmetric in k. Technologically such a device could prove useful in tunable optical isolation particularly in magnet-free integrated photonics platforms.

In conclusion, a technique for calculating the band structure of harmonically time-modulated system is presented. We show how tailored time-modulation can impart a directional synthetic momentum to the field causing unidirectional propagation in these periodic materials.

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