Multi-Fidelity Approach for Polynomial Chaos Based Statistical Analysis of Microwave Networks

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***Abstract* — In this paper, a novel polynomial chaos based approach for the fast statistical analysis of complex microwave structures is proposed. This approach leverages a highly efficient closed form low-fidelity model elicited from the high-dimensional model representation (HDMR) of the network. By cross-cutting the efficiency of this low-fidelity model with the accuracy of general high-fidelity simulations, the accuracy-CPU cost tradeoff for the statistical analysis can be achieved.**

I. INTRODUCTION

The generalized polynomial chaos (PC) theory is a widely accepted numerical approach for statistical analysis of circuits and systems [1]-[3]. In this approach, the uncertainty in a network response is modeled as a linear combination of polynomial basis functions of the input network parameters [1]-[3]. The coefficients of the expansion form the new unknowns of the network and can be evaluated via repeated black box solutions of the original network model [2],[3]. The main drawback with PC approaches is that the number of coefficients to be evaluated increases in a polynomial manner with the number of random parameters or dimensions. As a result, classical PC approaches are computationally prohibitive for statistical analysis of high-dimensional problems.

This paper presents a more efficient alternative to the traditional PC approaches. First, a closed form low-fidelity model of the network response is constructed using unidimensional PC expansions elicited from the first order terms of a high-dimensional model representation (HDMR). This low-fidelity model can be used to analytically emulate the network response at a fraction of the CPU costs required for a rigorous high-fidelity simulation. However, this efficiency comes at the cost of the low accuracy of the model caused by the negligence of the higher order HDMR terms. Next, a nonintrusive linear regression approach to evaluate the PC coefficients of the full-blown expansion is adopted. In this regression approach, the majority of the deterministic black box network solutions are determined by probing the closed-form low-fidelity model. In order to compensate for the lower accuracy of the low-fidelity model, only a limited number of high-fidelity simulations of the network response are added. In effect, the efficiency of the low-fidelity model is cross-cut with the accuracy of high-fidelity simulations. Notably the fraction of low-fidelity solutions to high-fidelity simulations can be easily tuned for the optimal accuracy-CPU cost tradeoff.

II. DEVELOPMENT OF PROPOSED MULTI-FIDELITY APPROACH

A general microwave network is considered where the input uncertainty is represented by n mutually uncorrelated random dimensions $\boldsymbol{\lambda}=[\lambda_1,\lambda_2,..,\lambda_n]$. Traditional PC approaches model the resultant uncertainty in the unknown current/voltage responses, $\mathbf{X}(t,\boldsymbol{\lambda})$, using a linear combination of orthogonal polynomials as

$$\mathbf{X}(t,\boldsymbol{\lambda}) = \sum_{k=0}^{P} \mathbf{X}_k(t)\Phi_k(\boldsymbol{\lambda}) \quad (1)$$

where $\Phi_k(\boldsymbol{\lambda})$ is the k^{th} multivariate polynomial, $\mathbf{X}_k(t)$ is the corresponding coefficient, and the number of terms $P+1 = (n+m)!/(n!m!)$, m being the maximum degree of the expansion. In this paper, a high dimensional model representation (HDMR) of the network of (1) is considered where any response $x(t,\boldsymbol{\lambda}) \in \mathbf{X}(t,\boldsymbol{\lambda})$ can be represented as a hierarchical superposition of functions describing the interactions among the random dimensions as [4]:

$$x(t,\boldsymbol{\lambda}) = x_0(t) + \sum_{i=1}^{n} x_i(t,\lambda_i) + \sum_{1\le i,j\le n} x_i(t,\lambda_i,\lambda_j) + \ldots + x_{12..n}(t,\lambda_i..\lambda_n) \quad (2)$$

In (2), x_0 is the mean value of $x(t,\boldsymbol{\lambda})$, $x_i(t,\lambda_i)$ represents the contribution of λ_i to $x(t,\boldsymbol{\lambda})$ acting alone, $x_{ij}(t,\lambda_i,\lambda_j)$ represents the pairwise contribution of λ_i and λ_j to $x(t,\boldsymbol{\lambda})$ etc. The first order terms of (2) can be expressed using cut-HDMR [4] as

$$x_i(t,\lambda_i) = x(t,\boldsymbol{\lambda})\big|_{\boldsymbol{\lambda}^{(0)}\setminus\lambda_i} - x_0(t) \quad (3)$$

where $\boldsymbol{\lambda}^{(0)}\setminus\lambda_i$ represents the vector where all $\boldsymbol{\lambda}$ except λ_i is 0. These first order terms can now be modeled as 1D PC expansionss

$$x_i(t,\lambda_i) \approx \sum_{k=1}^{m} x_k^{(i)}(t)\phi_k(\lambda_i) \quad (4)$$

where $x_i^{(k)}(t)$ represents the k^{th} coefficient and ϕ_k is the corresponding 1D basis. The coefficients of (4) can be evaluated using the pseudo-spectral collocation technique in conjunction with (3) [3]. Now, neglecting the higher order HDMR terms of (2), a low-fidleity PC representation of the response $x(t,\boldsymbol{\lambda})$ is obtained as

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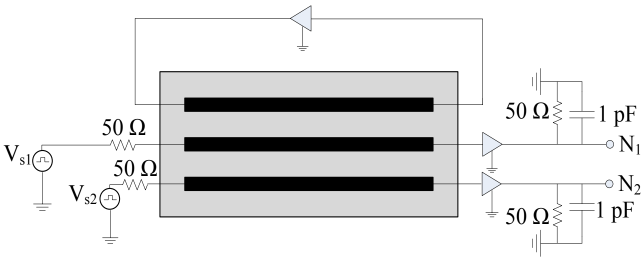

Fig. 1: Coupled transmission line network schematic

TABLE I
CHARACTERISTICS OF RANDOM PARAMETERS OF NETWORK OF FIG. 1

Random Parameters	Mean	Relative SD
ε_r (Relative permittivity of dielectric)	4.4	
σ (Metal conductivity)	5.8e7	
t (Thickness of metal)	5 µm	
w (Width of transmission lines)	180 µm	
h_1 (TL_1 height)	50 µm	10%
h_2 (TL_2 height)	70 µm	
h_3 (TL_3 height)	60 µm	
s (line spacing)	90 µm	
len (NMOS channel length)	0.1 µm	
wid (NMOS channel width)	10 µm	

$$x(t,\boldsymbol{\lambda}) \approx x_0(t) + \sum_{i=1}^{n}\sum_{k=1}^{m} x_k^{(i)}(t)\phi_k(\lambda_i) \tag{5}$$

The model of (5) is closed form in nature and can be used to analytically emulate the response instead of using a rigorous and time consuming high-fidelity simulation.

Next, to recover the coefficients of (1), the linear regression approach is chosen which typically requires $K = 2(P+1)$ black box model simulations of the network response [2]. In this paper, an iterative algorithm is used to find the fraction of K simulations performed using the low-fidelity model. This algorithm begins by initially assuming that all K simulations are performed using the low-fidelity model of (5). Thereafter, the number of high-fidelity network simulations is increased in steps of k nodes per iteration and the PC coefficients are reevaluated. The value of k is kept small enough (k = 10% of K) to minimize any overshoot beyond the optimal number of high-fidelity simulations. As the number of high-fidelity simulations increases per iteration relative to the number of low-fidelity simulations, the resultant improvement in the accuracy of the response variance is computed as

$$\eta = \frac{\int_0^{T_{\max}} \left(\sum_{i=1}^{P+1} \left(\hat{x}_i^{(r)}\right)^2 - \sum_{i=1}^{P+1} \left(\hat{x}_i^{(r-1)}\right)^2 \right) dt}{\int_0^{T_{\max}} \sum_{i=1}^{P+1} \left(\hat{x}_i^{(r)}\right)^2 dt} \tag{6}$$

where $\hat{x}$ are the computed PC coefficients and the superscript r is the iteration count. As the iteration count increases, the error in the numerator of η steadily decreases. Once the value of η falls below a prescribed tolerance, the iterations are halted.

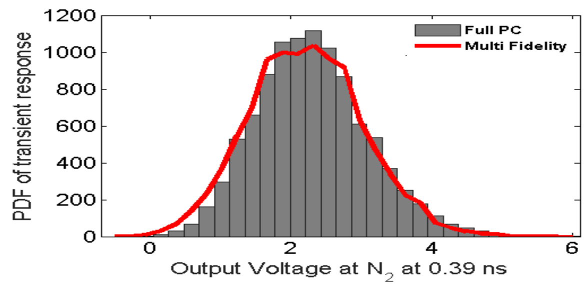

Fig. 2: Probability distribution function of transient response at node N_2 at timepoint of maximum standard deviation (t = 0.39 ns).

III. NUMERICAL RESULT AND DISCUSSION

To validate the proposed approach, the multicondutor transmission line (MTL) network of Fig. 1 is considered. The MTLs are terminated with cascode amplifiers modeled using SPICE level-49 CMOS transistors. The input waveform to lines 2 and 3 are trapezoidal pulses with rise/fall time T_r = 0.1 ns, pulse width T_w = 1 ns, and amplitude 5V. The uncertainty in the network is introduced via n = 10 random variables whose characteristics are shown in Table I. For this example, a Hermite PC expansion of degree m = 3 is required. The uncertainty quantification of the network is performed using two PC approaches – the proposed multi-fidelity approach and the traditional linear regression approach of [2]. The low-fidelity model requires only $(m+1)n+1$ = 41 SPICE simulations. In the multi-fidelity stage, only 30% of the $2(P+1)$ = 572 regression nodes (i.e., 171 nodes) requires a high-fidelity SPICE simulation translating to a speedup of more than 5 times over the traditional linear regression approach., The probability density of the transient response at N_2 evaluated at the time point when the standard deviation is maximum (0.39 ns) using the above methods and 10,000 samples is compared in Fig. 2. Despite the speedup, the proposed approach exhibits good accuracy relative to the traditional linear regression approach.

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