Improved Constraint NLMS Algorithm for Sparse Adaptive Array Beamforming Control Applications

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Abstract **—** In this paper, a new reweighted l_1-norm and an l_p-norm based normalized least mean square (NLMS) algorithms are developed for sparse adaptive array beamforming control applications. The proposed reweighted l_1-norm constrained NLMS (RL$_1$-CNLMS) and l_p-norm constrained NLMS (L$_P$-CNLMS) algorithms use the l_1-norm penalty and l_p-norm penalty to the conventional cost function of constrained normalized LMS (CLMS) algorithm to control the sparsity of the antenna array. What's more, in the derivation process, the gradient descent principle and Lagrange multiplier method are adopted to obtain the desired updating formulations. Computer simulations demonstrate that the superiority of proposed algorithms compared with other LMS based beamforming methods.

Index Terms **—** array beamforming, constrained LMS algorithm, l_1-norm constraint, l_p-norm constraint, sparse adaptive beamforming.

I. INTRODUCTION

Adaptive beamforming has drawn lots of attention due to its good performance, and it has been widely developed for wireless communications, radio astronomy, mobile communications, radar, sonar and other fields [1-2]. Adaptive beamformer can generate a main lobe in the interested direction to get a high gain, meanwhile, to form nulls to attenuate the interferences to obtain better the signal-to-interference-plus-noise ratio (SINR) [3].

The principle of adaptive beamforming algorithms is to match the signals of interest (SOI) and adaptively suppress the interferences by dynamically adjusting the array weight vectors. The linearly constrained minimum variance (LCMV) algorithm proposed by Frost [3] is a famous beamforming method which can realize the mentioned properties. In [4], under the assumption that array elements can be adjusted in real-time, the CLMS algorithm is developed as a normalized adaptive version of LCMV which can minimize the output power and reduce unwanted interferences with the object of keeping a maximum gain in the desired direction.

However, in some particular applications, especially in radar application, in order to realize the desired capacity, large arrays are always indispensable which attributes to the fact that big arrays are always restricted by the power supply and computation ability. Existing beamforming algorithms cannot solve this problem. Hence, as the development of sparse signal processing [5-14], and inspired by the Least Absolutely Shrinkage and Selection Operator [15] and Compressive Sensing [16], it is well worth to develop sparse adaptive beamforming algorithms to reduce the ratio of active elements in the antenna array, i.e., forcing the array weight vector towards sparsity [17-19].

Sparse signal processing technique can fully take advantage of the sparse characteristics existing in many situations, and it should have special advantages in both performance and convergence. In recent years. sparse signal processing has been widely studied. A great number of sparse LMS based algorithms have been developed for various sparse system identifications [5-8]. In these algorithms, it is no doubt that the zero-attracting LMS (ZA-LMS) which employs the l_1-norm penalty is representative. The ZA-LMS algorithm creates a modified updating formulation with a zero-attractor on all filter taps so as to force the inactive coefficients to zero quickly. To further accelerate the convergence speed, the reweighted ZA-LMS (RZA-LMS) is presented to take account different zero attractors for different taps.

Motivated by the ideas of sparse signal processing, a l_1-norm CNLMS (L$_1$-CNLMS) algorithm and weighted l_1-norm CNLMS (L$_1$-WCNLMS) are proposed in [17].

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Recently, many reweighted l_1-norm penalties and l_p-norm penalties are proposed and considered in [5-8]. Thus, it is possible to introduce these penalties into the cost function of the basic CLMS algorithm for obtaining a new beamformer to get a better performance.

In this paper, we develop a reweighted L_1-CNLMS (RL_1-CNLMS) algorithm and an l_p-norm based CNLMS (L_P-CNLMS) algorithm for sparse adaptive beamforming control applications. Simulation results demonstrate the proposed algorithms can get a better beam performance and use less antenna array elements, while the output SINR are also better than the existing algorithm in [17].

II. THE ARRAY PROCESSING MODEL

As is depicted in Fig. 1, a model of a planar antenna array which is composed of N omnidirectional antenna elements with a spacing of $\lambda/2$ is considered for discussing the adaptive beamforming algorithm, where λ denotes the operating frequency wavelength. Assuming that we have M+1 narrowband signals received by the antenna array including the SOI and interferences with the direction of θ_s and θ_i (i=1,2,…M). Then, receiving signals during kth snap can be written as:

$$\mathbf{x}(k) = \mathbf{a}_s \mathbf{s}(k) + \mathbf{a}_i \mathbf{i}(k) + \mathbf{n}(k). \quad (1)$$

In our notation, **a**, $\mathbf{s}(k)$, $\mathbf{i}(k)$ and $\mathbf{n}(k)$ are the steering matrix associated with the SOI as well as interferences, complex signal envelope vector and zero-mean white Gaussian noise vector, respectively. Note that the SOI, interferences and the noise are assumed to be statistically independent.

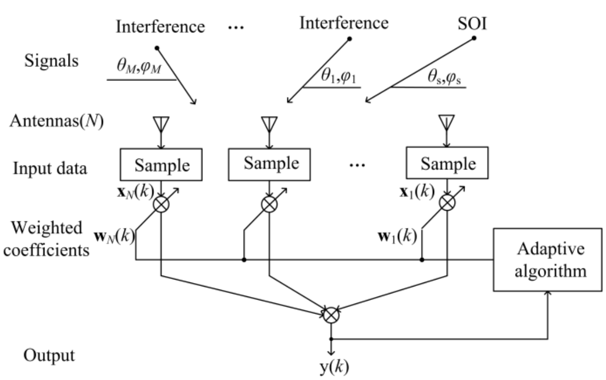

Fig. 1. Adaptive beamforming for planar antenna array.

In this case, one can write the SINR of the beamformer as:

$$\text{SINR} = \frac{p_s^2 |\mathbf{w}^{\text{H}} \text{a}_s|^2}{\mathbf{w}^{\text{H}} \mathbf{R}_{\text{n+i}} \mathbf{w}}, \quad (2)$$

where p_s^2 is the power of SOI, **w** is the weighted coefficient vector of the planar array with a dimension of N×1 and $\mathbf{R}_{\text{n+i}}$ is the interference-plus-noise covariance matrix which can be written as:

$$\mathbf{R}_{\text{n+i}} = E\left\{(\mathbf{i}(k)+\mathbf{n}(k))(\mathbf{i}(k)+\mathbf{n}(k))^{\text{H}}\right\}, \quad (3)$$

with $E\{\cdot\}$ representing the expectation operator and $(\cdot)^{\text{H}}$ stands for the Hermitian operator.

The output signal y(k) at time index k is given by:

$$\text{y}(k) = \mathbf{w}^{\text{H}} \mathbf{x}(k). \quad (4)$$

III. THE CNLMS ALGORITHMS FOR BEAMFORMING

A. The CLMS algorithm

The solution to the LCMV algorithm presented in [1] is expressed as:

$$\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^{\text{H}}\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{f}. \quad (5)$$

In (5), **R** is the covariance matrix of the input data. **C** and **f** are the constrained matrix, and the constrained vector, respectively, of whom the elements are associated to the SOI and interferences. Compared with the LCMV solution, the CLMS algorithm can adaptively provide a high gain for the SOI and effectively attenuate the interferences, which is to solve:

$$\min_{\mathbf{w}} E\left[|e_k|^2\right] \quad \text{subject to} \quad \mathbf{C}^{\mathbf{H}}\mathbf{w} = \mathbf{f}, \quad (6)$$

where $e_k = d_k - \mathbf{w}^{\text{H}}\mathbf{x}_k$ is the estimation error and d_k represents the expected output signal.

Make use of the Lagrange multiplier method, one can transform (6) into the following cost function:

$$L(k) = E\left[|e_k|^2\right] + \boldsymbol{\gamma}_1^{\text{H}}(\mathbf{C}^{\text{H}}\mathbf{w}_k - \mathbf{f}), \quad (7)$$

where $\boldsymbol{\gamma}_1$ is the Lagrange multiplier.

On the basis of the gradient descent principle, the update formulation can be constructed as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu \mathbf{g}_{\mathbf{w}} L(k), \quad (8)$$

where μ is the step size and $\mathbf{g}_{\mathbf{w}}L(k)$ is the gradient vector.

In this paper, we use the instantaneous estimate of the gradient vector for simply, which can be written as:

$$\mathbf{g}_{\mathbf{w}} L(k) = -2e_k^* \mathbf{x}_k + \mathbf{C}\boldsymbol{\gamma}_1. \quad (9)$$

Using the constraint in (6) and several straight-forward calculations, we can get the update function:

$$\mathbf{w}_{k+1} = \mathbf{P}\left[\mathbf{w}_k + \mu e_k^* \mathbf{x}_k\right] + \mathbf{f}_c, \quad (10)$$

where

$$\mathbf{P} = \mathbf{I}_{N\times N} - \mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathbf{H}}, \quad (11)$$

and

$$\mathbf{f}_c = \mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{f}. \quad (12)$$

B. The CNLMS algorithm

Note that the step size, also known as the convergence factor, is stationary in the CLMS algorithm. Hence, one can accelerate the convergence process by minimizing the instantaneous posteriori squared error with respect to the step size at snap k, which is to calculate [20]:

$$\frac{\partial[|e_{ap}(k)|^2]}{\partial \mu_k^*} = \frac{\partial\left[e_{ap}(k)e_{ap}^*(k)\right]}{\partial \mu_k^*} = 0, \quad (13)$$

where

$$e_{ap}\left(k\right)=e_k\left(1-\mu_k\mathbf{x}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k\right). \tag{14}$$

Then, we can get:

$$\mu_k=\frac{\mu_0}{\mathbf{x}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k+\xi_c}, \tag{15}$$

where $\xi_c>0$ is a small constant to prevent overflowing, and μ_0 is the step size used to implement this algorithm.

At last, we can get its updating equation:

$$\mathbf{w}_{k+1}=\mathbf{P}[\mathbf{w}_k+\mu_0\frac{e_k\mathbf{x}_k}{\mathbf{x}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k+\xi_c}]+\mathbf{f}_c. \tag{16}$$

C. The proposed new RL$_1$-CNLMS

In this paper, we develop a new RL$_1$-CNLMS algorithm for adaptive beamforming control application, which is to mimic:

$$\min_{\mathbf{w}} E\left[\left|e_k\right|^2\right] \quad \text{subject to} \quad \begin{cases}\mathbf{C}^{\mathbf{H}}\mathbf{w}_k=\mathbf{f};\\ \|\mathbf{h}_k\mathbf{w}_k\|_1=t,\end{cases} \tag{17}$$

where t denotes the constraint factor, and $\mathbf{h}_k$ is [6]:

$$\left[\mathbf{h}_k\right]_i=\frac{1}{\xi_{rl1}+|\mathbf{w}_{k-1}|_i},\ i=1,...,N \tag{18}$$

where $\xi_{rl1}>0$ is a small value similar with ξ_c in (15).

Similar to the basic CLMS algorithm, we can get the modified cost function applying the Lagrange multiplier method:

$$\begin{aligned}L_{rl1}(k)=&E\left[\left|e_k\right|^2\right]+\boldsymbol{\gamma}_1^{\mathbf{H}}(\mathbf{C}^{\mathbf{H}}\mathbf{w}_k-\mathbf{f})\\&+\gamma_{rl1}[\|\mathbf{h}_k\mathbf{w}_k\|_1\text{-}t],\end{aligned} \tag{19}$$

where $\boldsymbol{\gamma}_1$ and γ_{rl1} act as the Lagrange multipliers.

The instantaneous estimation for implementing the gradient of (19) is:

$$\mathbf{g}_{\mathbf{w}}L_{rl1}(k)=-2e_k^*\mathbf{x}_k+\mathbf{C}\boldsymbol{\gamma}_1+\gamma_{rl1}\mathbf{J}_{rl1}(k), \tag{20}$$

with

$$\mathbf{J}_{rl1}(k)=\frac{\mathrm{sgn}(\mathbf{w}_k)}{\xi_{rl1}+|\mathbf{w}_{k-1}|}, \tag{21}$$

where sgn($\cdot$) is an element-wise sign operator whose definition is:

$$\mathrm{sgn}(\mathrm{x})=\begin{cases}\dfrac{\mathrm{x}}{|\mathrm{x}|} & \mathrm{x}\neq 0;\\ 0 & \text{elsewhere.}\end{cases} \tag{22}$$

Based on the principle of gradient descent concepts shown in (8), we can get the final update equation that is written as:

$$\mathbf{w}_{k+1}=\mathbf{w}_k\text{-}\mu\mathbf{g}_{\mathbf{w}}L_{rl1}(k), \tag{23}$$

where $\mathbf{g}_{\mathbf{w}}L_{rl1}(k)$ is given in (20).

Now, it turns to solve the Lagrange multipliers. Under the circumstance that the algorithm has converged, we have $\mathbf{w}_{k+1}=\mathbf{w}_k$, then the constraints in (17) can be rewritten as:

$$\begin{cases}\mathbf{C}^{\mathrm{H}}\mathbf{w}_{k+1}=\mathbf{C}^{\mathrm{H}}\mathbf{w}_k=\mathbf{f},\\ \mathbf{J}_{rl1}(k)\mathbf{w}_{k+1}=\mathbf{J}_{rl1}(k)\mathbf{w}_k=\|\mathbf{h}_k\mathbf{w}_k\|_1=t.\end{cases} \tag{24}$$

After substituting (20) into (23), and pre-multiplying (23) by $\mathbf{C}^{\mathrm{H}}$ and $\mathbf{J}_{rl1}$, we can get the expressions for $\boldsymbol{\gamma}_1$ and γ_{rl1}:

$$\begin{cases}\boldsymbol{\gamma}_1=\mathbf{G}(2\mathrm{e}_k^*\mathbf{x}_k-\gamma_{rl1}\mathbf{J}_{rl1}(k)),\\ \gamma_{rl1}=(\dfrac{-2}{n\mu})(t-\mathbf{J}_{rl1}^{\mathrm{H}}(k)\mathbf{w}_k)+\dfrac{2e_k^*\mathbf{s}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k}{n},\end{cases} \tag{25}$$

with

$$\begin{cases}\mathbf{G}=(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}}\\ n=\|\mathbf{P}\mathbf{s}_k\|_2^2.\end{cases} \tag{26}$$

Putting $\boldsymbol{\gamma}_1$ and γ_{l1} into (23), and considering the normalizing method in [20], we can derive the final updating equation for the proposed RL$_1$-CNLMS:

$$\mathbf{w}_{k+1}=\mathbf{w}_k+\mu_k e_k^*\mathbf{V}+(t-\mathbf{J}_{rl1}^{\mathrm{H}}(k)\mathbf{w}_k)(\frac{\mathbf{P}\mathbf{J}_{rl1}(k)}{m}), \tag{27}$$

where, for simply, we use the notations as below:

$$\begin{cases}q=\mathbf{J}_{rl1}^{\mathrm{H}}(k)\mathbf{P}\mathbf{x}_k,\\ m=\mathbf{J}_{rl1}^{\mathrm{H}}(k)\mathbf{P}\mathbf{J}_{rl1}(k),\\ \mu_k=\dfrac{\mu_0\left[e_k-(t-\mathbf{J}_{rl1}^{\mathrm{H}}(k)\mathbf{w}_k)(\dfrac{\mathbf{P}\mathbf{J}_{rl1}(k)}{m})\mathbf{x}_k\right]}{e_k\mathbf{V}^{\mathrm{H}}\mathbf{x}_k+\xi_{rl1}},\\ \mathbf{P}=\mathbf{I}_{N\times N}-\mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathbf{H}},\\ \mathbf{V}=\mathbf{P}(\mathbf{x}_k-\dfrac{q\mathbf{J}_{rl1}(k)}{m}).\end{cases} \tag{28}$$

IV. THE Lp-CNLMS ALGORITHM

To further improve the adaptive beamforming performance of the designed beamformer, we develop an L$_p$-CNLMS algorithm. Inspired by the fact for the corresponding sparse constraint, the more it is closer to l_0-norm, the better result we will get. Thus, as we have known from the field of sparse system identification [6-9], the l_p-norm penalty which can obtain better results than l_1-norm is considered as a new constraint in the CNLMS algorithm to further improve the estimation behavior of adaptive beamformers.

The cost function of the L$_p$-CNLMS algorithm with $0<p<1$ is presented [6]:

$$\begin{aligned}L_{lp}(k)=&E\left[\left|e_k\right|^2\right]+\boldsymbol{\gamma}_1^{\mathbf{H}}(\mathbf{C}^{\mathbf{H}}\mathbf{w}_k-\mathbf{f})\\&+\gamma_{lp}[\|\mathbf{w}_k\|_{lp}\text{-}t].\end{aligned} \tag{29}$$

One can get the gradient instantaneous estimation for $L_{lp}(k)$ case, which is expressed as:

$$\mathbf{g}_{\mathbf{w}}L_{lp}(k)=-2e_k^*\mathbf{x}_k+\mathbf{C}\boldsymbol{\gamma}_1+\gamma_{lp}\mathbf{J}_{lp}(k), \tag{30}$$

where

$$\mathbf{J}_{lp}(k)=\frac{(\|\mathbf{w}_k\|_{lp})^{1-p}\,\mathrm{sgn}(\mathbf{w}_k)}{\xi_{lp}+|\mathbf{w}_k|^{1-p}}. \tag{31}$$

Note that the only difference between (20) and (30) are the $\mathbf{J}_{rl1}$ and $\mathbf{J}_{lp}$ terms. In this case, one can easily obtain the final updating function of the L$_p$-CNLMS algorithm by considering the $\mathbf{J}_{rl1}$ term like the equation

(26) to obtain:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_k e_k^* \mathbf{V} + (t - \mathbf{J}_{lp}^{\mathrm{H}}(k)\mathbf{w}_k)(\frac{\mathbf{P}\mathbf{J}_{lp}(k)}{m}), \quad (32)$$

where

$$\begin{cases} q = \mathbf{J}_{lp}^{\mathrm{H}}(k)\mathbf{P}\mathbf{x}_k, \\ m = \mathbf{J}_{lp}^{\mathrm{H}}(k)\mathbf{P}\mathbf{J}_{lp}(k), \\ \mu_k = \dfrac{\mu_0 \left[e_k - (t - \mathbf{J}_{lp}^{\mathrm{H}}(k)\mathbf{w}_k)(\dfrac{\mathbf{P}\mathbf{J}_{lp}(k)}{m})\mathbf{x}_k \right]}{e_k \mathbf{V}^{\mathrm{H}}\mathbf{x}_k + \xi_{lp}}, \\ \mathbf{P} = \mathbf{I}_{N\times N} - \mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathbf{H}}, \\ \mathbf{V} = \mathbf{P}(\mathbf{x}_k - \dfrac{q\mathbf{J}_{lp}(k)}{m}). \end{cases} \quad (33)$$

V. SIMULATION RESULTS

In this section, experiments are set up to evaluate the effectiveness and improvement of the proposed algorithms. The SOI and interferences are QPSK signals from the azimuth of 90°, 22°, 62°, 120° and 147°, respectively, which are received by the 91-elements hexagonal array (HA). The interference-to-noise ratio (INR) is 30 dB and the initialized step size for L_1-WCNLMS, RL_1-CNLMS, CNLMS and L_P-CNLMS are 5×10^{-2}, 2×10^{-3}, 5×10^{-3} and 5×10^{-4}, respectively; while the constraint factor t is set to 0.8 uniformly. The iteration times are 1.2×10^{-4}, while the parameters γ and ξ are 5 and 5×10^{-3}.

Figure 2 illustrates the beam patterns of the proposed algorithms in comparison with the existing algorithms. It can be seen from the figure that our proposed algorithms can form nulls corresponding interferences while generate nearly identical main lobe in the direction of SOI. What's more, the side lobe level (SLL) is lower than the algorithm developed in [17], but a little higher against the non-sparse algorithm CNLMS.

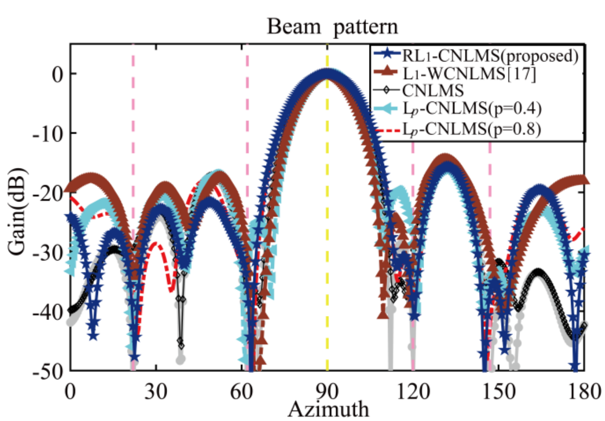

Fig. 2. Beam patterns of the proposed algorithms versus the CNLMS algorithm and the existing algorithm in [17]. Yellow line stands for the SOI, pink lines are interferences.

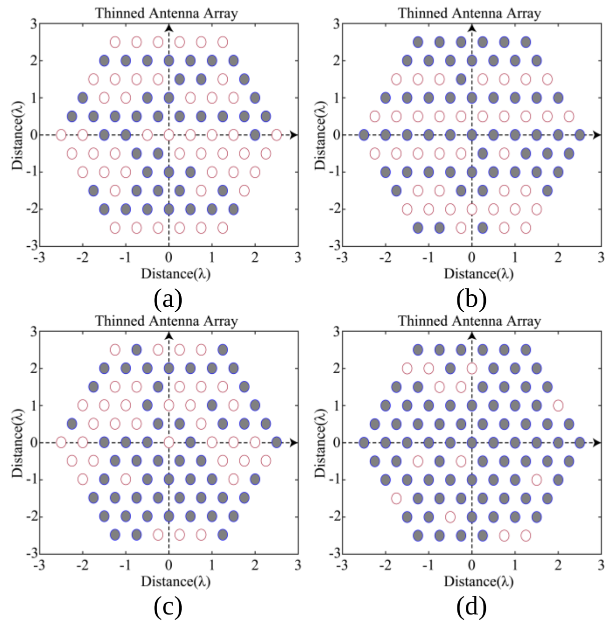

Fig. 3. Sparse arrays thinned by the proposed algorithms and the algorithm developed in [17]. (a) L_P-CNLMS algorithm with p=0.8, (b) L_P-CNLMS algorithm with p=0.4, (c) RL_1-CNLMS algorithm, and (d) algorithm in [17].

Figure 3 shows the sparse arrays thinned by using the proposed algorithms and the algorithm in [17]. As the figure indicates, all the algorithms can achieve sparse adaptive beamforming successfully. However, it is clearly that the beam patterns of the proposed algorithms turn off much more active antennas in comparison with the algorithm in [17] under the same iteration times. In addition, the L_P-CNLMS has a better performance than that of the RL_1-CNLMS algorithm since it can effectively exploit the sparseness of the antenna array. Thus, our proposed adaptive beamformer can reduce the power supply via utilizing less antenna elements to get nearly same performance in the HA beamforming.

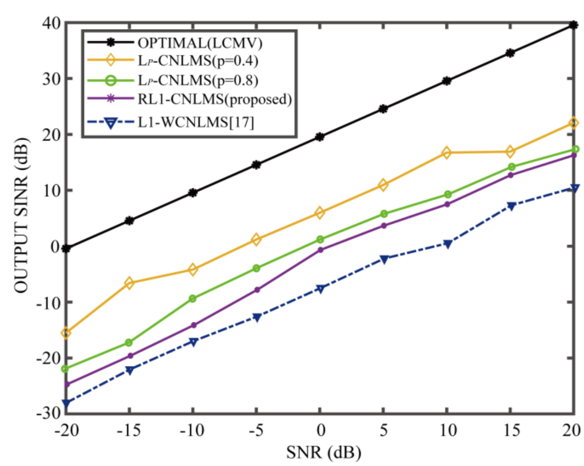

Fig. 4. Output SINR versus the input SNR.

In terms of the output SINR performance shown in Fig. 4, the proposed algorithms can obviously obtain a better SINR results with the same SNR. In addition, the L_p-CNLMS algorithm is superior to the RL_1-CNLMS. What's more, if p goes closer to 0, we will get better SINR performance, but its beam is getting worse. Thus, it is a trade-off for practical applications. Moreover, since we aim to develop sparse antenna array, resulting in an inferior output SINR which should be improved in the future.

VI. CONCLUSION

In this paper, a RL_1-CNLMS algorithm and an L_P-CNLMS algorithm have been proposed for sparse adaptive beamforming control applications. The proposed algorithms can reconstruct the main beam in the direction of SOI and provide nulls to reduce the influences from the interferences. Besides, they can achieve better performance than the existing sparse beamformer by using much less antenna elements. In terms of the output SINR, our proposed algorithms also have a good property. However, they still have some weaknesses that need for further study, such as the high SLL. Additionally, in the model, we neglect the influence of mutual coupling, which may lead to estimate error and need future investigate either. Also, we will consider the sparse beam scanning antenna arrays in the future studies in the MIMO antenna arrays [21-23].

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