Theory and Simulation of Linearized Force Coefficients for Active Magnetic Bearings with Multiple Magnetic Poles

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Abstract **─** Active magnetic bearing (AMB) is a kind of typical mechatronic product which is widely used due to its high performance. Since the electromagnetic force is strongly nonlinear, the force-displacement coefficient and the force-current coefficient need to be linearized in the process of controlling the AMB. This paper analyzes the three most common magnetic bearings and gives their linearized force formulas. Simulations show that the formulas have high precision. In addition, the source of errors is also analyzed in this paper.

Index Terms **─** Active magnetic bearing, ANSYS Maxwell simulation, error analysis, force coefficient, multiple magnetic poles.

I. INTRODUCTION

Industrially, active magnetic bearings have been widely used because of the excellent properties, such as high speed, long system life, adaptability to extreme temperature and corrosion condition [1]. Nevertheless, due to the open-loop instability and multiple nonlinearities of the AMB system, applicable feedback is required making the rotor stable to operate [2]. In the control process, the force-current coefficient and force-displacement coefficient of the bearing are very important quantities, which directly affect the control accuracy and system performance.

For the magnetic bearing with eight magnetic poles, Schweitzer et al. gave the linearization formula [3]. This formula is widely accepted, for example in [4,5]. But in this paper, a more accurate coefficient including an extra angle factor is given. Three types of magnetic bearings researched in this paper were introduced in [6], but corresponding linearization results were not given. In [7], the force-current coefficient of a 16-pole magnetic bearing was analyzed and the result was verified experimentally. Unlike the arc-shaped magnetic pole surface in this paper, in [8], the magnetic pole surface is considered to be plane, which will introduce an additional angle factor. For the papers mentioned above, there is no complete set of unified formulas for the magnetic bearings of each structure. Especially for magnetic bearings with different magnetic pole structures and distributions, the amount of air gap changes between the rotor and the stator are not the same as the rotor moves, resulting in a more complex formula for the linearized force.

This paper gives the linearization results of the AMB force including the bearings with eight, twelve, and sixteen magnetic poles, and carries out simulation verification using ANSYS Maxwell model. Material saturation, hysteresis, and magnetic leakage have all been ignored, and a new linear material has been set up to represent the main characteristics of the rotor and stator. This paper illustrates that for a particular structure of a magnetic bearing, its force-displacement coefficient and force-current coefficient can be calculated according to the formulas given in this paper. The simulation results show that these formulas have higher accuracy. Moreover, it should be noted that the model and theoretical calculations are obtained under very ideal conditions. However, for actual magnetic bearing models, the results obtained in this paper can be concluded that the factors neglected in this paper have little influence on the accuracy of magnetic bearing force calculation. The formulas in this paper can reflect the main aspects. Therefore, the formulas and the modeling method proposed in this paper have remarkable meaning and can be used to solve engineering problems, such as the control of unbalancing rotor and the coaxiality detection of the AMB system.

This paper consists of five sections. Section 2 gives the formulas for electromagnetic force linearization. Section 3 analyzes the two kinds of force coefficients of three types of magnetic bearings respectively. Section 4 analyzes the source of errors for linearization forces. Section 5 is the conclusion.

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II. LINEARIZED MAGNETIC FORCE FORMULA

Figure 1 is the schematic diagram of a couple of magnetic poles and a rotor. For a magnetic bearing, the force between the rotor and the stator depends on the magnetic induction in the air gap, which is generated by the current in the coil [9]. The two magnetic poles are respectively enwound with coils of different directions to form a closed magnetic field in the rotor, magnetic poles, and the air gaps, which generate force effect to the rotor.

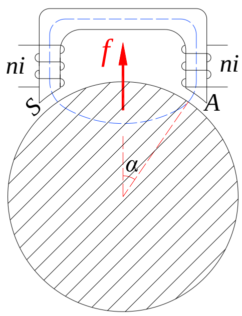

Fig. 1. A couple of magnetic poles and a rotor.

According to [3], the force generated by a couple of magnetic poles can be expressed as follows:

$$f = k\frac{i^2}{s^2}\cos\alpha\ , \qquad (1)$$

where i is the coil current, s the air gap width between the magnetic pole and the rotor, α the angle between the magnetic pole center and the vertical direction, k the intermediate quantity including the vacuum permeability μ_0, the number of turns n of the coil, and the area A of the magnetic pole. For the magnetic bearing of this structure, $k=\mu_0 n^2 A$, $\mu_0=4\pi\times10^{-7}$ N·A^{-2}.

Suppose the initial distance between the rotor and the magnetic pole is s_0. When the rotor moves up a distance x, the distance between the magnetic pole and the rotor is shortened to s_0-$x\cos\alpha$. Notice that the vertical direction is defined as x direction in this paper and only the vertical rotor displacement is discussed. An identical couple of magnetic poles is located at the lower symmetrical side of the rotor. The upper and lower magnetic poles adopt a differential driving mode. The current flowing through the coil is composed of bias current i_0 and control current i_x. Therefore, the resultant force in the vertical direction is:

$$f = k\left(\frac{(i_0+i_x)^2}{(s_0-x\cos\alpha)^2} - \frac{(i_0-i_x)^2}{(s_0+x\cos\alpha)^2}\right)\cos\alpha\ . \qquad (2)$$

According to the Taylor formula for the multivariate function, we have:

$$f = f(i_0,s_0) + f_i'(i_0,s_0)(i-i_0) + f_x'(i_0,s_0)(s-s_0) + \cdots . (3)$$

After eliminating high-order terms, the formula becomes:

$$f_+ = k\left(\frac{i_0^2}{s_0^2} + \frac{2i_0}{s_0^2}\cdot i_x + \frac{2i_0^2}{s_0^3}\cos\alpha\cdot x\right)\cos\alpha\ , \qquad (4)$$

$$f_- = k\left(\frac{i_0^2}{s_0^2} + \frac{2i_0}{s_0^2}\cdot(-i_x) + \left(-\frac{2i_0^2}{s_0^3}\cos\alpha\right)\cdot x\right)\cos\alpha\ . \qquad (5)$$

Then,

$$f = \frac{4ki_0}{s_0^2}\cos\alpha\cdot i_x + \frac{4ki_0^2}{s_0^3}\cos^2\alpha\cdot x\ , \qquad (6)$$

where the force-current coefficient and force-displacement coefficient are:

$$k_i = \frac{4ki_0}{s_0^2}\cos\alpha,\ k_x = \frac{4ki_0^2}{s_0^3}\cos^2\alpha\ . \qquad (7)$$

III. ANALYSIS OF DIVERSE BEARINGS

A. Bearing with eight magnetic poles

Figure 2 is the schematic diagram of the force of magnetic bearing with eight magnetic poles, where the rotor displacement and control current are drawn in one picture. For the sake of clarity, the air gaps between the rotor and the magnetic poles are abnormally amplified while the practical air gaps are usually in the millimeter order of magnitude. Such an alteration has a significant influence on the calculation accuracy, which will be explained in the following section.

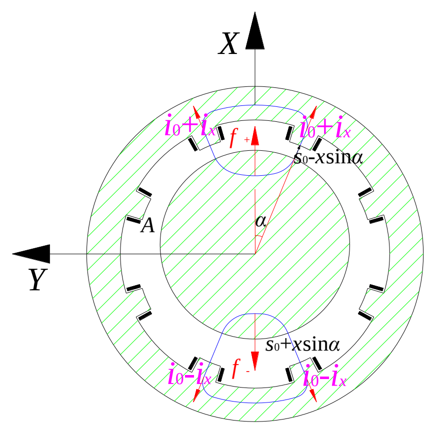

Fig. 2. Force schematic of bearing with eight poles.

A two-dimensional ANSYS Maxwell model is built, as shown in Fig. 3. In the model, the rotor radius is 104 mm, the air gap is 1 mm, the angle of magnetic pole with respect to the rotor center is 11°, α=22.5°. There is a coil enwinding on each of the magnetic poles to supply the current. For a two-dimensional figure, the width of a magnetic pole represents area A, because the axial length is set to 1m by default. So, the magnetic pole area is:

$$A = \alpha r = \frac{11^\circ}{180^\circ}\pi\cdot 0.105 \approx 0.0202\text{m}^2\ . \qquad (8)$$

A new material, Steel_1, with relative permeability and bulk conductivity of 10,000 and 2,000,000 $S\cdot m^{-1}$, respectively, is provided, and the material of stator and rotor is set to this "new" material. The N-pole and S-pole of magnetic poles can be altered by setting the winding direction of the coil so each of the two pairs of magnetic poles in the vertical direction can form a closed magnetic field to generate force to the rotor.

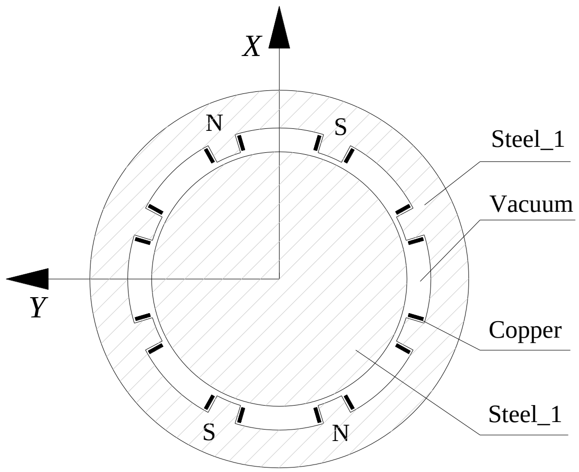

Fig. 3. ANSYS Maxwell model of eight-pole bearing.

Consider the two coefficients separately, i.e., when calculating the force-current coefficient, let x=0; when calculating the force-displacement coefficient, let i_x=0. For the sake of simplicity, only the vertical direction is considered, so two pairs of magnetic poles in the horizontal direction do not supply current. The maximum length of the grid in the model is limited to 1mm, and other parameters are set to default.

1) Force-current coefficient

Take n=100, i_0=5A, i_x=0.1A, so in Maxwell $NI_U=n\cdot(i_0+i_x)$=510A, $NI_D=n\cdot(i_0+i_x)$=490A; x=0. The theoretical value and the calculation result are 469.04N and 474.03N respectively, which the error is 1.06%.

2) Force-displacement coefficient

Take n=100, i_0=5A, x=0.1mm, so in Maxwell $NI_U=NI_D=n\cdot i_0$=500A. The theoretical value and the calculation result are 2167N and 2178N respectively, which the error is 0.51%.

B. Bearing with twelve magnetic poles

Figure 4 is the schematic diagram of the force of magnetic bearing with twelve magnetic poles where the angles of large and small magnetic poles are θ_L=30° and θ_S=15° respectively, and the angle between two magnetic poles is θ_A=10°.

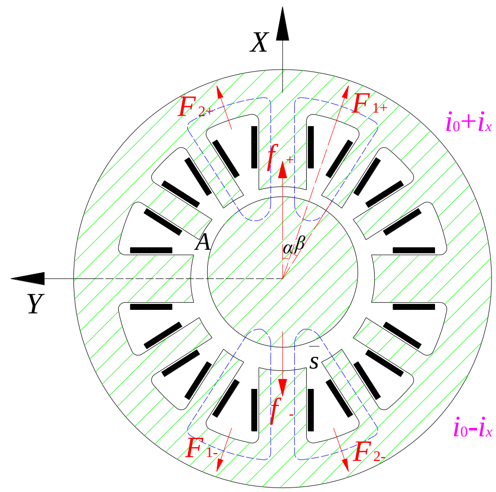

Fig. 4. Force schematic of bearing with twelve poles.

In ANSYS Maxwell model, the radius of the rotor is 109mm and the air gap is 1mm. As shown in Fig. 4, half of a large magnetic pole and a small magnetic pole generate a complete magnetic field, so A is the area of a small magnetic pole whose value is:

$$A = \alpha r = \frac{15^\circ}{180^\circ}\pi\cdot 0.11 \approx 0.0288\text{m}^2 . \qquad (9)$$

The calculation of the coefficients should be divided into two steps. In the first step, a small magnetic pole and half of a large magnetic pole generate forces F_1 and F_2; in the second step, F_1 and F_2 are merged into a vertical force F.

Unlike the bearing with eight magnetic poles, the changes in the air gaps at the large and small magnetic poles are not the same when the rotor moves. When there is a displacement x of the rotor in the vertical direction, the average air gaps for large and small poles are $s_1=s_0\pm x\cdot\cos(\theta_L/4)$ and $s_2=s_0\pm x\cdot\cos(\theta_L/2+\theta_A+\theta_S/2)$ respectively. Hence, for a complete magnetic circuit, the average air gap is:

$$\begin{aligned}\bar{s} &= s_0 \pm \frac{x}{2}\left(\cos\frac{\theta_L}{4}+\cos\left(\frac{\theta_L}{2}+\theta_A+\frac{\theta_S}{2}\right)\right) \\ &= s_0 \pm x\cos\alpha\cos\beta.\end{aligned} \qquad (10)$$

Then F_1 or F_2, whose direction has an angle α with the vertical axis is:

$$f = k\left(\frac{(i_0+i_x)^2}{(s_0 - x\cos\alpha\cos\beta)^2} - \frac{(i_0-i_x)^2}{(s_0 + x\cos\alpha\cos\beta)^2}\right)\cos\beta . \qquad (11)$$

According to the Taylor formula, we have:

$$f = \frac{4ki_0}{s_0^2}\cos\beta\cdot i_x + \frac{4ki_0^2}{s_0^3}\cos^2\beta\cos\alpha\cdot x , \qquad (12)$$

where the force-current coefficient is the same as the previously obtained one while the force-displacement coefficient contains an extra cosα. The ANSYS Maxwell model is shown in Fig. 5.

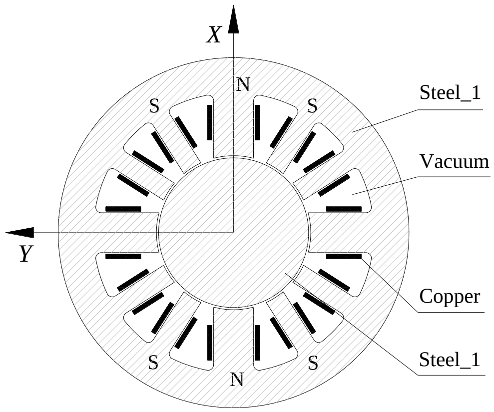

Fig. 5. ANSYS Maxwell model of twelve-pole bearing.

1) Force-current coefficient

Take n=100, i_0=5A, i_x=0.1A, so in Maxwell NI_U=$n\cdot(i_0+ i_x)$=510A, NI_D= $n\cdot(i_0+ i_x)$=490A; x=0. The theoretical value and the calculation result are 1328.0N and 1310.7N respectively, which the error is 1.30%.

2) Force-displacement coefficient

Take n=100, i_0=5A, x=0.1mm, so in Maxwell NI_U=NI_D=$n\cdot i_0$=500A. The theoretical value and the calculation result are 6090.0N and 5998.4N respectively, which the error is 1.50%.

C. Bearing with sixteen magnetic poles

Figure 6 is the schematic diagram of the force of magnetic bearing with sixteen magnetic poles, where the angles of large and small magnetic poles are θ_L=18° and θ_S=9° respectively, and the angle between two magnetic poles is θ_A=9°. Hence, β=0.5×(θ_S+θ_A)=9°, α=θ_L+θ_A=27°.

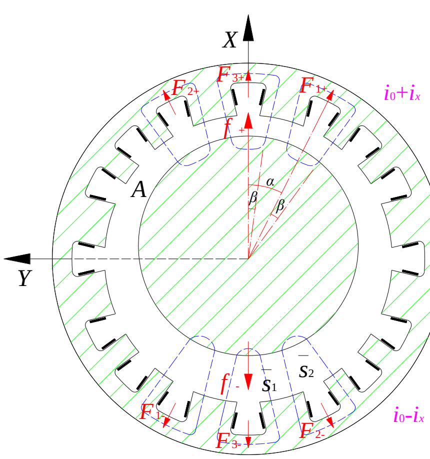

Fig. 6. Force schematic of bearing with sixteen poles.

As shown in Fig. 6, the force generated by the bearing with 16 magnetic poles consists of two parts: half of a large magnetic pole and one small magnetic pole form two magnetic fields, and the direction of the force between its direction and the vertical axis is α; two halves of the two large magnetic poles form another magnetic field. The total force is the superposition of the three.

In ANSYS Maxwell model, the radius of the rotor is 109mm and the air gap is 1mm. A is the area of a small magnetic pole whose value is approximate 0.0173m^2.

The calculation of the coefficients should be divided into three steps. In the first step, a small magnetic pole and half of a large magnetic pole generate a pair of forces F_1 and F_2; in the second step, two halves of the two large magnetic poles generate F_3; in the third step, F_1, F_2, and F_3 generate a vertical force F. Fig. 7 is the ANSYS Maxwell model.

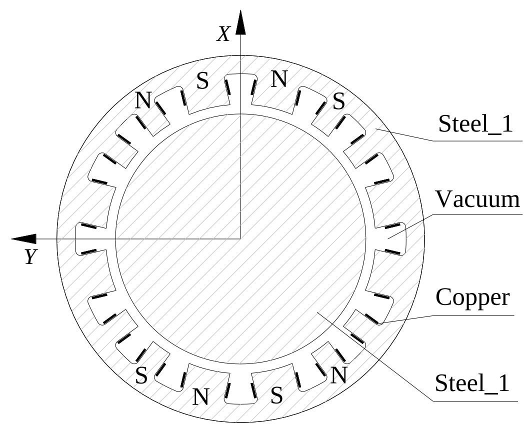

Fig. 7. ANSYS Maxwell model of sixteen-pole bearing.

1) Force-current coefficient

Take n=100, i_0=5A, i_x=0.1A, so in Maxwell NI_U=$n\cdot(i_0+ i_x)$=510A, NI_D= $n\cdot(i_0+ i_x)$=490A; x=0. Then,

$$F_1 = F_2 = F_3 = \frac{4\mu_0 n^2 A i_0}{s_0^2}\cos\beta \cdot i_x\,, \qquad (13)$$

$$f=(F_1+ F_2)\cos\alpha+F_3. \qquad (14)$$

The theoretical value and the calculation result are 1328.0N and 1310.7N respectively, which the error is 1.30%.

2) Force-displacement coefficient

Similar to the bearing with 12 poles, when the rotor has a displacement x in the vertical direction, the changes in the air gap at different poles are different. For the middle magnetic circuit, the average air gap is $\overline{s}_1 = s_0 \pm x\cos\beta$. For the magnetic circuit on both sides, the average air gap of large magnetic poles is s_{2L}=s_0±$x$cos(α-θ_A/2-θ_L/4), and the average air gap of small magnetic poles is $s_{2S}=s_0\pm x\cos(\alpha+\beta)$; therefore, the average air gap:

$$\begin{aligned}\bar{s}_2 &= s_0 \pm \frac{x}{2}\left(\cos\left(\alpha-\frac{\theta_A}{2}-\frac{\theta_L}{4}\right)+\cos\left(\alpha+\beta\right)\right). \\ &= s_0 \pm x\cos\alpha\cos\beta\end{aligned} \quad (15)$$

Analogy available, when $i_x=0$, take $n=100$, $i_0=5$A, $x=0.1$mm, so in Maxwell $NI_U=NI_D=n\cdot i_0=500$A. Then,

$$F_1 = F_2 = \frac{4ki_0^2}{s_0^3}\cos^2\beta\cos\alpha\cdot x\ , \quad (16)$$

$$F_3 = \frac{4ki_0^2}{s_0^3}\cos^2\beta\cdot x\ , \quad (17)$$

$$f=(F_1+F_2)\cos\alpha+F_3. \quad (18)$$

The theoretical value and the calculation result are 5488.1N and 5569.2N respectively, which the error is 1.5%.

IV. ERROR ANALYSIS

A. Air gap

The air gap is an important parameter of the magnetic bearing. Hence, the influence of the air gap on the computational accuracy is taken into considered. In theory, the larger the air gap and the smaller the rotor displacement, the better the linearization of the model, and the more accurate the theoretical and computational results. However, several air gap values are given and it is found that this was not the case.

Taking a bearing with twelve poles as an example, in the simulation, the air gap value is adjusted by changing the radius of the rotor. Ten simulations are performed, where $n=100$ and $i_0=5$A. Table 1 gives the comparison of theoretical and ANSYS results. In this section, theoretical value T is regarded as the true value and error e represents the degree of deviation of ANSYS result C, i.e., $e=(T-A)/T\times100\%$, which e might be positive or negative.

Table 1: Effect of air gap on calculation error of 12-pole bearing

Air Gap	k_x (x=0.1mm, i_x=0)			k_i (i_x=0.1A, x=0)		
	Theory/N	ANSYS/N	Error	Theory/N	ANSYS/N	Error
1mm	6090.0	5998.4	-1.50%	1328.0	1310.7	-1.30%
2mm	761.25	769.27	1.05%	332	338.71	2.02%
3mm	225.56	232.28	2.98%	147.56	153.09	3.75%
4mm	95.16	99.453	4.51%	83	86.856	4.65%
5mm	48.72	51.519	5.75%	53.12	55.743	4.97%
6mm	28.194	30.098	6.75%	36.889	38.637	4.74%
7mm	17.755	19.097	7.56%	27.102	28.227	4.15%
8mm	11.895	12.826	7.83%	20.75	21.418	3.22%
9mm	8.3539	9.0675	8.54%	16.395	16.723	2.00%
10mm	6.0900	6.6234	8.76%	13.28	13.351	0.53%

The two types of errors are plotted in a graph, and the trends can be seen, as shown in Fig. 8.

From Fig. 8, we can see that for the force-displacement coefficient, the error has a significant upward trend with the increase of the air gap value. This is probably because when the air gap is too large, part of the magnetic induction lines will close directly through the vacuum between the magnetic poles without passing through the rotor. However, for the force-current coefficient, the error first increases to a maximal value, and then decreases as the air gap increases. The increasing trend of force-current error has the same reason with force-displacement error. However, as the air gap continues to increase, the total amount of magnetic induction lines passing through the rotor decreases, resulting in a significant drop in force. Moreover, the residual magnetic induction lines are sparser. Therefore, the proportion of the residual magnetic induction line that does not pass through the rotor reduces, which reduces the error. It also can be seen that too small air gap does not improve the calculation accuracy.

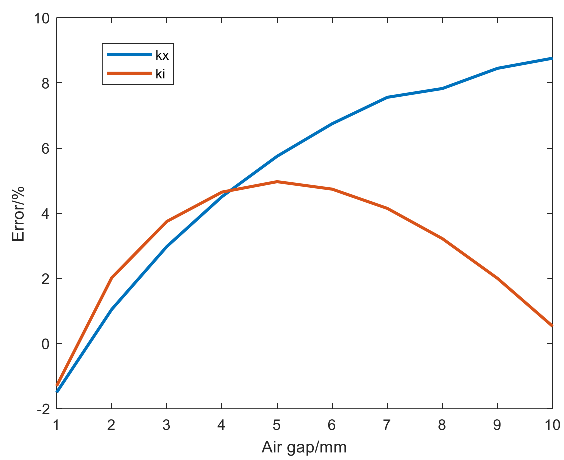

Fig. 8. Effect of air gap on errors of k_i and k_x.

However, the air gap width in this paper is not universal for all magnetic bearings, because for larger bearings, the tolerable air gap values become correspondingly larger. For the twelve-pole bearing in this paper, the circumferential distance of the two poles at the air gap is 0.0288m. As can be seen from the figure, when the air gap value is 1.5mm, the errors are small. A general ratio $\delta = s_0/l$ can be defined for the air gap s_0 and the partial circumferential distance l between two poles at the air gap. In this paper, the ratio of the two is 5.2%. Therefore, in order to ensure good computational accuracy, it is recommended that the δ not be larger than 5% to ensure both types of errors are close to zero.

B. Rotor displacement

Obviously, as the rotor displacement increases, the linearization formulas will no longer be accurate. For this reason, the scope of application of the force-displacement formulas is studied in this paper. From Section 4.A, taking 12 magnetic-pole bearing as an example, take the air gap as 1.5mm, n=100, i_0=5A, and analyze the force-displacement coefficient computational accuracy. The rotor displacements, theoretical and ANSYS results are listed in Table 2. The error trend is shown in Fig. 9.

Table 2: Effect of rotor displacement on calculation error of 12-pole bearing

Rotor Displacement	Theory/N	ANSYS/N	Error	Rotor Displacement	Theory/N	ANSYS/N	Error
0.025mm	451.11	447.32	-0.84	0.3mm	5413.3	5742.4	6.08
0.05mm	902.22	895.95	-0.70	0.4mm	7217.8	8084.9	12.0
0.075mm	1353.3	1347.1	-0.46	0.5mm	9022	10864	20.4
0.1mm	1804	1802	-0.11	0.6mm	10827	14298	32.1
0.125mm	2255.6	2261.9	0.28	0.7mm	12631	18718	48.2
0.15mm	2706.7	2728.3	0.80	0.8mm	14436	24661	70.9
0.175mm	3157.8	3202.4	1.41	0.9mm	16240	33054	104
0.2mm	3608.9	3685.7	2.13	1mm	18044	45621	153

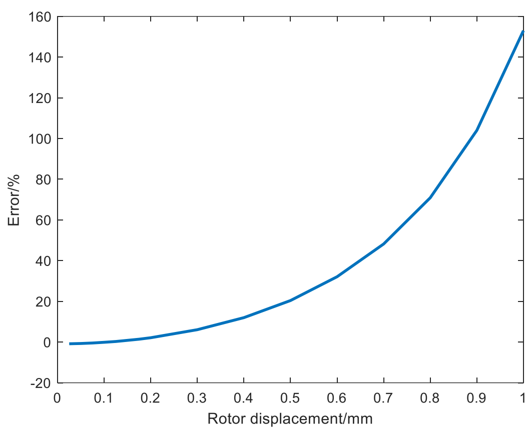

Fig. 9. Effect of rotor displacement on error of k_x.

From the table, it can be seen that when the rotor displacement exceeds 0.3 mm, the error will be greater than 6% and will increase sharply as the rotor displacement increases. Define a general ratio $\varepsilon = x/s_0$ for the rotor displacement x and air gap width s_0. In this paper, it can be considered that the linearization formula has an effective ε limit of 20%. Hence, for all sizes of bearings, the rotor displacement coefficient ε should not exceed this value.

V. CONCLUSION

In this paper, the linearized force-current and force-displacement formulas of magnetic bearings with eight, twelve, and sixteen poles are given. Simulation shows that the linearized formulas has high accuracy. The work in this paper shows that for some magnetic bearings of specific structures, the linearized coefficients can be accurately calculated based on their parameters. The source of errors is also analyzed and two referenced parameters are given to meet the engineering needs.

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