Identification of the Structural Deviations Impacting the Dynamics of a Flexible Multispan Rotor on Full Electromagnetic Suspension

Viktor F. Ovchinnikov, Mikhail Y. Nikolaev, Vasily N. Litvinov, and Denis V. Kapitanov

Research Institute of Mechanics
Lobachevsky State University of Nizhny Novgorod, Russia
minick@mech.unn.ru

Abstract **—** An algorithm for accounting for identification of the structural deviations impacting the dynamics of a flexible multispan rotor on full electromagnetic suspension in an actively developed computer model of dynamics of a flexible rotor on active magnetic bearings is presented. The algorithm is illustrated by applying it against the scale model of a rotor on active magnetic bearings.

Index Terms **—** Active magnetic bearing, computer model of dynamics of a flexible rotor on active magnetic bearings, dynamics influenced by structural deviations, flexible rotor.

I. INTRODUCTION

There are rather strict requirements maintained for the vibration level of rotor systems, as the lower vibration level is, the higher are rotor's running qualities and reliability. The problem of lowering the vibration level is relevant for all rotor systems, especially for the systems suspended on active magnetic bearings (AMB) as the rigidity of AMB is substantially lower compared to traditional sliding bearings, thus same level of impact on the rotor on AMB leads to higher vibration level.

The main cause of rotor vibration during normal operation is the imbalance, so to minimize it there is always a mandatory balancing procedure performed during the assembly of the rotor. Though residual imbalance is still always present, it could be partially compensated for after the rotor is installed in place.

Today a multispan flexible rotor on AMB is seen as a tool for solving relevant issues of energy efficiency and energy savings in the nuclear power engineering [1, 2]. Such rotors could also be utilized in vertical axial wind power plants allowing for efficient transformation of wind energy into electricity [3-5].

The multispan flexible rotor is intended to be a core part of the turbomachine of new nuclear power plant having high-temperature gas-cooled reactor. During the balancing procedure of the scale model of the rotor on AMB [6] it was found that certain structural parameters, despite being in tolerable limits according to design documentation, produce substantial effects on the dynamics of the rotor and thusly on the procedure of identification of the residual imbalance. Discovered effects showed that the previously developed method of balancing the rotor [7, 8] needs to be improved taking into account identification of the structural deviations affecting dynamics of the multispan flexible rotor on full electromagnetic suspension.

II. THE DESCRIPTION OF THE IDENTIFICATION METHOD

The method includes both numerical and experimental parts. The numerical one uses a discrete mathematical model of dynamics of a multispan flexible rotor on AMB [9, 10 and their references] adapted for the rundown mode. The input parameters of the model could be divided in two groups. First ones are the parameters with known values, and second ones are the parameters which are unpredictable and cannot be reliably computed but have to be obtained by solving the reverse problem.

The experimental part is based on utilizing the regular control system of AMB which allows for simultaneous recording of the following parameters of dynamics of the rotor:

- rotation angle and angular speed of the rotor;
- displacements of the rotor in the cross-sections where horizontal sensors are installed;
- currents in the AMB coils.

The oscillograms of the sensors are the input data for solving the reverse problem and for defining the unknown input parameters of the model.

To minimize the dependency of the solution of the reverse problem from the accuracy of measuring the experimental data it is advised to minimize the number of defined parameters. This is done by selecting and processing modes which are affected only by the desired parameters. One of such modes is the rundown mode during which no external forces except the ones from axial and radial AMB are impacting the rotor.

To solve the reverse problem portions of the rundown recordings where rotation frequency could be considered constant are used; such portions could be

Submitted On: June 29, 2018
Accepted On: December 14, 2018

1054-4887 © ACES

seen as stable modes with constant angular speed. The portions that are close to critical rotation frequencies are excluded because at such frequencies dynamics of the rotor is impacted by inner and outer dissipation forces which are very hard to measure for complex structures.

A. The model of dynamics of a multispan flexible rotor

During the formulation of the discrete mathematical model of dynamics of a multispan flexible rotor on AMB the rotor is taken as elastic heterogeneous rod with piecewise characteristics with circular sections. For description of deformations and displacements of each of the rotor sections the oscillation equations of elastic rod is used (the Timoshenko model of a beam [11]). These elements are coupled by three types of concentrated elements: rigid joint, elastic joint and support point of the rotor (AMB). To create the mathematical model of the machine certain combination of the aforementioned basic elements is used.

In the mechanical model of the rotor Cartesian coordinate system *0xyz* is used for describing the movements of the rotor. *0x* axis is taken as vertical one coinciding with the rotation axis of the unstrained rotor. It is assumed that for all the rotor characteristics and for the radial AMB axial symmetry conditions stand. Current position of the elements of the rotor is defined by: linear displacement ***U***; vector of angular displacements of the normal cross-section ***Θ***; angular displacements incurred by own rotation with angular speed ω. The deformation of material of the rotor elements creates the tension in the normal cross-section which are statically equivalent to inner torque ***M*** and inner force ***Q*** applied in the center of the bending. Vector ***Q*** includes intersecting stress, and ***M*** includes bending torque in two directions. As the rotor element moves it is being affected by a distributed force ***q*** applied on the axis of the element.

When projected to horizontal axes *0y* and *0z*, the motion equations of the element of the rotor including gyroscopic forces are:

$$
\begin{aligned}
&\frac{\partial Q_y}{\partial x} = m\frac{\partial^2 U_y}{\partial t^2} - q_y,\\
&\frac{\partial M_z}{\partial x} = -Q_y + \rho I\frac{\partial^2 \Theta_z}{\partial t^2} + 2\omega\rho I\frac{\partial \Theta_y}{\partial t},\\
&\frac{\partial \Theta_z}{\partial x} = \frac{1}{EI}M_z, \quad \frac{\partial U_y}{\partial x} = \Theta_z + \frac{1}{GFk^*}Q_y,\\
&\frac{\partial Q_z}{\partial x} = m\frac{\partial^2 U_z}{\partial t^2} - q_z,\\
&\frac{\partial M_y}{\partial x} = Q_z + \rho I\frac{\partial^2 \Theta_y}{\partial t^2} - 2\omega\rho I\frac{\partial \Theta_z}{\partial t},\\
&\frac{\partial \Theta_y}{\partial x} = \frac{1}{EI}M_y, \quad \frac{\partial U_z}{\partial x} = -\Theta_y + \frac{1}{GFk^*}Q_z.
\end{aligned}
\tag{1}
$$

Here ρ, E, G are density, modulus of elasticity and shear modulus of the material of the element. F, I are area and axial moment of inertia of the normal cross-section of the element. $m=\rho\cdot F$ is the distributed mass of the tube. k^* is the coefficient accounting for nonuniformity of distribution of tangential stress over the cross-section of the element. x is the coordinate along the rotation axis; t is time. Lower index stands for projection on respective axis (*0y* or *0z*).

The presented model of transverse oscillations of the rotor is used to develop the method of identification of imbalance characteristics when the following are present: non-orthogonality of disc of the axial AMB to the rotation axis; misalignment of the rotor elements in the elastic joints of the elements (elastic clutches). The studied effects lead to amplitude-constant harmonic impact on the rotor on the frequency of the rotor revolution. If the only effects present in the system are these, and the rotor and AMB are axisymmetric, then the rotor movements in *0xy* and *0xz* planes will be harmonic and identical except that *0xz* movements will lag by a quarter of the period. Using complex representation of the bending forms of the rotor in the stationary forced oscillation mode the equations (1) could be written as follows:

$$
\begin{aligned}
&Q_y = Q(x)\cdot \exp j\omega t, \quad M_z = M(x)\cdot \exp j\omega t,\\
&\Theta_z = \Theta(x)\cdot \exp j\omega t, \quad U_y = U(x)\cdot \exp j\omega t,\\
&q_y = q(x)\cdot \exp j\omega t, \quad \mu_z = \mu(x)\cdot \exp j\omega t,\\
&\Theta_y = -\Theta(x)\cdot \exp j(\omega t - \pi/2) = j\Theta_z,\\
&\frac{dQ}{dx} = -\omega^2\cdot m\cdot U - q,\\
&\frac{dM}{dx} = -Q - 3\cdot\omega^2\rho I\cdot\Theta,\\
&\frac{d\Theta}{dx} = \frac{1}{EI}M,\\
&\frac{dU}{dx} = \Theta + \frac{1}{GFk^*}Q.
\end{aligned}
\tag{2}
$$

To solve the differential equations (2) we used the expansion of the solution by the basis of orthogonal functions $Q_k(x), M_k(x), \Theta_k(x), U_k(x)$ defined by solving the boundary problem of the rotor where the dependency between the inner forces and movements at the elements is expressed by:

$$
\begin{aligned}
&\frac{dQ_k}{dx} = -\omega_k^2\cdot m\cdot U_k,\\
&\frac{dM_k}{dx} = -Q_k - 3\cdot\omega_k^2\cdot\rho I\cdot\Theta_k,\\
&\frac{d\Theta_k}{dx} = \frac{1}{EI}M_k,\\
&\frac{dU_k}{dx} = \Theta_k + \frac{1}{GFk^*}Q_k.
\end{aligned}
\tag{3}
$$

The form number k corresponds to Eigen frequency ω_k. Equations (3) differ from same equations for Eigen oscillation modes of a nonrotating rotor by a factor of "3" in the second addendum in the right-hand member of the second equation. This difference is caused by the impact of gyroscopic forces. It is assumed that the AMB control system forms a linear control law of generating the force depending on the rotor displacement [12], so the boundary conditions placed in the cross-sections where radial AMB are installed correspond to the rotor resting on elastic supports. The rigidity of these supports is defined by the parameters of the AMB and the used control laws. Also the forms account for the impact from axial AMB which is equivalent to the one from springs of negative rigidity [10] appearing on rotor inclinations. The rigidity of such springs is defined by the structure of the axial AMB and by the forces they generate.

The shafting in question is taken as L elastically coupled rotors. Elastic joints are located at cross-sections with coordinate $x = x_i$ ($i = \overline{1,L}$). It is assumed that at each elastic joint there are linear and angular displacements of axes of the elements of rotors, which are characterized by components U_{1i}^0, U_{2i}^0 and $\Theta_{1i}^0, \Theta_{2i}^0$ in the coordinate system attached to the rotor. The shafting has N radial AMB in the cross-section with coordinates $x = x_n$ which are characterized by rigidity c_n ($n = \overline{1,N}$). The shafting is supported in the vertical direction by N_0 axial AMB placed at $x = x_\nu$ $(\nu = \overline{1,N_0})$. Due to non-orthogonality of the disc of axial AMB to rotation axis the rotor is impacted by a constant torque which is defined by M_1^ν, M_2^ν components in the coordinate system attached to the rotor. The rotor structure is characterized by the imbalance which creates a distributed force along $0y$ axis:

$$q_y = \omega^2 m[e_1(x)\cdot\cos\varphi - e_2(x)\cdot\sin\varphi]. \tag{4}$$

Here $e_i(x)$ are components of the eccentricity vector in the coordinate system attached to the rotor ($i=1,2$); φ is the rotation angle of the rotor $\left(\frac{d\varphi}{dt} = \omega\right)$.

To compose the numerical discrete model of dynamics of the rotor, the movements of the rotor along the $0y$ axis are expanded by eigenmodes:

$$\begin{aligned} U_y &= \sum_{k=1}^{K} a_k U_k(x),\ \Theta_z = \sum_{k=1}^{K} a_k \Theta_k(x), \\ M_z &= \sum_{k=1}^{K} a_k M_k(x),\ Q_y = \sum_{k=1}^{K} a_k Q_k(x). \end{aligned} \tag{5}$$

Here a_k are time-dependent expansion coefficients filling the role of generalized coordinates in the discrete model; K is the amount of eigenmodes used in the approximation of the solution. By using Lagrange equations we have defined the discrete model of movement equations of the rotor:

$$\begin{aligned}
& m_0 \frac{d^2 a}{dt^2} + m_0 \Omega_K a = \alpha \cdot D^*(d, I_0) \\
& + \omega^2 [f_1 \cos\phi - f_2 \sin\phi] + R_1 \cos\phi - R_2 \sin\phi, \\
& D^*(d, I_0) = c\cdot d + D(d, I_0), \\
& \alpha = \{\alpha_{kn}\}, \quad \alpha_{kn} = U_k(x_n), \\
& c = diag(c_1, ..., c_N), \quad \Omega_K = diag(\omega_k^2), \\
& f_j = \{f_{kj}\}, \quad f_{kj} = \int_0^l m\cdot e_j(x)\cdot U_k(x) dx, \\
& k = \overline{1,K}, \quad j = 1,2, \\
& R_1 = \Theta_0 \cdot M_2 + Q_0 \cdot U_1^0 + M_0 \cdot \Theta_2^0, \\
& R_2 = -\Theta_0 \cdot M_1 + Q_0 \cdot U_2^0 - M_0 \cdot \Theta_1^0, \\
& M_1 = \left(M_j^1, ..., M_j^\nu, ..., M_j^{N_0}\right)^T, \\
& U_j^0 = \left(U_{j1}^0, ..., U_{ji}^0, ..., U_{jL}^0\right)^T, \\
& \Theta_j^0 = \left(\Theta_{j1}^0, ..., \Theta_{ji}^0, ..., \Theta_{jL}^0\right)^T, \quad j = 1,2, \\
& \Theta_0 = \begin{vmatrix} \Theta_1(x_1) & \dots & \Theta_1(x_{N_0}) \\ \dots & \dots & \dots \\ \Theta_K(x_1) & \dots & \Theta_K(x_{N_0}) \end{vmatrix}, \\
& Q_0 = \begin{vmatrix} Q_1(x_1) & \dots & Q_1(x_L) \\ \dots & \dots & \dots \\ Q_K(x_1) & \dots & Q_K(x_L) \end{vmatrix}, \\
& M_0 = \begin{vmatrix} M_1(x_1) & \dots & M_1(x_L) \\ \dots & \dots & \dots \\ M_K(x_1) & \dots & M_K(x_L) \end{vmatrix}.
\end{aligned} \tag{6}$$

Here $a=(a_1,...,a_K)$ is the K-dimensioned vector of generalized coordinates; d is the N-dimensioned vector representing displacements of the rotor in the cross-sections where radial AMB are placed; m_0 is the mass of the rotor. $D(d, I_0)$ is the vector of active forces generated by AMB; I_0 is the $4N$-dimensioned vector describing the currents in the AMB coils. R_j is the vector of generalized forces accounting for non-orthogonality of discs of axial AMB to rotation axis and misalignment in the elastic clutches. Upper index "T" stands for operation of transposing of a matrix. It is worth noting that equations (6) also describe the movements of the rotor along the $0z$ axis.

B. Mathematical statement of the identification problem

It is assumed that transversal movements of the rotor are monitored by P sensors $(P \geq N)$ placed at the cross-sections with coordinates x_p $(p = \overline{1,P})$. The case of $P=N$ corresponds to the situation when the machine has only regular sensors of the control system of the AMB. As shown in (5), dynamic movements of the rotor in the cross-sections of the sensors $U = (U^1, U^2, ..., U^p)^T$ are bound to the introduced vector of generalized coordinates by the equation:

$$U = \beta \cdot a, \quad \beta = \{\beta_{pk}\}, \quad \beta_{pk} = U_k(x_p),$$
$$k = \overline{1,K}, \; p = \overline{1,P}. \tag{7}$$

The mathematical problem of identification of the imbalance, forces and torques from the axial AMB, misalignment characteristics in the elastic clutches is placed by utilizing the equations (6) and (7) and by using known (experimentally measured) vector functions *U(t)* and *I(t)* and to define the components of the vectors $f_j, M_j, U_j^0, \Theta_j^0$ (j=1,2). The vector of active forces $D(d, I_0)$ is computed using known properties of the AMB, measured currents in the coils of the AMB, and rotor displacements in the cross-sections where axial AMB are installed.

To exclude the errors caused by the presence of high-frequency noise in the real-world data it makes sense to consider equations (6) and (7) in the frequency region. Forced oscillations of the system on the constant rotation frequency could be viewed as harmonic as a first approximation:

$$a(t) = a^{(1)} \cos\varphi + a^{(2)} \sin\varphi,$$
$$U(t) = U_1 \cos\varphi + U_2 \sin\varphi. \tag{8}$$

The vector $D^*(d, I_0)$ has complex time dependency because the control system is nonlinear in nature, but it is possible to extract harmonic part on the rotation frequency of the rotor, and as a first approximation the following is true:

$$D^*(t) = D_1 \cos\varphi + D_2 \sin\varphi. \tag{9}$$

After substitution of (8) and (9) to (6) and (7) we can get the following:

$$\begin{aligned} & m_0 \cdot \left(\Omega_K - \omega^2 \cdot E_K\right) \cdot a^{(1)} - \alpha^T \cdot D_1 = \\ & \quad \omega^2 \cdot f_1 + \Theta_0 \cdot M_2 + Q_0 \cdot U_1^0 + M_0 \cdot \Theta_2^0, \\ & m_0 \cdot \left(\Omega_K - \omega^2 \cdot E_K\right) \cdot a^{(2)} - \alpha^T \cdot D_2 = \\ & \quad -\omega^2 \cdot f_2 + \Theta_0 \cdot M_1 - Q_0 \cdot U_2^0 + M_0 \cdot \Theta_1^0, \\ & U_1 = \beta \cdot a^{(1)}, \quad U_2 = \beta \cdot a^{(2)}. \end{aligned} \tag{10}$$

By elimination of vectors $a^{(1)}, a^{(2)}$ we can get matrix equations against unknown vectors $f_j, M_j, U_j^0, \Theta_j^0$ (j=1,2):

$$\begin{aligned} & U_1 - H \cdot \alpha^T \cdot D_1 = \omega^2 \cdot H \cdot f_1 + H \cdot \Theta_0 \cdot M_2 \\ & + H \cdot Q_0 \cdot U_1^0 + H \cdot M_0 \cdot \Theta_2^0, \\ & U_2 - H \cdot \alpha^T \cdot D_2 = -\omega^2 \cdot H \cdot f_2 + H \cdot \Theta_0 \cdot M_1 \\ & - H \cdot Q_0 \cdot U_2^0 + H \cdot M_0 \cdot \Theta_1^0, \\ & H = \beta \cdot G, \quad G = diag\left(\frac{1}{m_0 \cdot \left(\omega_k^2 - \omega^2\right)}\right), \\ & A(\omega) \cdot g_j = l_j(\omega), \\ & l_j(\omega) = U_j - H \cdot \alpha^T \cdot D_j, \quad j = 1,2, \\ & A(\omega) = \left(\omega^2 \cdot H, H \cdot \Theta_0, H \cdot Q_0, H \cdot M_0\right), \\ & g_1 = \left(f_1^T, M_2^T, U_1^{0^T}, \Theta_2^{0^T}\right)^T, \\ & g_2 = \left(-f_2^T, M_1^T, -U_2^{0^T}, \Theta_1^{0^T}\right)^T. \end{aligned} \tag{11}$$

Equations (11) define the connection between the structural deviations and measured characteristics of the movements of the rotor.

The *P* amount in equations in (11) is usually less than the amount of unknown quantities ($K+N^0+2L$). The unknown are components of the vectors $f_j, M_j, U_j^0, \Theta_j^0$. Also, some of the equations might be linearly dependent, so to solve the practical problem it is necessary to perform measurements over a series of frequencies $\omega^{(i)}$, combine the equations for all those series and solve the combined equations set as a whole:

$$A(\omega^{(i)}) \cdot g_j = l_j(\omega^{(i)}), \quad i = \overline{1, I^*}, \quad j = 1,2. \tag{12}$$

Here I* is the amount of taken portions of recordings of the rotor rundown. It is obvious that, when we increase the amount of experimental data, the accuracy of identification of residual imbalance of the rotor also increases. However, in such case the amount of equations in the set (12) is going to be bigger than amount of unknown quantities (overdetermined system), and due to noises and errors in the measurement some of the equations will become contradictory. To solve such problems there are special mathematical methods based on least-squares method [13].

III. VERIFICATION OF THE COMPUTER IMPLEMENTATION OF THE METHOD

The described method was implemented in the software to identify the residual imbalance of a rotor. Verification of the method and the implementation was conducted against the shafting comprised by two elastically coupled rotors 140 mm in diameter (see Fig. 1).

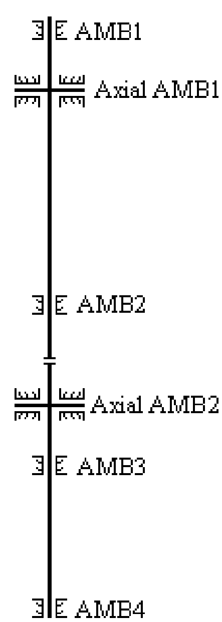

Fig. 1. Scheme of the experimental shafting.

Mass of the upper rotor is 600 kg, length is 4.86 m. Mass of the lower rotor is 420 kg, length is 3.37 m. Centers of radial AMS are located at 0.05 m, 4.62 m, 5.4 m and 8.18 m from the upper end of the shafting. Axial AMB are located at 0.35 m and 5.09 m. Outer radii of the discs of the axial AMB are 0.235 m. Rigidity of radial AMB is 800 N/mm. Elastic clutch shear stiffness is 40 N/mm, bending stiffness 700 N/mm. Computed lowest critical frequencies are (in Hz): 7.18, 9.51, 13.02, 13.81, 29.32, 54.47, 72.27, 139.50. Operational rotation frequency of the shafting is 60 Hz.

For verification the numerical model of the shafting had the following structural deviations introduced:

- Imbalance of 6.042 kg*mm in the upper rotor, placed at 0.025 m from the upper end; directed at angle $\psi = 0^0$ in the coordinate system attached to the rotor.
- Imbalance 1.813 kg*mm in the lower rotor placed at 6.625 m from the upper end of the shafting; directed at $\psi = 270^0$.
- Non-orthogonality of the disc of axial AMB1 to the rotation axis with angle being Θ^*=0.0005 rad. The direction of horizontal rotation of the disc in the coordinate system attached to the rotor is described as $\psi = 0^0$.
- Non-orthogonality of the disc of axial AMB2 to the rotation axis with angle being Θ^*=0.0008 rad. Directed at $\psi = 90^0$.
- Linear shear of two rotors' axes in the elastic clutch is 1 mm. The direction of the shear in the coordinate system attached to the rotor is described as $\psi = 0^0$.
- Angular shear of two rotors' axes in the elastic clutch is 0.1 rad. The shear's plane is characterized by $\psi = 90^0$.

Using the computer model of dynamics of multispan flexible rotor on AMB we had conducted the calculations of spin-up mode of the shafting with acceleration of 0.25 Hz/s to rotation frequency 70 Hz. Figure 2, black curves depict dependencies of displacement amplitudes from rotation frequency at the cross-sections where the radial AMBs are installed.

It is obvious from the picture that displacement amplitudes are close to the clearance in the retainer bearings (0.4 mm) when passing critical frequencies. This is unacceptable for the rotors on AMB as, based on recommendations [14], rotor displacements should be no more than half of the clearance in the retainer bearings. Therefore, for the shafting in question it is necessary to conduct certain measures aimed at reducing the impact on the rotor dynamics from the structural deviations. It could be done based on the knowledge of the properties of the aforementioned deviations determined by the identification method shown above. In this example displacement sensors are installed at AMB cross-sections as well as at 2.42 m and 6.61 m from the upper end of the shafting. The identification method used 13 portions of the rundown recordings (with rotation frequency, Hz): 3.0, 4.0, 5.0, 8.0, 12.0, 16.0, 20.0, 25.0, 35.0, 40.0, 45.0, 50.0, 60.0, 65.0. Each portion is 16.0 m long. The identification results for the components of the vector of the modal imbalance f^*_{kj} and comparison with the "precise" values f_{kj}, computed using (6), are presented in the Table 1.

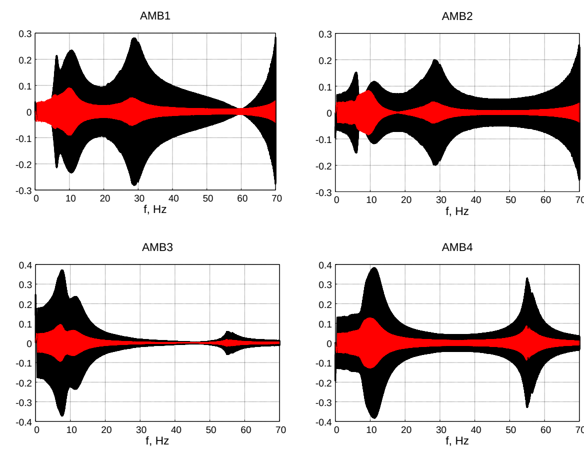

Fig. 2. Dependency of the displacement amplitudes from rotation frequency. Black curves: deviations are present; red curves: deviations are compensated by reducing the imbalance and misalignments.

Table 1: Comparison of the identified and the "precise" imbalance

k	"Precise" Values, kg*mm		Identified, kg*mm		Difference	
	f_{k1}	f_{k2}	f^*_{k1}	f^*_{k2}	Δf_k	δ_k, %
1	-5.63	0.11	-5.39	0.29	0.3	5.3
2	0.81	2.81	0.40	3.14	0.53	18.0
3	-11.82	0.49	-11.42	-0.80	1.35	11.4
4	-3.29	-0.95	-3.50	-1.30	0.41	11.9
5	15.47	0.02	15.22	-0.01	0.25	1.6
6	0.07	-3.27	0.11	-3.11	0.17	5.0
7	-14.49	0.11	-14.27	-1.31	1.44	9.9
8	-13.99	-0.02	-13.66	1.20	1.26	9.0

As modal imbalance, generalized forces, torques from axial AMB, linear and angular shears of rotors' axes in the elastic clutch are vector values. The difference in Table 1 is computed using the following formula:

$$\Delta A = \sqrt{\left(A_1^* - A_1\right)^2 + \left(A_2^* - A_2\right)^2},$$

$$\delta = 100 \cdot \Delta A / A, \quad A = \sqrt{A_1^2 + A_2^2},$$

where it is clear that for all modes except k=2, the relative difference is less than 12%. For mode k=2 high level of relative difference is caused by small value of the modal imbalance – absolute difference for this mode is on par with other modes.

Table 2 presents the values of torques M_j^{ν} from axial AMB calculated by using the given structural parameters as well as identified using the method presented.

Table 2: Comparison of the identified and the "precise" torques

ν	"Precise" Value, N*m		Identified, N*m		Difference	
	M_1^{ν}	M_2^{ν}	$M_1^{*\nu}$	$M_2^{*\nu}$	ΔM_{ν}	δ_{ν}, %
1	0	108.3	-2.3	125.5	17.4	16.0
2	-123.3	0	-121.4	13.5	13.6	11.1

Table 3 presents linear and angular shears of rotors' axes in the elastic clutch U_j^0, Θ_j^0 and respective properties U_j^{*0}, Θ_j^{*0} retrieved by identification.

The results that are displayed in Tables 1-3 show that the developed method presented in this paper could be satisfactory used to identify structural deviations impacting the dynamics of the rotor. The information obtained by the method allows implementing specific measures for compensating or minimizing those deviations. The measures include mechanical work for improving the physical rotor structure (placing balance weights, straightening of the elastic clutch) and using control laws for AMB that compensate for the effect of harmonic forces caused by structural deviations [10].

Table 3: Comparison of the "precise" and the identified shear

Shear	"Precise"		Identified		Difference	
	j=1	j=2	j=1	j=2	Δ	δ, %
U_j^0, mm	1,0	0	0.961	0.006	0.04	3.9
Θ_j^0, rad	0	0.1	0.0017	0.075	0.025	25.0

For the example used in the verification by utilizing the data from the tables it is possible to: reduce the imbalance by 88%; reduce the non-orthogonality angle of upper axial AMB by 84%, and to lower axial AMB by 88%; reduce linear and angle shears in the elastic clutch by 96% and 75%, respectively. The displacement amplitudes after such measures are depicted on Fig. 2 by red curves. These red curves show that in all the frequency range during spin-up the displacements are no more than 130 microns. In the stable operation mode displacements at the cross-sections of AMB are no more than 30 microns. Such level of displacement according to [14] is in the "A" zone which is typical vibration for new machines being introduced in the operation.

IV. CONCLUSION

We have developed the numerical-experimental method to identify structural deviations of a multispan flexible rotor on full electromagnetic suspension. Examples of the structural deviations taken into account are: rotor imbalance, non-orthogonality of the discs of the axial AMB to the rotation axis, misalignment of the elements of the rotor in the elastic clutch. Such structural deviations are creating harmonic forces with the frequency equal to rotation frequency. The identification method is based on solving the reverse dynamics problem: from experimentally measured rotor displacement we calculate the properties of the forces which caused those displacements. The model and the software implementation are verified against the sample shafting comprised by two rotors coupled by an elastic clutch.

ACKNOWLEDGMENT

The research was performed with financial support from Russian Science Fund (Grant No. 16-19-10279).

REFERENCES

[1] S. Lei, Y. Suyuan, Y. Guojun, S. Zhengang, and X. Yang, "Technical design and principle test of active magnetic bearings for the compressor of HTR-10GT," in *Proc. HTR 2010, Prague, Czech Republic*, no. 285, Oct.18-20, 2010.

[2] F. M. Mitenkov, N. G. Kodochigov, A. V. Vasyaev, V. F. Golovko, N. N. Ponomarev-Stepnoi, N. E. Kukharkin, and A. Y. Stolyarevskii, "High-temperature gas-cooled reactors - energy source for industrial production of hydrogen," *Atomic Energy*, vol. 97-6, pp. 432-446, 2004.

[3] F. M. Mitenkov, A. S. Chistov, V. F. Ovchinnikov, M. Y. Nikolaev, E. V. Kiryushina, V. N. Litvinov, E. V. Fadeeva, and O. G. Savikhin, "Electromagnetic suspension in vertical axial wind-driven generators," *Journal of Machinery Manufacture and Reliability*, vol. 44, no. 3, pp. 195-199, 2015.

[4] N. X. Wang, J. G. Zhang, G. and P. Ding, "Influence of magnetic bearing stiffness on rotor in wind turbine generator," *Applied Mechanics and Materials*, vol. 150, pp. 57-62, Jan. 2012.

[5] F. M. Mitenkov, V. F. Ovchinnikov, M. Y. Nikolaev, E. V. Kiryushina, V. N. Litvinov, E. V. Fadeeva, and A. S. Chistov, "The effect of wind speed on dynamics of vertical-axial rotor of wind power unit with electromagnetic suspension," *Problems of Strength and Plasticity*, vol. 78, pp. 5-12, 2016.

[6] I. V. Drumov, N. G. Kodochigov, S. E. Belov, D. S. Znamensky, et al., "Studies of the electro-magnetic suspension system for the GT-MHR turbo machine rotor model," *Proceedings of HTR*,

Prague, Czech Republic, no. 41, pp. 1-7, Oct. 18-20, 2010.

[7] F. M. Mitenkov, V. V. Znyshev, Y. V. Kiryushina, M. Y. Nikolaev, and V. F. Ovchinnikov, "Specific features of the dynamics of a vertical flexible rotor with an electromagnetic suspension," *Journal of Machinery Manufacture and Reliability*, vol. 36, no. 5, pp. 397-399, 2007.

[8] V. Ovchinnikov, M. Nikolaev, and V. Litvinov, "Algorithm for compensation of residual imbalance of a flexible rotor on active magnetic bearings," *Cybernetics and Physics*, vol. 6, no. 2, pp. 76-79, 2017.

[9] V. F. Ovchinnikov, M. Y. Nikolaev, and V. N. Litvinov, "Algorithm for accounting for inner damping in a computer model of dynamics of a flexible rotor on active magnetic bearings," *Applied Computational Electromagnetics Society Journal*, vol. 32, no. 8, pp. 726-730, 2017.

[10] V. F. Ovchinnikov, M. Y. Nikolaev, V. N. Litvinov, N. G. Kodochigov, and I. V. Drumov, "The impact of design deviations on dynamic characteristics of a flexible multispan rotor on complete electro-magnetic suspension," *Thermal Engineering*, vol. 65, no. 9, pp. 589-596, 2018.

[11] S. Timoshenko, D. H. Young, and W. Weaver, Jr., *Vibration Problems in Engineering*. New York, Chichester, Brisbane, Toronto, Singapore, John Wiley & Sons, 1974.

[12] D. V. Balandin and M. M. Kogan, "Controlling the motion of a vertical rigid rotor rotating in electromagnetic bearings," *Izv. Ross. Akad. Nauk. Teor. Sist. Upr.*, no. 5, pp. 3-17. 2011.

[13] G. H. Golub and C. F. Van Loan, *Matrix Computations*. JHU Press, 1996.

[14] ISO 14839-2:2004 Mechanical vibration - Vibration of rotating machinery equipped with active magnetic bearings - Part 2: Evaluation of vibration (IDT).

Viktor Ovchinnikov graduated from Lobachevsky State University of Nizhny Novgorod. Leading Researcher of Research Institute of Mechanics of Lobachevsky State University of Nizhny Novgorod. Ph.D. (Physics and Mathematics). Field of interest: studying and modelling dynamics of flexible-body systems.

Mikhail Nikolaev graduated from Lobachevsky State University of Nizhny Novgorod. Leading Researcher at Research Institute of Mechanics of Lobachevsky State University of Nizhny Novgorod. Ph.D. (Engineering Science). Field of interest: studying and modelling dynamics of complex unique systems.

Vasily Litvinov graduated from Lobachevsky State University of Nizhny Novgorod. Junior Researcher at Research Institute of Mechanics of Lobachevsky State University of Nizhny Novgorod. Field of interest: studying and modelling dynamics of magnetic and mechanical systems.

Denis Kapitanov graduated from Lobachevsky State University of Nizhny Novgorod. Researcher at Research Institute of Mechanics of Lobachevsky State University of Nizhny Novgorod. Ph.D. (Physics and Mathematics). Field of interest: studying and modelling dynamics of elastic systems.