On the Application of Continuity Condition in the Volume-Surface Integral Equation for Composite Closed PEC-Electrical Anisotropy Objects

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Abstract **─** The validity of the use of continuity condition (CC), combined with the volume-surface integral equation (VSIE), is studied when it is explicitly enforced on the closed perfect electric conductor (PEC)-electrical anisotropy interfaces. It is found that if the standard magnetic field integral equation (MFIE) is involved in the VSIE to model the closed PEC surfaces, the solution might be inaccurate, especially when the CC is enforced. The reason for this phenomenon is discussed, and two previously reported approaches are adopted to improve the accuracy of MFIE. Numerical results show that whether the CC is enforced or not, the improvement of the MFIE will result in more accurate VSIE solution.

Index Terms **─** Continuity condition, electrical anisotropy, method of moments (MoM), volume-surface integral equation (VSIE).

I. INTRODUCTION

Electromagnetic (EM) problems involving anisotropic dielectrics and perfect electric conductors (PECs) are of great interest in the field of EM simulation. The development of new materials has created an urgent need for accurate EM solvers for analyzing the EM radiation or scattering properties of composite PEC-complex dielectric objects. Among the numerous numerical calculation methods, the volume-surface integral equation (VSIE) [1], in conjunction of the method of moments (MoM) [2], is one of the most competitive methods to analyze the general composite objects involving both PECs and dielectrics. In addition to its advantages, the VSIE suffers from large number of unknowns since three dimensional discretization of volumetric dielectrics is required to model the dielectrics. However, lots of the composite objects are composed of PECs and dielectrics in any arbitrarily contact. For these kinds of problems, the continuity condition (CC):

$$\hat{n}(\vec{r})\bullet\vec{D}(\vec{r}) = \rho_s(\vec{r}) = -\frac{\nabla\bullet\vec{J}_S(\vec{r})}{j\omega}, \tag{1}$$

that relates the electric flux density and the surface electric current can be explicitly enforced on the PEC-dielectric interfaces to eliminate the associated volume unknowns as well as to reduce the memory usage, and the larger the size of contact surface, the more saving of the memory is expected [1, 3-6]. In (1), $j=\sqrt{-1}$, ω is angular frequency, $\vec{D}$ is the electric flux density in the dielectrics, and $\vec{J}_S$ and ρ_s are the equivalent surface electric current and charge density on the PEC surfaces, respectively. Some previous articles have focus on the use of CC. In [1, 3, 4], how the CC is adopted in the VSIE was discussed. However, whether the PEC surfaces are open or closed was not considered, and the validity was not studied rigorously. In [5], for the higher-order Legendre basis functions with the property of orthogonality, the CC can be explicitly enforced on any PEC-electrical isotropy interfaces. Nevertheless, when the lower-order basis functions are adopted, whether the use of CC is still valid was not discussed. In [6], the validity of the use of CC was investigated. It is stated that if the involved PEC surfaces are open, $\vec{J}_S$ is actually the summation of currents densities residing on both sides [12]. In other words, the single combined current $\vec{J}_S$ only has mathematical significance but no physical meaning. In this case, the explicit enforcement of CC in the VSIE might lead to inaccurate results. Besides, [6] also provides a convenient way to embed it into the context of the multilevel fast multipole algorithm (MLFMA).

Nevertheless, the previous articles focused on the objects involving electrical isotropic dielectrics. As we know, the properties of anisotropic dielectrics are very different from the isotropic ones: for inhomogeneous isotropic dielectrics, the constitutive relation between $\vec{D}$ and the electric field $\vec{E}$ is $\vec{D}(\vec{r})=\varepsilon(\vec{r})\vec{E}(\vec{r})$.The equivalent

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volume electric current is defined as $\vec{J}_V(\vec{r})=[\varepsilon(\vec{r})-\varepsilon_0]\vec{E}(\vec{r})$, where the permittivity $\varepsilon(\vec{r})$ is a scalar value with the free space permittivity ε_0 [1]. On the contrary, for the inhomogeneous anisotropic dielectrics, the constitutive relation is changed into $\vec{D}(\vec{r})=\bar{\bar{\varepsilon}}(\vec{r})\cdot\vec{E}(\vec{r})$ with $\vec{J}_V(\vec{r})=\left[\bar{\bar{\varepsilon}}(\vec{r})-\varepsilon_0\bar{\bar{I}}\right]\cdot\vec{E}(\vec{r})$, while $\bar{\bar{\varepsilon}}(\vec{r})$ is a tensor, and $\bar{\bar{I}}$ is the identity tensor. From the above, although the use of CC in the VSIE for the composite PEC-electrical isotropy objects has been well verified, we still want to know whether the CC is valid when the involved dielectrics are electrical anisotropic. Because (1) conforms to the current continuity equation that is independent of the type of medium, the CC can also be adopted to the PEC-electrical anisotropy interfaces. In addition, for the SIE part of VSIE, the electric field integral equation (EFIE) is commonly adopted since it can be used to model both the open and closed PEC surfaces. For the closed PEC surfaces of composite object, the magnetic field integral equation (MFIE) can be added to the EFIE to form a well-conditioned combined field integral equation (CFIE). But since the standard MFIE is inaccurate to some extent [7-9], the application of MFIE might have a negative effect on the accuracy of VSIE solution, especially when the CC is enforced.

In this paper, the validity of explicit enforcement of CC for the objects containing PEC-electrical anisotropy interfaces is investigated. In addition, when the MFIE is involved in modeling the PEC surfaces, the calculation accuracy of the VSIE with or without enforcing the CC is discussed. Furthermore, both approaches shown in [8, 9] are adopted to improve the accuracy of MFIE, and the numerical results show that this improvement can provide more accurate VSIE solution, especially when the CC is enforced.

II. THEORY AND FOMULATIONS

Consider an arbitrary PEC surface S, wholly or partially covered by electrical anisotropic dielectrics with permittivity tensor $\bar{\bar{\varepsilon}}(\vec{r})$ occupying a region V, as shown in Fig. 1. For the convenience of analysis, it is assumed that this composite object is suspended in free space with permittivity ε_0 and permeability μ_0, and illuminated by a plane EM wave $\left[\vec{E}^i(\vec{r}),\vec{H}^i(\vec{r})\right]$ at an arbitrary angle (θ^i, φ^i).The scattering field $\left[\vec{E}^s(\vec{r}),\vec{H}^s(\vec{r})\right]$ is the superposition of fields produced by the equivalent surface electric current $\vec{J}_S$ on S and the equivalent volume electric current $\vec{J}_V$ in V as:

$$\begin{cases}\vec{E}^s(\vec{r})=\vec{E}^s_S(\vec{r})+\vec{E}^s_V(\vec{r})\\ \vec{H}^s(\vec{r})=\vec{H}^s_S(\vec{r})+\vec{H}^s_V(\vec{r})\end{cases}, \tag{2}$$

with

$$\begin{cases}\vec{E}^s_T(\vec{r})=-j\omega\vec{A}_T(\vec{r})-\nabla\varphi_T(\vec{r})\\ \vec{H}^s_T(\vec{r})=\dfrac{1}{\mu_0}\nabla\times\vec{A}_T(\vec{r})\end{cases} \quad T\in S \ or \ V. \tag{3}$$

The vector potential $\vec{A}_T$ and scalar potential φ_T are expressed as the convolutions of equivalent electric current or its divergence and the Green's function as:

$$\begin{cases}\vec{A}_T(\vec{r})=\mu_0\int_T\vec{J}_T(\vec{r}')\dfrac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}dT'\\ \varphi_T(\vec{r})=\dfrac{j}{\omega\varepsilon_0}\int_T\nabla'\cdot\vec{J}_T(\vec{r}')\dfrac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}dT'\end{cases}. \tag{4}$$

On the PEC surfaces S, the EFIE is formed based on the PEC boundary condition that requires vanishing the tangential component of total electric field as:

$$\hat{n}(\vec{r})\times\vec{E}(\vec{r})=\hat{n}(\vec{r})\times\left[\vec{E}^i(\vec{r})+\vec{E}^s(\vec{r})\right]=0 \quad \vec{r}\in S\,. \tag{5}$$

Furthermore, for the closed PEC surfaces, the MFIE:

$$\vec{J}_S(\vec{r})-\hat{n}(\vec{r})\times\vec{H}^s(\vec{r})=\hat{n}(\vec{r})\times\vec{H}^i(\vec{r}) \quad \vec{r}\in S^+, \tag{6}$$

where $\vec{r}\in S^+$ means that the field point $\vec{r}$ approaches to S from outside, can be added to the EFIE to form the well-conditioned CFIE as:

$$CFIE=\alpha EFIE+(1-\alpha)\eta_0 MFIE\,. \tag{7}$$

In (7), α ($0\leqslant\alpha\leqslant1$) is a real constant, and η_0 is the intrinsic impedance in the free space. Obviously, when α=1 or 0, the CFIE degrades into EFIE or MFIE.

The total electric field in the regions V is a superposition of the incident and scattering electric fields which can be written as the so-called volume integral equation (VIE):

$$\vec{E}(\vec{r})=\left[\bar{\bar{\varepsilon}}(\vec{r})\right]^{-1}\cdot\vec{D}(\vec{r})=\vec{E}^i(\vec{r})+\vec{E}^s(\vec{r}) \quad \vec{r}\in V\,. \tag{8}$$

Thus, (7) and (8) can be combined together to build the CFIE-VIE which is a second-kind VSIE form to solve EM problems of composite objects involving closed PEC surfaces and electrical anisotropic dielectrics.

Using the Galerkin's MoM, the VSIE is converted into a matrix equation. In the implementation of this paper, the lower-order RWG basis function [10] and SWG basis function [11] are used to disperse $\vec{J}_S$ on the PEC surface and $\vec{D}$ in the dielectric region as:

$$\begin{cases}\vec{J}_S(\vec{r})=\sum_{i=1}^{N_S}I_i^S\vec{f}_i^S(\vec{r})\\ \vec{D}(\vec{r})=\dfrac{1}{j\omega}\sum_{i=1}^{N_V}I_i^V\vec{f}_i^V(\vec{r})\end{cases}, \tag{9}$$

respectively. In (9), N_S and N_V are the numbers of the RWG basis functions $\vec{f}_i^S$ and SWG basis functions $\vec{f}_i^V$, while I_i^S and I_i^V are the corresponding unknown expansion coefficients, respectively. Dispersing $\vec{D}$ instead of $\vec{J}_V$ can hold the continuity of normal component which is consistent with the boundary condition on dielectric interfaces [11]. In this case,

$$\begin{aligned}\vec{J}_V(\vec{r})&=\left[\bar{\bar{\varepsilon}}(\vec{r})-\varepsilon_0\bar{\bar{I}}\right]\cdot\left[\bar{\bar{\varepsilon}}(\vec{r})\right]^{-1}\cdot\vec{D}(\vec{r})\\ &=\frac{1}{j\omega}\sum_{i=1}^{N_V}I_i^V\left\{\bar{\bar{I}}-\varepsilon_0\left[\bar{\bar{\varepsilon}}(\vec{r})\right]^{-1}\right\}\cdot\vec{f}_i^V(\vec{r})\end{aligned}. \tag{10}$$

At the exterior boundary of dielectrics, since $\vec{D}$ is not necessarily zero, a "half" SWG basis function associated with only one tetrahedron needs to be defined [11]. However, if an exterior face of the only tetrahedron is terminated by a PEC triangular patch as well as this triangular patch exactly coincides with the exterior face, this "half" SWG function can be removed by using the CC. As mentioned in [3], according to (1) and (9), the coefficient I_p^V associated to the pth "half" SWG basis function $\vec{f}_p^S$ can be directly calculated by:

$$I_p^V = -\frac{\sum_{k\in M_S} I_k^S \nabla\bullet \vec{f}_k^S(\vec{r})}{\hat{n}(\vec{r})\bullet \vec{f}_p^V(\vec{r})}. \tag{11}$$

In (11), M_s is a set of RWG basis functions index defined on the corresponding PEC triangle. That is to say, when the CC is enforced, according to (11), the equivalent volume electric current associated to the PEC-electrical anisotropy interfaces (denoted by $\vec{J}_{VS}$) can be directly calculated by the $\vec{J}_S$ defined on the corresponding PEC surface. It is worth to mention that for the numerator of (11), since $\vec{f}_k^S$ is the RWG basis function defined over triangles, $\nabla\cdot\vec{f}_k^S$ is a constant over the whole triangle area [10]; while for the denominator, $\vec{f}_p^V$ is the SWG basis function defined over tetrahedrons, so $\hat{n}\cdot\vec{f}_p^V$ is also a constant over the tetrahedron surface [11].

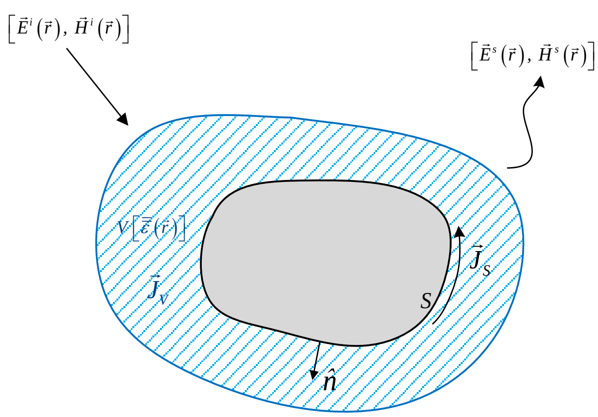

Fig. 1 Composite PEC-electrical anisotropy object under plane EM wave illumination.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we will present the bistatic radar cross section (RCS) results of a PEC sphere coated with homogeneous electrical anisotropic dielectric, while the target residual error in iterative solvers is fixed to 0.001. The Gaussian quadrature rule with 4/5 sampling points is applied to the inner or outer triangle/tetrahedron integrations during calculating the interactions between the testing and basis functions.

The radius of the PEC sphere is 0.5λ (λ is the wavelength in free space), the coating thickness is 0.075λ, and the permittivity tensor of the electrical anisotropy is:

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} 3 & -j & 0 \\ j & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}. \tag{12}$$

After discretization with an appropriate average mesh size, the total number of triangles, tetrahedrons and unknowns are 1,380, 5,955 and 15,565, respectively. The coated PEC sphere is illuminated by a θ-polarized plane wave with the incident angle θ^i=0, φ^i=0, and the observation range is $0\leqslant\theta\leqslant 180°$ and φ=0. Both the CFIE-VIE and that enforced the CC (CC-CFIE-VIE) with different α values are used during the calculation, while the results are shown in Fig. 2. The exact result from Mie series is also given in this figure as a reference. It is seen that the numerical results agree well with the exact result in most angles. However, over the valley range (in this case, 137~143°), the difference of these results is evident, as shown in Fig. 3. For α=1, the results with and without CC are in excellent agreement, and the average difference of the results obtained from the CFIE-VIE and CC-CFIE-VIE over the valley range is about 0.23 dB. However, for α=0.5, the average difference of the results over the valley range is about 0.80 dB. While for α=0, this difference extends to about 0.94 dB. On the other hand, if more sampling points are used over the outer triangular integrations in the MFIE (from 4 to 16) and keep other parameters unchanged, the average differences over the valley range for α=0.5 and α=0 are reduced to 0.31 dB and 0.61 dB, respectively, as shown in Fig. 4. This phenomenon is similar to that stated in [8]: for the type of the SIE part of the VSIE, the EFIE is known to give accurate $\vec{J}_S$ with the use of RWG basis function for the PEC surfaces with arbitrary planar triangulations. On the contrary, for the MFIE, as mentioned in [7], two possible reasons may lead to the inaccuracy: 1) Overlooking the mild logarithmic singularity in the field integration with an insufficiently number of integration points inside the testing triangles; 2) The section of improper solid angle expression for the observation points. Therefore, when MFIE is involved in the CFIE-VIE (i.e., $\alpha\neq 1$), the obtained $\vec{J}_S$ on the PEC surfaces may not be so accurate, leading to inaccurate $\vec{J}_{VS}$. Thus, when $\alpha\neq 0$, the results from CFIE-VIE and CC-CFIE-VIE have a certain difference, and the greater proportion of MFIE occupied (i.e., the closer α is to 1), the larger the difference will be.

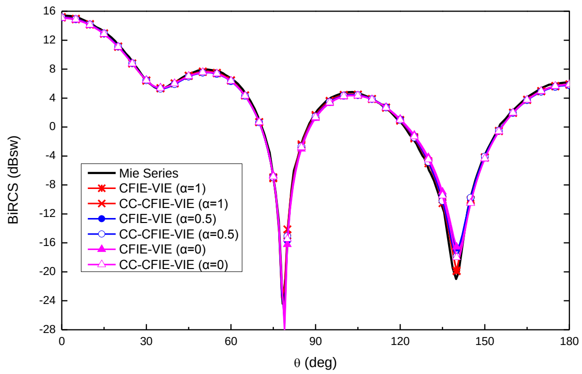

Fig. 2. Bistatic RCS for a PEC sphere of radius 0.5λ coated with 0.075λ thick homogeneous electrical anisotropic dielectric at $\varphi=0°$, illuminated by a θ-polarized plane wave with the incident angle $\theta^i=0$, $\varphi^i=0$.

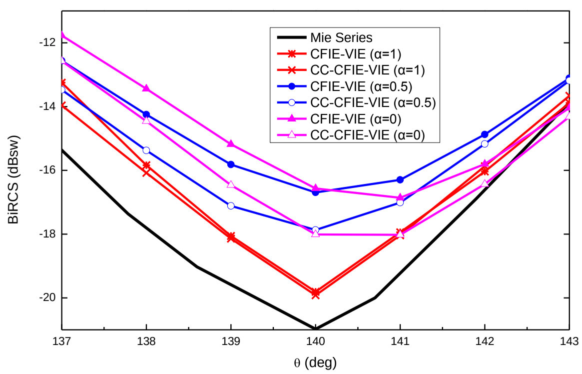

Fig. 3. Enlarged Fig. 2 at $137° \leqslant \theta \leqslant 143°$.

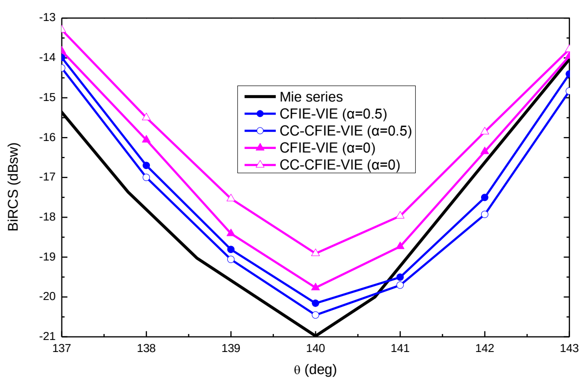

Fig. 4. Bistatic RCS of the coated PEC sphere at $137° \leqslant \theta \leqslant 143°$ and $\varphi=0°$, with 16 sampling points over the outer triangular integrations in the MFIE.

To verify the above conclusion, both of the two approaches stated in [8] and [9] are adopted to modify the standard MFIE (denoted by mMFIE): 1) extracting the logarithmic singularity during the computation of the outer integrals; 2) correcting the external solid angle of the surface at observation points. The detailed approaches can be found in [8, 9]. After the two modifications, when mMFIE is involved in the CFIE-VIE (denoted by mCFIE-VIE), more accurate $\vec{J}_S$ as well as $\vec{J}_{VS}$ are expected, and so do the numerical results. The same coated PEC sphere is calculated while the results are shown in Fig. 5. During the calculation, the Gaussian quadrature rule with 4 sampling points is applied to all the inner/outer triangular integrations. It is observed that compared to Fig. 3, the accuracy of the results have been greatly improved. In addition, the results with and without CC have a better agreement. The average differences of the results obtained from the CFIE-VIE and CC-CFIE-VIE over the valley range for $\alpha=0.5$ and $\alpha=0$ are reduced to 0.29 dB and 0.32 dB, respectively. From this calculation, we may draw the conclusion that accurate evaluating the matrix elements related to the MFIE part can improve the accuracy of CFIE-VIE solution, especially when the CC is enforced.

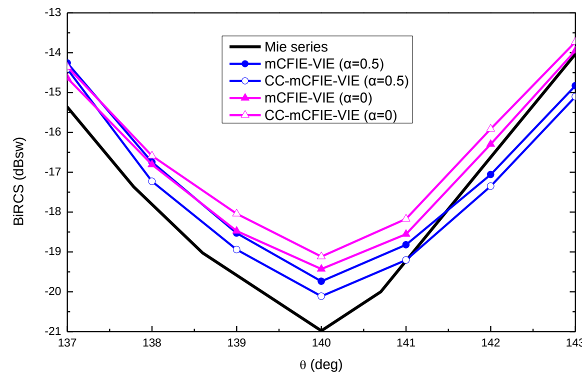

Fig. 5. Bistatic RCS of the coated PEC sphere at $137° \leqslant \theta \leqslant 143°$ and $\varphi=0°$, with the adoption of mMFIE.

IV. CONCLUSION

In this paper, the validity of explicit enforcement of CC for PEC objects coated electrical anisotropic dielectrics is investigated. It is found that possible inaccuracy may arise if the standard MFIE to model closed PEC surfaces is involved in the VSIE used, especially when the CC is enforced on the PEC-electrical anisotropy interfaces. After modifying the standard MFIE using the previously reported approaches, when the CC is enforced, more accurate equivalent surface electric current and the volume electric current associated to the PEC-electrical anisotropy interfaces are obtained.

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REFERENCES

[1] W. C. Chew, J. M. Jin, E. Michielssen, and J. M. Song, *Fast and Efficient Algorithms in Computational Electromagnetics.* MA: Artech House, Boston, 2001.

[2] R. F. Harrington, *Field Computation by Moment Methods*. MacMillan, New York, 1968.

[3] C. C. Lu and W. C. Chew, “A coupled surface-volume integral equation approach for the calculation of electromagnetic scattering from composite metallic and material targets,” *IEEE Trans. Antennas Propag.*, vol. 48, no. 12, pp. 1866-1868, Dec. 2000.

[4] K. Xiao, S .L. Chai, and L. W. Li, “Comparisons of coupled VSIE and noncoupled VSIE formulations,” *Journal of Electromagnetic Waves & Applications*, vol. 25, no. 10, pp. 1341-1351, 2011.

[5] O. S. Kim, P. Meincke, O. Breinbjerg, and E. Jørgensen, “Solution of volume-surface integral equations using higher-order hierarchical Legendre basis functions,” *Radio Science*, vol. 42, no. 4, pp. RS4023, 10.1029/2006RS003584.

[6] M. He, J. Liu, B. Wang, C. Zhang, and H. Sun, “On the use of continuity condition in the fast solution of volume-surface integral equation,” *IEEE Antennas and Wireless Propagation Letters*, vol. 16, pp. 625-628, 2017.

[7] Ö. Ergül and L. Gürel, “Investigation of the inaccuracy of the MFIE discretized with the RWG basis functions,” *Antennas & Propagation Society International Symposium*, vol. 3, pp. 3393-3396, Dec. 2004.

[8] L. Gürel and Ö. Ergül, “Singularity of the magnetic-field integral equation and its extraction,” *IEEE Antennas and Wireless Propagation Letters*, vol. 4, pp. 229-232, 2005.

[9] J. M. Rius, E. Úbeda, and J. Parrón, “On the testing of the magnetic field integral equation with RWG basis functions in method of moments,” *IEEE Trans. Antennas Propag.*, vol. 49, no. 11, pp. 1866-1868, Dec. 2001.

[10] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Trans. Antennas Propag.*, vol. 30, no. 5, pp. 409-418, May 1982.

[11] D. H. Schaubert, D. R. Wilton, and A. W. Glisson, “A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies,” *IEEE Trans. Antennas Propag.*, vol. 32, no. 1, pp. 77-85, Jan. 1984.

[12] E. H. Neuman and M. R. Schrote, “On the current distribution for open surfaces,” *IEEE Trans. Antennas Propag.*, vol. 31, no. 3, pp. 1550-1553, May 1983.