EMC Simulation Based on FDTD Analysis Considering Uncertain Inputs with Arbitrary Probability Density

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Abstract **─** Stochastic Galerkin Method, a prevailing uncertainty analysis method, has been successfully used in today's EMC simulation, in order to consider non-ideality and unpredictability in actual circumstance. In this case, the inputs of the simulation are no longer certain values, but random variables with corresponding probability density distribution. This paper focuses on the arbitrary probability density cases at inputs. Two constructing orthogonal basis methods, the Wiener Haar expansion and the Stieltjes procedure, are generalized into the Stochastic Galerkin Method which is combined with the Finite Difference Time Domain analysis. With the help of the Feature Selective Validation, the quantitative precision comparison of the proposed methods in different cases (the probability density function is continuous or discontinuous) can be presented in detail.

Index Terms **─** Arbitrary probability density, EMC simulation, FDTD analysis, stochastic Galerkin method, uncertainty analysis.

I. INTRODUCTION

Recent years, the Electromagnetic Compatibility (EMC) community usually takes uncertainty factors of input parameters into account, in order to improve the reliability of calculation results. The uncertainty factors may come from the lack of knowledge, manufacturing tolerance and so forth. For example, the uncertainty into material or excitation source is always considered, thanks to the complexity of the electromagnetic environment. Meanwhile, Finite Difference Time Domain (FDTD) analysis is a powerful tool in EMC simulation [1, 2]. In order to analyze this uncertainty in EMC simulation, many uncertainty analysis methods have been generalized into the FDTD analysis in recent studies [3-6].

Among the existing references, the Monte Carlo Method (MCM) is conventionally widely adopted [3, 4]. In MCM, the uncertain inputs are sampled in terms of their distributions. At each sampling point, a certain simulation will be performed. The final uncertainty analysis result should be the statistical characteristic of the results in every point. As confirmed by [3] and [4], MCM has been proved accurate, though its computational efficiency presents quite low [4]. Anyway, thanks to its high accuracy, the results given by the MCM are always treated as the reference data to evaluate the precision of other uncertainty analysis methods in the theoretical research.

Other uncertainty analysis methods have been also successfully used in EMC simulation based on the FDTD analysis, such as Stochastic Collocation Method (SCM) [5], Method of Moments (MOM) [6] and Stochastic Galerkin Method (SGM) [7]. The SCM is based on the multidimensional Lagrange Interpolation theorem, and the interpolation errors might be brought in the results. Thus, the SCM can hardly guarantee high accuracy when the output results are not smoothness enough [5]. In MOM, the first order Taylor series expansion is applied to calculate the expectation and the standard deviation of the outputs. However, the precision of MOM tends to be very poor when the magnitude is large in inputs or outputs [6].

The SGM is rooted in the generalized Polynomial Chaos (gPC) expansion theory, and it has been attached much attention to in recent research thanks to its high accuracy, though the realization of it is quite complex comparing with the SCM and the MOM [7, 8]. In reference [7] and reference [8], the SGM is generalized into the FDTD analysis to solve the stochastic Maxwell's equations, the shielding effectiveness analysis with uncertain materials and 3D sphere scattering calculation with uncertain geometric parameters are presented. It is proved that the accuracy of the SGM is highly consistent with that of the MCM in these calculation examples.

In using the SGM, the uncertain inputs must be

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modeled by the random variables with corresponding probability density distributions. However, in previous studies [7, 8], the distributions of the uncertain inputs are all supposed the common distributions, like the Uniform distributions or the Gaussian distributions. Little attempts in considering the arbitrary probability density distributions have been made, which greatly limits the application of the SGM.

In this paper, the Stieltjes procedure [9, 10] and the Wiener Haar expansion [11] are proposed to construct the orthogonal basis of the SGM when considering the uncertain inputs with arbitrary probability density distributions. A published example in reference [12] is brought in, and the uncertain inputs are assumed the stochastic excitation source with arbitrary probability density distributions. The uncertainty analysis results given by the MCM are regarded as the reference data, in order to evaluate the accuracy of two proposed methods in different cases.

The structure of the paper is as follows: Section II gives a brief introduction of the SGM mechanism combined with FDTD analysis. Stieltjes procedure and Wiener Haar expansion in constructing orthogonal basis of the SGM are presented respectively in Section III. The accuracy comparison by using FSV is demonstrated in Section IV. Section V provides the conclusion part of this paper.

II. THE STOCHASTIC GALERKIN METHOD MECHANISM

In the real electromagnetism environment, lack of knowledge or manufacturing tolerance may cause the uncertainty in material parameters or geometric parameters in EMC simulation model, and they can be called random events. Conventional deterministic FDTD analysis are not capable of dealing with this uncertainty, since that some input parameters are no longer certain values.

The random variables can be used to model the random events, and they are expressed as:

$$\xi(\theta)=\left\{\xi_1(\theta),\ \xi_2(\theta),\ \cdots,\ \xi_M(\theta)\right\}, \tag{1}$$

where θ represents the random events. $\xi_i(\theta)$ is a random variable with its own distribution depending on the random events. $\xi(\theta)$ is the random variable vector, and M is the number of the random variables in the vector.

The one-dimensional Maxwell's equations in discrete version by using Finite Difference Time Domain Method are given as:

$$H_y^{n+1/2}(i+1/2)=H_y^{n-1/2}(i+1/2)+\gamma\left(E_z^n(i+1)-E_z^n(i)\right), \tag{2}$$

and

$$E_z^{n+1}(i)=\alpha E_z^n(i)+\beta\left(H_y^{n+1/2}(i+1/2)-H_y^{n+1/2}(i-1/2)\right). \tag{3}$$

The direction of propagation is along the x axis. H_y stands for the magnetic field intensity in y axis, and E_z is the electric field intensity in z axis. Where n presents the Discrete-time, and i stands for the Discrete-space. α, β and γ are the constant values which are calculated by material parameters, time interval and space interval.

Suppose the position of the excitation source is $i=ks$, equation (3) can be arranged as:

$$\begin{aligned}&E_z^{n+1}(ks)=\alpha E_z^n(ks)+\\&\beta\left(H_y^{n+1/2}(ks+1/2)-H_y^{n+1/2}(ks-1/2)\right).\end{aligned} \tag{4}$$

If the excitation source is uncertain, the random variable vector ξ would be introduced into the Maxwell's equations and (4) can be re-written as:

$$\begin{aligned}&E_z^{n+1}(ks,\xi)=\alpha E_z^n(ks,\xi)+\\&\beta\left(H_y^{n+1/2}(ks+1/2,\xi)-H_y^{n+1/2}(ks-1/2,\xi)\right),\end{aligned} \tag{5}$$

where $E_z^{n+1}(ks,\xi)$ and $E_z^n(ks,\xi)$ are the excitation source, and they are uncertain inputs. Obviously, the output parameters $H_y^{n+1/2}(ks+1/2,\xi)$ and $H_y^{n+1/2}(ks-1/2,\xi)$ would be influenced by the uncertain inputs.

In Stochastic Galerkin Method (SGM), the uncertainty analysis results, namely the uncertain output parameters, should be expressed as the form of the polynomial of the random variables at first. It can be expressed as:

$$\begin{aligned}&H_y^{n+1/2}(ks+1/2,\xi)=h_0(ks+1/2)\varphi_0(\xi)+\\&h_1(ks+1/2)\varphi_1(\xi)+h_2(ks+1/2)\varphi_2(\xi),\end{aligned} \tag{6}$$

and

$$\begin{aligned}&H_y^{n+1/2}(ks-1/2,\xi)=h_0(ks-1/2)\varphi_0(\xi)+\\&h_1(ks-1/2)\varphi_1(\xi)+h_2(ks-1/2)\varphi_2(\xi),\end{aligned} \tag{7}$$

where $\varphi_i(\xi)$ is the Chaos polynomial (or named orthogonal basis), and it is depended on the distribution of the random variable in (1). $h_i(ks+1/2)$ and $h_i(ks-1/2)$ are the coefficients to be determined later. When these coefficients are calculated, equation (6) and equation (7) are the final uncertainty analysis results what we want.

The polynomials given in (6) and (7) are orthogonal to each other, and their relationship can be presented as:

$$\left\langle\varphi_i,\ \varphi_j\right\rangle=\left\langle\varphi_i^2\right\rangle\delta_{ij}, \tag{8}$$

where δ_{ij} represents the Kronecker function and satisfies:

$$\delta_{ij}=\begin{cases}1 & (i=j)\\0 & (i\neq j)\end{cases}. \tag{9}$$

The inner product calculation $\langle .\ ,\ .\rangle$ is defined as:

$$\left\langle\varphi_i,\ \varphi_j\right\rangle=\int\varphi_i(\xi)\varphi_j(\xi)w(\xi)d\xi, \tag{10}$$

where $w(\xi)$ is the weight function which can be obtained by calculating the joint probability density of the random variables in (1).

By substituting equation (6) and (7) into equation (5), it can be rearranged to obtain:

$$\begin{aligned} & E_z^{n+1}(ks,\xi) = \alpha E_z^n(ks,\xi) \\ & +\beta\left(h_0(ks+1/2)-h_0(ks+1/2)\right)\varphi_0(\xi) \\ & +\beta\left(h_1(ks+1/2)-h_1(ks+1/2)\right)\varphi_1(\xi) \\ & +\beta\left(h_2(ks+1/2)-h_2(ks+1/2)\right)\varphi_2(\xi). \end{aligned} \tag{11}$$

The inner product calculation $\langle \varphi_0(\xi)\ ,\ .\rangle$ is performed on the both sides of equation (11), and we can get:

$$\begin{aligned} & \left\langle E_z^{n+1}(ks,\xi),\varphi_0(\xi)\right\rangle = \alpha\left\langle E_z^n(ks,\xi),\varphi_0(\xi)\right\rangle + \\ & \beta\left(h_0(ks+1/2)-h_0(ks+1/2)\right)\left\langle \varphi_0(\xi),\varphi_0(\xi)\right\rangle + \\ & \beta\left(h_1(ks+1/2)-h_1(ks+1/2)\right)\left\langle \varphi_1(\xi),\varphi_0(\xi)\right\rangle + \\ & \beta\left(h_2(ks+1/2)-h_2(ks+1/2)\right)\left\langle \varphi_2(\xi),\varphi_0(\xi)\right\rangle. \end{aligned} \tag{12}$$

According to the relationship in (8), (13) can be translated into:

$$\begin{aligned} & \left\langle E_z^{n+1}(ks,\xi),\varphi_1(\xi)\right\rangle = \alpha\left\langle E_z^n(ks,\xi),\varphi_1(\xi)\right\rangle \\ & +\beta\left(h_1(ks+1/2)-h_1(ks+1/2)\right), \end{aligned} \tag{13}$$

where $\left\langle E_z^{n+1}(ks,\xi),\varphi_0(\xi)\right\rangle$ and $\left\langle E_z^n(ks,\xi),\varphi_0(\xi)\right\rangle$ are the constants that can be calculated by (10):

$$\begin{aligned} & \left\langle E_z^{n+1}(ks,\xi),\varphi_1(\xi)\right\rangle = \alpha\left\langle E_z^n(ks,\xi),\varphi_1(\xi)\right\rangle \\ & +\beta\left(h_1(ks+1/2)-h_1(ks+1/2)\right), \end{aligned} \tag{14}$$

and

$$\begin{aligned} & \left\langle E_z^{n+1}(ks,\xi),\varphi_2(\xi)\right\rangle = \alpha\left\langle E_z^n(ks,\xi),\varphi_2(\xi)\right\rangle \\ & +\beta\left(h_2(ks+1/2)-h_2(ks+1/2)\right). \end{aligned} \tag{15}$$

The process of SGM mechanism is shown from equation (11) to equation (15). Obviously, the uncertain equation (5) is translated into three certain equations, namely equation (13), equation (14) and equation (15). Admittedly, conventional Finite Difference Time Domain Method can be carried out in these three equations, and the coefficients $h_i(ks+1/2)$ and $h_i(ks-1/2)$ in (6) and (7) will be obtained. Sampling the random variables in (6) and (7) in terms of their distributions, the statistical property of the results can be easily got. For example, expectation, variance, the worst case value, the probability density curve and so forth. These statistical properties can stand for the uncertainty analysis outputs.

III. STIELTJES PROCEDURE AND WIENER HAAR EXPANSION

This section shows the schemes in constructing orthogonal basis $\varphi_i(\xi)$ in (6) or (7). It is usually the first step of the SGM. It is worth noting that the random variables in this paper are all in arbitrary probability density form.

A. Stieltjes procedure

The Stieltjes Procedure is firstly applied into the uncertainty analysis of the Computational Fluid Mechanics in reference [9]. For simplicity, only the construction of one-dimensional orthogonal basis is presented, since that the high-dimensional basis can be obtained by performing the tensor products calculation. The three-term recurrence relation in the Stieltjes Procedure is presented as:

$$\begin{aligned} & \varphi_{i+1}(\xi) = (\xi - a_i)\varphi_i(\xi) - b_i\varphi_{i-1}(\xi), \quad i = 0,1,\ldots, \\ & \varphi_0(\xi) = 1, \quad \varphi_{-1}(\xi) = 0, \end{aligned} \tag{16}$$

where a_i and b_i are recurrence coefficients, which can be calculated by:

$$a_i = \frac{\left\langle \xi\varphi_i(\xi),\varphi_i(\xi)\right\rangle}{\left\langle \varphi_i(\xi),\varphi_i(\xi)\right\rangle}, \tag{17}$$

and

$$b_0 = \left\langle \varphi_0(\xi),\varphi_0(\xi)\right\rangle, \quad b_i = \frac{\left\langle \varphi_i(\xi),\varphi_i(\xi)\right\rangle}{\left\langle \phi_{i-1}(\xi),\varphi_{i-1}(\xi)\right\rangle}. \tag{18}$$

The inner product calculation is same as the equation (10). The orthogonal basis in the Stieltjes Procedure is in form of polynomial, and it satisfies the relationship in (8).

If the order of the polynomial is higher, the results will be more accurate, but the simulation time will be longer. Furthermore, it is worth mentioned that the number of high-dimensional orthogonal basis is exponential times of that of one-dimensional orthogonal basis. Thus, although the Stieltjes Procedure can be performed infinitely, the order is usually less than 10.

B. Wiener Haar expansion

The Wiener Haar Expansion is another orthogonal basis constructing method. Similarly, the high-dimensional basis is tensor products of one-dimensional basis, so only one-dimensional case is presented.

Considering the one-dimensional random variable ξ, the relationship between probability density function $pdf(\xi)$ and distribution function $p(\xi)$ are shown as:

$$pdf(\xi) = \begin{cases} \dfrac{dp(\xi)}{d\xi} > 0 & \forall\xi \in (A,B) \\ 0 & \forall\xi \notin (A,B) \end{cases}, \tag{19}$$

where A and B are the boundary of random variable.

The distribution function $p(\xi)$ satisfies the character as:

$$y = p(\xi) \in [0,1] \to \xi = p^{-1}(y) \in [A,B]. \tag{20}$$

It indicates that the distribution function value is one-to-one correspondence with the interval [0, 1].

The Haar wavelet function can implement the orthogonal decomposition in the interval [0, 1]. Using this particular character, the orthogonal basis can be obtained as:

$$X(\xi \in [A,B]) = X_0\chi_{0,0}(p(\xi)) + \sum_{j=0}^{\infty}\sum_{k=0}^{2^j-1} X_{j,k}\psi_{j,k}(p(\xi)), \tag{21}$$

where $X(\xi \in [A,B])$ represents the output parameters under solved, like $H_y^{n+1/2}(ks+1/2,\xi)$ in (5). $\chi_{0,0}(p(\xi))$ is the orthogonal basis which is structured by Haar Father Wavelet, and $\psi_{j,k}(p(\xi))$ is the orthogonal basis given by Haar Mother Wavelet. X_0 and $X_{j,k}$ are coefficients like $h_i(ks+1/2)$ in (6), and they can be calculated by

$$\begin{aligned} X_0 &= \int_0^1 X(\xi)\chi_{0,0}(p(\xi))dp(\xi) \\ &= \int_{[A,B]} X(\xi)pdf(\xi)d\xi, \end{aligned} \tag{22}$$

and

$$\begin{aligned} X_{j,k} &= \int_0^1 X(\xi)\psi_{j,k}(p(\xi))dp(\xi) \\ &= \int_{[A,B]} X(\xi)\psi_{j,k}(p(\xi))pdf(\xi)d\xi. \end{aligned} \tag{23}$$

It is obviously seen that the orthogonal basis of the Wiener Haar Expansion is in the form of Haar Wavelet function. Thus, the convergence rate of the Wiener Haar Expansion must be different from that of the Stieltjes Procedure in different cases. For example, if the PDF function of the random variable is smooth enough, and the Stieltjes Procedure will perform better due to its polynomial decomposition. On the contrary, the Wiener Haar Expansion is more suitable for the PDF function with some transient peaks or some gaps, thanks to the nature of the Haar Wavelet function.

In next section, the performance of Stieltjes Procedure and Wiener Haar Expansion using in SGM is presented, taking the MCM results as the reference data.

IV. ACCURACY COMPARISON BY USING FEATURE SELECTIVE VALIDATION

In this example, a Gaussian pulse with uncertain parameters is introduced into a one dimensional problem space. The example comes from a calculation model in reference [12], and it is electromagnetic wave propagation problem.

Only one dielectric slab is contained in the model, and Fig. 1 represents the geometry construction of this model. The length of the space is 1m, and the location of the slab is from $x=0.1\text{m}$ to $x=0.2\text{m}$. The relative permittivity of the slab is supposed $\varepsilon_r = 4$, and other spaces are supposed full of vacuum. The output result is the absolute value of the frequency response of the electric field component, which is recorded at $x=0.5\text{m}$. A Gaussian pulse is simulated from the point $x=0.7\text{m}$ in the space, and it is a probably 1 V/m uncertain input excitation. The input excitation should be

$$E_z(x=0.7,t) = E_0(\xi)\exp\left(\frac{-(t_0-t)^2}{2\beta^2}\right), \tag{24}$$

where $E_0(\xi)$ is the maximum amplitude of the pulse, and it is an uncertain input parameter which is modeled by the random variable ξ. t_0 stands for the onset time delay of the pulse, t represents the time and β is the width of the pulse at half its maximum height.

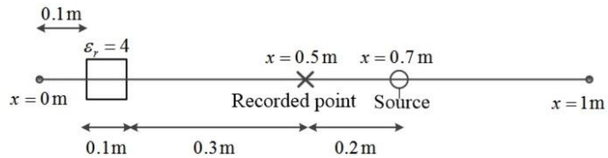

Fig. 1. The model of one dimensional problem space [12].

In this simulation, the Finite Difference Time Domain (FDTD) method is proposed to calculate the outputs. The FDTD cell size is $\Delta x = 0.005\text{m}$, and the FDTD time step is supposed $\Delta t = 8.33\text{ps}$. Meanwhile, $t_0 = 40\Delta t$ and the duration is $\beta = 5\sqrt{2}\Delta t$. This excitation is a broad Gaussian pulse, and it guarantees that the electric field can be calculated over a broad frequency range. The time response of the electric field is recorded from 0 to 41.67ns.

The uncertain input $E_0(\xi)$ satisfies:

$$E_0(\xi) = (0.5+0.5\xi)\times E_m, \tag{25}$$

where E_m is the electric field intensity, $E_m = 1\text{V/m}$. ξ is the random variable with arbitrary probability density, which will be given in different cases in the following texts.

A. Continuous probability density case

In this case, the continuous PDF of the random variable is given as:

$$PDF_c(\xi) = \begin{cases} \frac{1}{2}\sin(\frac{3\pi}{2}\xi) + (1-\frac{1}{3\pi}), & 0 \le \xi \le 1 \\ 0, & \text{others} \end{cases}. \tag{26}$$

We call it continuous thanks to the values are concentrated near only one place, 0.5. Figure 2 presents this continuous PDF curve.

Stochastic Galerkin Method with Stieltjes procedure (SP-SGM), Stochastic Galerkin Method with Wiener Haar expansion (WHE-SGM) and Monte Carlo Method (MCM) are proposed to perform the uncertainty analysis. The results given by MCM are regarded as the reference data to test the precision of the other two methods. 20000 times samplings of MCM have been done in order to make sure that the MCM has reached the convergence.

Figure 3 shows the expectation of uncertainty analysis results, and Fig. 4 gives the standard deviation information.

Feature Selective Validation (FSV) has proved its successful applications in credibility evaluation of CEM results [13, 14]. By using FSV, the difference between the simulation results under evaluated and the reference data can be quantified. Total-GDM, a key value in FSV, reflects the quantitative description of validity evaluation. Total-GDM value is lower, and it means that the simulation results perform better. Table 1 presents a one-to-one correspondence between Total-GDM and the qualitative description. More details about the FSV can be found in [14].

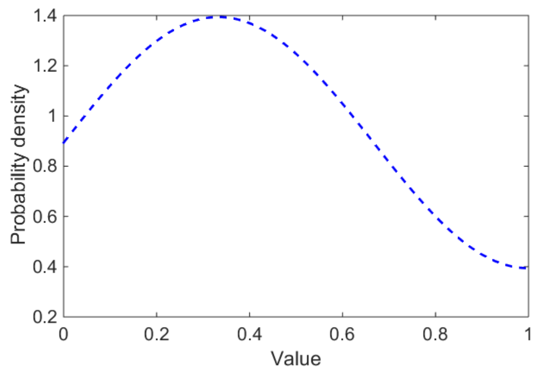

Fig. 2. Continuous PDF curve.

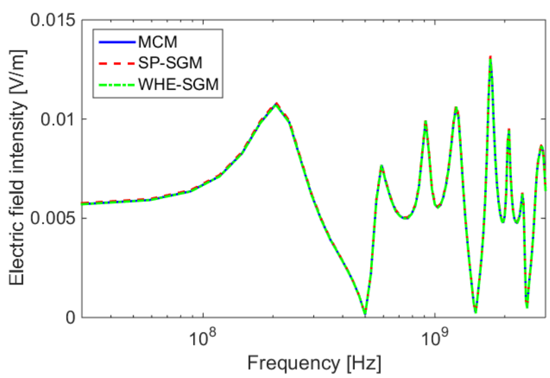

Fig. 3. The expectation results in continuous probability density case.

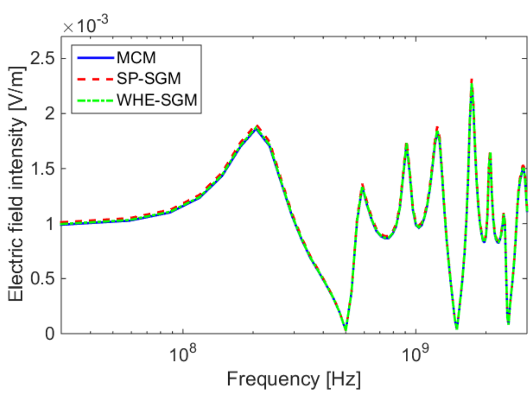

Fig. 4. The standard deviation results in continuous probability density case.

The Total-GDM results of SP-SGM and WHE-SGM in Fig. 3 and Fig. 4 are given in Table 2. It is shown that all the values in Table 2 are less than 0.1. Thus, it indicates that all the results are "Excellent" match with the results given by MCM according to Table 1. Furthermore, it indicates the conclusion that both SP-SGM and WHE-SGM can be as good as the MCM in this continuous probability density case.

Table 1: Relationship between Total-GDM and Quantitative Description [14]

Total-GDM (Quantitative)	**FSV Interpretation (Qualitative)**
Less than 0.1	Excellent
Between 0.1 and 0.2	Very Good
Between 0.2 and 0.4	Good
Between 0.4 and 0.8	Fair
Between 0.8 and 1.6	Poor
Greater than 1.6	Very Poor

Table 2: The Total-GDM values in continuous probability density case

	SP-SGM	**WHE-SGM**
Expectation	3.32×10^{-3}	6.47×10^{-4}
Standard deviation	0.026	0.027

The simulation time of the MCM is 1.35 hours, SP-SGM takes 1.14 minutes, and WHE-SGM uses 2.32 minutes. Thus, it is proved that the computational efficiencies of SP-SGM and WHE-SGM are much higher than that of MCM. The reason is that MCM needs thousands of times of the FDTD analysis, in order to make sure the calculation is converged. In this example, 2000 times are used. SP-SGM or WHE-SGM only needs one augmented FDTD analysis, so the simulation time is the several times of one common FDTD analysis. Thus, SP-SGM and WHE-SGM are in high computational efficiency.

Furthermore, the computational efficiency of SP-SGM is a little better than WHE-SGM. The reason is that SP-SGM needs less chaotic polynomials than WHE-SGM. Thus, the augmented FDTD analysis of the SP-SGM is easier than that of the WHE-SGM.

B. Discontinuous probability density case

The discontinuous PDF of the random variable should be:

$$PDF_{unc}(\xi)=\begin{cases}4, & 1\le\xi\le1.2\\ 0.075\times[-2(\xi-17)^2+8(\xi-17)-6], & 18\le\xi\le20.\\ 0, & \text{others}\end{cases}\quad(27)$$

In contrast to the continuous case in (26), the values are dispersed in two places. One is near the value 1.1, and the other is near the value 19. Figure 5 shows the discontinuous PDF curve. In some cases, if a material parameter is uncertain due to the lack of knowledge and it has two kinds of inherent discrete states, the random variable can be presented by such discontinuous PDF since that we cannot judge which state it should be. Thus, the discontinuous PDF form is also frequently appeared in uncertainty analysis.

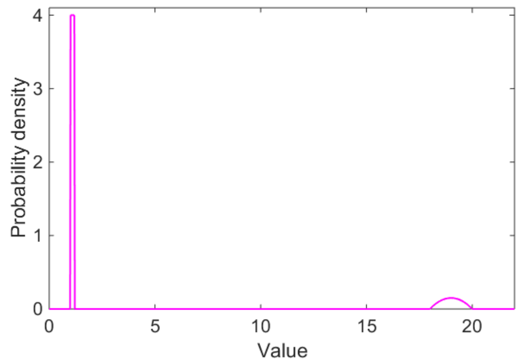

Fig. 5. The discontinuous PDF curve.

In the similar way, MCM, SP-SGM and WHE-SGM are undertaken for the uncertainty analysis. Fig. 6 and Fig. 7 give the expectation results and the standard deviation results in discontinuous probability density case.

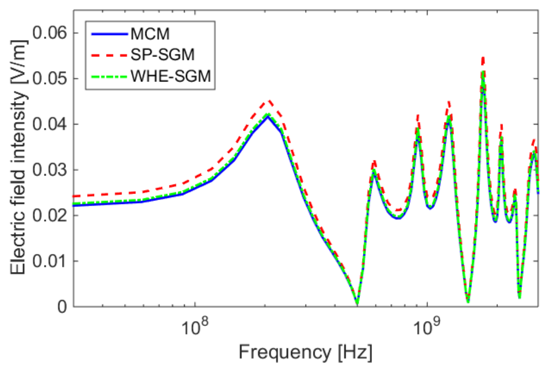

Fig. 6. The expectation results in discontinuous probability density case.

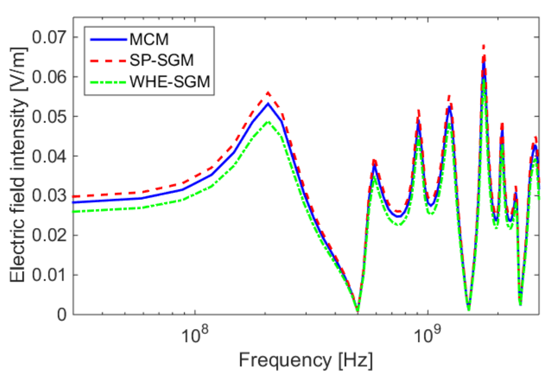

Fig. 7. The standard deviation results in discontinuous probability density case.

The Total-GDM results in Fig. 6 and Fig. 7 are presented in Table 3. As to the expectation results, the SP-SGM receives a “Very good” evaluation and the WHE-SGM has an “Excellent” result. It indicates that the WHE-SGM does better than the SP-SGM in expectation results in discontinuous probability density case. Considering the standard deviation results, the SP-SGM is an “Excellent” match with the MCM. On the contrary, the WHE-SGM only presents a “Very good” match. Consequently, the SP-SGM does better in standard deviation calculation.

It is worth noting that the expectation is the basic results of the uncertainty analysis results. Admittedly, the importance of the expectation results outweighs that of the standard deviation results. As a whole, the WHE-SGM performs better in discontinuous probability density case.

No matter SP-SGM or WHE-SGM, the simulation results in the discontinuous case are not good enough as that in the continuous case. The reason is that the discontinuous PDF leads the more complex uncertainty analysis outputs, so the truncation of the chaotic polynomials may cause a little bigger error.

Table 3: The Total-GDM values in discontinuous probability density case

	SP-SGM	**WHE-SGM**
Expectation	0.122	0.027
Standard deviation	0.069	0.112

The simulation time of the MCM is 1.59 hours, SP-SGM takes 1.47 minutes, and WHE-SGM uses 2.89 minutes. Thus, it would give the same conclusion with the continuous case in computational efficiency.

V. CONCLUSION

In this paper, two constructing orthogonal basis methods, Stieltjes procedure and Wiener Haar expansion, are applied into the Stochastic Galerkin Method (SGM) in order to perform uncertainty analysis with the arbitrary probability density inputs in Electromagnetic Compatibility simulation based on the Finite Difference Time Domain analysis. By simulating an electromagnetic wave propagation example with stochastic excitation source included, the following conclusions can be obtained according to the Feature Selective Validation results.

Firstly, when Probability Density Function is continuous, both Stochastic Galerkin Method with Stieltjes procedure (SP-SGM) and Stochastic Galerkin Method with Wiener Haar expansion (WHE-SGM) can provide the accurate results like the Monte Carlo Method (MCM). But the computational efficiency of the SP-SGM is a little better than that of the WHE-SGM.

Secondly, when Probability Density Function is discontinuous, the WHE-SGM does better in calculating the expectation of the uncertainty analysis results, and the SP-SGM can give more accurate in the variance information. To sum up, the WHE-SGM performs better than the SP-SGM in this case.

Finally, it also indicates that the computational efficiencies of both SP-SGM and WHE-SGM are much better than that of MCM.

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