Uncertainty Analysis of the EMC Simulation Based on the Non-Intrusive Galerkin Method

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Abstract **─** Recently, as a high-efficient uncertainty analysis method, the Stochastic Galerkin Method has been widely applied in EMC simulations. In this method, the original solver must be changed during uncertainty analysis. Thus, the realization of the Stochastic Galerkin Method may become impossible in some cases. In this paper, a novel method named Non-Intrusive Galerkin method is proposed in order to sove this problem. The performance of the proposed method can be clearly shown by calculating a published example.

Index Terms **─** Non-Intrusive Galerkin Method, Stochastic Galerkin Method, Uncertainty Analysis, EMC simulation.

I. INTRODUCTION

In actual engineering environment, uncertainties exist extensively as the random changes of the geometry condition or operating parameters. In order to model such situation precisely, stochastic modeling techniques have been widely introduced into the Electromagnetic Compatibility (EMC) simulation [1]. In this case, the inputs of the EMC models are uncertain parameters, and many uncertainty analysis methods have been introduced to deal with the variability in model inputs.

The Monte Carlo Method (MCM) is the most widely used uncertainty analysis method, and it has been testified accurate in the EMC simulation [2, 3]. However, low computational efficiency makes the MCM uncompetitive. The Method of Moments [4] and the Perturbation Method [5] are another two uncertainty analysis methods, and both of them cannot achieve high accuracy.

In recent years, the Stochastic Galerkin Method (SGM), which is based on the generalized Polynomial Chaos (gPC) expansion theory [6, 7], has caught some researchers' attention in EMC field. In the references [8, 9], the SGM is presented to solve the stochastic Transmission Line Model. The crosstalk calculation of the random cables is presented in [8], and the field line coupling affection simulation with uncertain parameters in field is given in [9]. In the references [1, 10], the SGM is introduced into the finite-difference time-domain (FDTD) method to solve the stochastic Maxwell's equations. In the examples of the recent research, the SGM shows good accuracy and high efficiency [6-10].

However, during the uncertainty analysis of the SGM, the original solver must be changed. There is no doubt that it will be difficult to realize the SGM when the solver becomes complex. Especially in some cases, the solver can't be changed like the EMC software, and the using of the SGM will become impractical. This paper presents a novel method named Non-Intrusive Galerkin Method (NIGM), which improves the SGM by using the Numerical integration [11]. After calculating a published example in reference [12], the performance of the proposed method can be shown obviously by the use of the Feature Selective Validation [13].

The structure of the paper is as follows. Section II employs a brief description of the Stochastic Galerkin Method; the Non-Intrusive Galerkin Method can be seen in Section III; algorithm validation is presented in Section IV; Section V provides a summary of this paper.

II. THE STOCHASTIC GALERKIN METHOD

In the traditional EMC simulation, all the input parameters are supposed certain. However, in some cases, some input parameters need to be regarded as uncertain parameters, in order to improve the reliability of the EMC simulation results. If the inputs of the models are uncertain, the output parameters will be no longer deterministic too. And how to obtain such outputs is what the uncertainty analysis methods do.

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The Maxwell's Equations in 1D are taken for example, as (1) and (2) shown. It is the wave propagating in a linear isotropic homogeneous material along the z-axis,

$$-\frac{\partial E_x(z,t)}{\partial z} = \mu \frac{\partial H_y(z,t)}{\partial t}, \tag{1}$$

$$-\frac{\partial H_y(z,t)}{\partial z} = \varepsilon \frac{\partial E_x(z,t)}{\partial t} + \sigma E_x(z,t), \tag{2}$$

where, E_x (z, t) represents the electric field intensity orienting in the x direction, at a position z and time t. Similarly, H_y (z, t) is the magnetic field orienting in the y direction. The symbols μ, ε, and σ stand for the permeability, permittivity and conductivity of the medium in which the electromagnetic fields propagate.

It is obviously that E_x (z, t) and H_y (z, t) are output parameters which we are interested in. The material parameters, like μ and ε, are the input parameters.

Suppose that the input parameters μ and ε are uncertain because of the lack of the knowledge. Thus, it is suitable to use the Uniform distribution parameters or the Gaussian distribution parameters to replace input parameters, rather than use a certain estimated value. In this situation, the lack of the knowledge is called a random event θ. Several random variables can be used to model the random event θ as (3) shown,

$$\xi(\theta) = \{\xi_1(\theta),\ \xi_2(\theta),\ \cdots\ ,\ \xi_n(\theta)\}, \tag{3}$$

where, $\xi(\theta)$ is the random variable space which is made up by the random variables. $\xi_i(\theta)$ is the random variable, and every variable has its own distribution.

After modeling by the random variables, the stochastic Maxwell's equations are obtained like (4) and (5):

$$-\frac{\partial E_x(z,t,\xi)}{\partial z} = \mu(\xi) \frac{\partial H_y(z,t,\xi)}{\partial t}, \tag{4}$$

$$-\frac{\partial H_y(z,t,\xi)}{\partial z} = \varepsilon(\xi) \frac{\partial E_x(z,t,\xi)}{\partial t} + \sigma E_x(z,t,\xi). \tag{5}$$

The parameters $\mu(\xi)$ and $\varepsilon(\xi)$ are uncertain inputs. Thus, the output parameters H_y (z, t, ξ) and E_x (z, t, ξ) will be influenced by the uncertainty of the inputs.

According to the gPC theory, firstly, the output parameters are expressed in the form of polynomial as (6) shown,

$$H_y(z,t,\xi) = h_0\varphi_0(\xi) + h_1\varphi_1(\xi) + h_2\varphi_2(\xi), \tag{6}$$

where, $\varphi_i(\xi)$ is the Chaos polynomial which is determined by the Askey rule, shown in Table 1, and it is in the form of the polynomial of the random variables. More details about the Askey rule can be seen in reference [6, 7]. The coefficient h_i is under calculated. In a word, the solving process of the gPC theory is a kind of Undetermined Coefficients method.

The polynomials provided by the Askey rule satisfy the perpendicularity to each other like (7) and (8):

$$\langle \varphi_i,\ \varphi_j \rangle = \langle \varphi_i^2 \rangle \delta_{ij}, \tag{7}$$

$$\delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases}. \tag{8}$$

The inner product computation in (7) can be seen in (9):

$$\langle \varphi_i,\ \varphi_j \rangle = \int \varphi_i(\xi)\varphi_j(\xi) w(\xi) d\xi, \tag{9}$$

where, $w(\xi)$ is the weight function, which can be obtained by calculating the joint probability density of the random variables.

Table 1: The Askey rule

Random Variables	**Wiener-Askey Chaos**	**Support**
Gaussian	Hermite-chaos	$(-\infty,+\infty)$
Gamma	Laguerre-chaos	$[0,+\infty)$
Beta	Jacobi-chaos	$[a,b]$
Uniform	Legendre-chaos	$[a,b]$

The Galerkin projection equations by using the SGM can be given by (10) and (11):

$$-\frac{\partial}{\partial z}\begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} M_{\mu 0,0} & M_{\mu 1,0} & M_{\mu 2,0} \\ M_{\mu 0,1} & M_{\mu 1,1} & M_{\mu 2,1} \\ M_{\mu 0,2} & M_{\mu 1,2} & M_{\mu 2,2} \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix}, \tag{10}$$

$$-\frac{\partial}{\partial z}\begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} M_{\xi 0,0} & M_{\xi 1,0} & M_{\xi 2,0} \\ M_{\xi 0,1} & M_{\xi 1,1} & M_{\xi 2,1} \\ M_{\xi 0,2} & M_{\xi 1,2} & M_{\xi 2,2} \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix} + \sigma \begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix}, \tag{11}$$

where, e_i and h_i are the coefficients of the Chaos polynomial, which are under calculated. And they are the abbreviation of $e_i(z,\ t)$ and $h_i(z,\ t)$. $M_{\mu i,j}$ means the product computation result of $\langle \mu(\xi)\varphi_i(\xi),\ \varphi_j(\xi) \rangle$. Similarly, $M_{\varepsilon i,j}$ means the product computation result of $\langle \varepsilon(\xi)\varphi_i(\xi),\ \varphi_j(\xi) \rangle$.

Obviously, the original solver is changed due to the Galerkin projection in SGM process. And after calculating the coefficients of (6), the uncertainty analysis results can be easily acquired by sampling in terms of the distributions of the random variables in (3). More details about the SGM can be seen in the references [6-10].

Due to the Galerkin Process, the solver must be changed during the SGM. This character severely limits the application of the SGM. In next section, another coefficient calculating method is given, and the original solver would not be changed in this method.

III. THE NON-INTRUSIVE GALERKIN METHOD

In the NIGM, numerical integration is introduced to improve the SGM.

The inner product computation with $\varphi_0(\xi)$ is carried out in both sides of (6), and (12) is obtained,

$$\begin{aligned}\langle \varphi_0(\xi),\ H_y(t,z,\xi)\rangle &= h_0\langle \varphi_0(\xi),\varphi_0(\xi)\rangle \\ &+ h_1\langle \varphi_0(\xi),\varphi_1(\xi)\rangle + h_2\langle \varphi_0(\xi),\varphi_2(\xi)\rangle.\end{aligned} \tag{12}$$

Using the orthogonal property of the Chaos polynomial in (7), $\langle \varphi_0(\xi),\varphi_0(\xi)\rangle = 1$, $\langle \varphi_0(\xi),\varphi_1(\xi)\rangle = 0$, and $\langle \varphi_0(\xi),\varphi_2(\xi)\rangle = 0$ can be gotten. Then (13) is obtained:

$$e_0 = \langle \varphi_0(\xi),\ H_y(t,z,\xi)\rangle. \tag{13}$$

In a similar way, the coefficients calculation can be replaced by the integration calculations, shown as (14):

$$e_k = \int_{a(\xi)}^{b(\xi)} \varphi_k(\xi)\ H_y(t,z,\xi) w(\xi) d\xi\ , \tag{14}$$

where, $a(\xi)$ and $b(\xi)$ are the lower bound and the upper bound of the integration calculation.

In the NIGM, Numerical integration is provided to calculate the integration in order to remove the random variables. Numerical integration is an approximate numerical method, which is used to calculate the integration when the integrand function is in a complex form. By this way, the coefficients calculation turns to be (15):

$$e_k = \sum_{i=1}^{n+1} A_i \varphi_k(NI_i) H_y(t,z,NI_i), \tag{15}$$

where, the integration points NI_i satisfy the character $a(\xi) \le NI_1 < NI_2 < \cdots < NI_{n+1} \le b(\xi)$, and A_i stands for the integration weight of the Numerical integration. The integration points are chosen according to the Gaussian Quadrature formula [11], and the total number of the integration points is supposed $n + 1$. $H_y(t, z, NI_i)$ presents making the certain EMC simulation result in point NI_i. Thus, the uncertainty ξ in input is replaced by the certain value NI_i. In another word, the random variable disappears. The value of $\varphi_0(NI_i)$ can be easily calculated by substituting NI_i into it.

Due to the numerical integration, the uncertain EMC simulation is replaced by several certain EMC simulations in numerical integration points NI_i. Thus, a steady EMC solver is enough for the NIGM, and the solver can be regarded as a 'black box'. There is no doubt that the realization of the NIGM is much easier than the SGM, and the NIGM can be introduced into the EMC software to make the uncertainty analysis.

IV. ALGORITHM VALIDATION

In this section, two typical examples are simulated in order to present the performance of the NIGM. Because of the high accuracy of the MCM, the uncertainty analysis results given by the MCM will be regarded as the standard data.

The first example is one-dimension wave propagation example published in reference [12], as shown in Fig. 1. The certain EMC simulation method FDTD is used to solve the Maxwell's equations.

The space step of the FDTD is $1.5\times10^{-2} m$ and the time step is $5.0\times10^{-11} s$. The number of discrete points in the electric field intensity is 151, and it is 150 in the magnetic field intensity. The sine excitation source is in the first discrete point with the amplitude $2.7\times10^{-3} V/m$ and the frequency $1.0\times10^{9} Hz$. The total number of the time steps is 100.

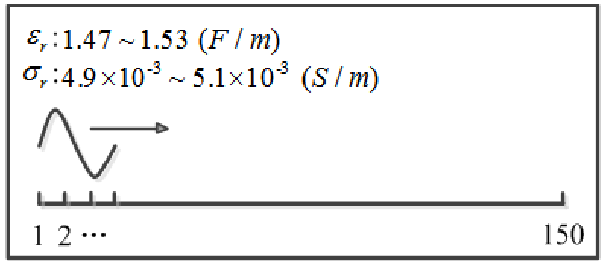

Fig. 1. One-dimension wave propagation model with uncertain inputs in medium parameters.

The dielectric coefficient ε_r and the conductivity σ_r are supposed to be the uncertain parameters, and they are both in the Uniform distribution. The dielectric coefficient obeys $U[1.47,\ 1.53](F/m)$, and the conductivity is $U[4.9\times10^{-3},\ 5.1\times10^{-3}](S/m)$.

Such random event in the inputs can be modeled by two random variables ξ_1 and ξ_2 like (16) and (17). Both of the variables are in the Uniform distribution with the bound [-1, 1]:

$$\varepsilon_r = \varepsilon_r^* \times (1 + 0.02\times\xi_2), \tag{16}$$

$$\sigma_r = \sigma_r^* \times (1 + 0.02\times\xi_1), \tag{17}$$

where, the mean values of the input parameters are $\varepsilon_r^* = 1.5\ (F/m)$ and $\sigma_r^* = 5\times10^{-3}\ (S/m)$.

For the SGM, six terms of the Chaos polynomial are given in (18). It is the tensor product of one-dimension Chaos polynomial, more details can be found in [9]:

$$\begin{aligned}
\varphi_0(\xi) &= 1 \\
\varphi_1(\xi) &= \sqrt{3}\times\xi_1 \\
\varphi_2(\xi) &= \sqrt{3}\times\xi_2 \\
\varphi_3(\xi) &= \frac{\sqrt{5}}{2}\times(3\times\xi_1^{\,2} - 1) \\
\varphi_4(\xi) &= 3\times\xi_1\times\xi_2 \\
\varphi_5(\xi) &= \frac{\sqrt{5}}{2}\times(3\times\xi_2^{\,2} - 1)
\end{aligned} \tag{18}$$

According to [11], the numerical integration points in the NIGM are given by the Table 2, and the integration weight A_i is also presented.

The Probability Density Function (PDF) curve of the electric field intensity at the 100^{th} discrete point simulation results are shown as Fig. 2.

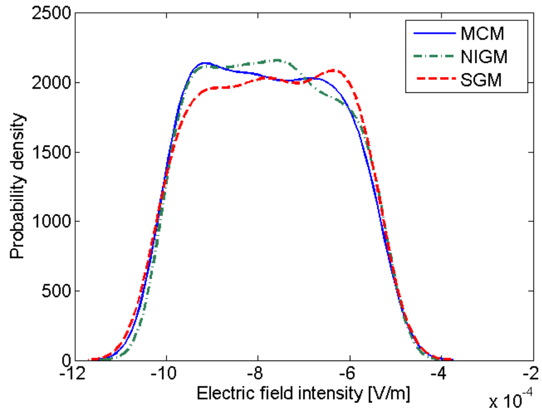

Fig. 2. The PDF of electric field intensity at the 100th discrete point.

Table 2: The numerical integration points (ξ_1, ξ_2)

Number	**1**	**2**	**3**
Point	$\left(-\frac{\sqrt{15}}{5},-\frac{\sqrt{15}}{5}\right)$	$\left(-\frac{\sqrt{15}}{5},\ 0\right)$	$\left(-\frac{\sqrt{15}}{5},\frac{\sqrt{15}}{5}\right)$
Weight	$\frac{25}{324}$	$\frac{40}{324}$	$\frac{25}{324}$
Number	**4**	**5**	**6**
Point	$\left(0,-\frac{\sqrt{15}}{5}\right)$	$(0,0)$	$\left(0,\frac{\sqrt{15}}{5}\right)$
Weight	$\frac{40}{324}$	$\frac{64}{324}$	$\frac{40}{324}$
Number	**7**	**8**	**9**
Point	$\left(\frac{\sqrt{15}}{5},-\frac{\sqrt{15}}{5}\right)$	$\left(\frac{\sqrt{15}}{5},0\right)$	$\left(\frac{\sqrt{15}}{5},\frac{\sqrt{15}}{5}\right)$
Weight	$\frac{25}{324}$	$\frac{40}{324}$	$\frac{25}{324}$

The results of the MCM are treated as the standard data, and 10,000 times of certain EMC simulations were done to make sure the convergence of the MCM.

Furthermore, the Fig. 3 shows the expectation value of all 200 discrete points, and the Fig. 4 presents the variance information. The expectation value means the most possible value, and the variance information presents the magnitude of the uncertainty.

Using the Feature Selective Validation, the Total Global Difference Measure (Total-GDM) values are shown in Table 3.

According to the qualitative rule in the Feature Selective Validation, if the Total-GDM value is less than 0.1, it means that the simulation results are in the 'Excellent' level. Thus, all the four values in Table 3 are in 'Excellent' level, and it is proved that the accuracy of the NIGM and the SGM in this example is the same as the MCM. It is worth noting that though the Total-GDM value of the SGM is less than that of the NIGM, the accuracy of the SGM and the NIGM is still in the same level. That is the opinion of the Feature Selective Validation [13].

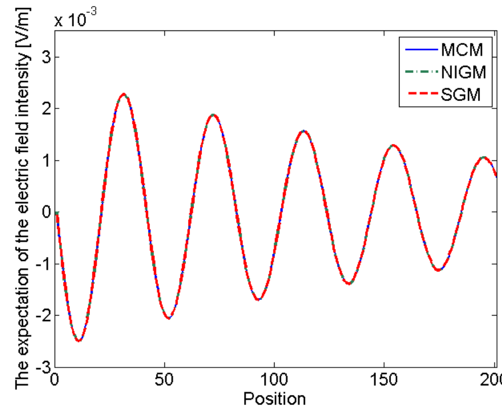

Fig. 3. The expectation values of all 200 discrete points.

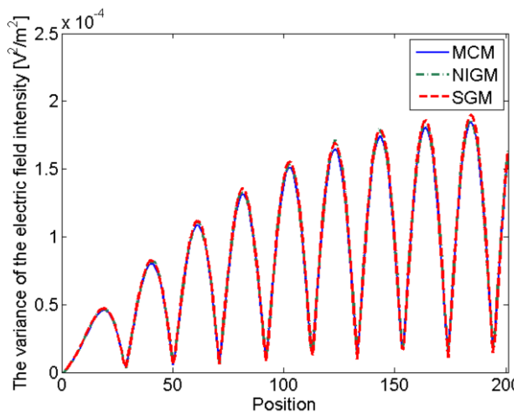

Fig. 4. The variance values of all 200 discrete points.

Table 3: The Total-GDM value of the results

Results	**SGM**	**NIGM**
Expectation	0.006	0.01
Variance	0.037	0.05

The simulation time of the MCM is 63.2 *s*, and the SGM takes 1.12 *s*. The NIGM wastes 0.65 *s*. It is proved that the computational efficiency of the SGM and the NIGM are in the same level, and much better than the MCM.

The second example is the shielding effectiveness calculation of a Perfect Electric Conductor (PEC) box with the random hole. In actual situation, the position of the hole and the size of the hole might be uncertain because of the existence of the manufacturing tolerance. Thus, the inputs of simulation model must be random in order to model the situation better.

In this example, the EMC software, CST Studio Suite, is applied to calculate the shielding effectiveness. As the solver of the software is not open source, the SGM cannot be used in this case. Thus, only the results of the

NIGM and the MCM are presented.

The material of the box is supposed the PEC, other space is supposed vacuum. The size of the solution space is $0.2m\times0.2m\times0.2m$. If the step of is supposed $2\times10^{-3}m$, the space can be described like $100\times100\times100$. The box is in the shape of cube located in the middle of the solution space, the length of side is $0.12m$. The wall thickness is $6\times10^{-3}m$, 3 times of space step.

The shielding effectiveness calculation at 100 MHz is obtained by the NIGM and the MCM. The position of the excitation source is at the point (6, 64, 64) of the space. And the excitation source is sinusoidal electric field source at single frequency 100 MHz. The shielding effectiveness reference point is in the middle of the solution space, that is the point (50, 50, 50). The model is given in Fig. 5.

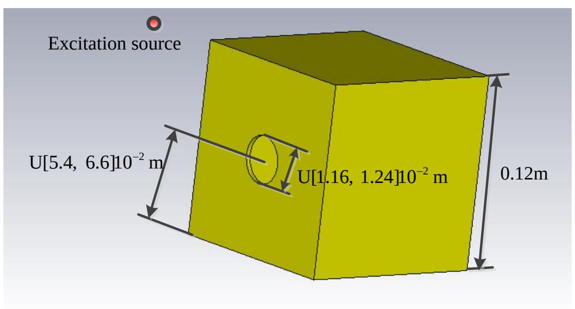

Fig. 5. The model of the PEC box with an uncertain hole on the surface.

The random variables for modeling the random event are similar as (16) and (17) shown. Thus, the numerical integration points and the weight are same as the Table 2 shown. Figure 6 gives shielding effectiveness results of the NIGM and the MCM.

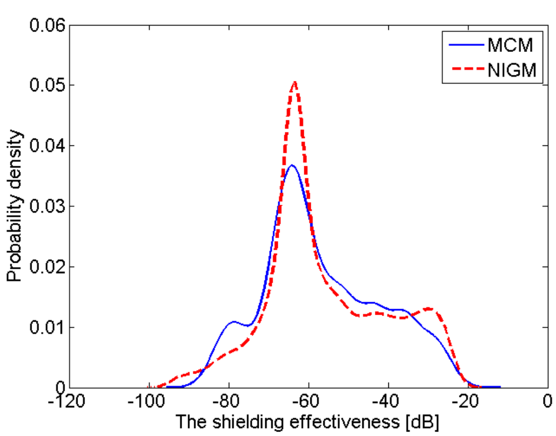

Fig. 6. The shielding effectiveness results given by the NIGM and the MCM.

In this example, the result is in the form of PDF. The PDF curve has the character that the area surrounded by the curve and the horizontal axis is 1. That means that the PDF curve will be 'wane and wax', unlike the results in Fig. 3 and Fig. 4. Thus, judging the accuracy of the simulation result by means of comparing the difference of two PDF curves is not reasonable. Furthermore, the Feature Selective Validation is not suitable at the same time.

The mean value and the variance comparison of the NIGM and the MCM are proposed. The mean value of shielding effectiveness values calculated by the MCM is -56.7 dB, and the variance information of the MCM is 14.6 dB^2. Meanwhile, the mean value of the results given by the NIGM is -55.8 dB, and the variance information of the NIGM is 15.3 dB^2. The error in mean value is 1.6%, and that in variance is 4.8%.

It is demonstrated that the NIGM is as accurate as the MCM in the second example.

The certain simulation times of the MCM are 5,000, and these of the NIGM are only 9. It is seen that the computational efficiency of the NIGM is much higher than the MCM.

In short, the NIGM is as accurate as the MCM and the SGM. Like the SGM, the NIGM owns high computational efficiency, and much better than the MCM. Like the MCM, the NIGM can realize the uncertainty analysis without changing the original solver. Thus, the application scope of the NIGM is much wider than the SGM.

V. CONCLUSION

This paper proposed a novel method named the Non-Intrusive Galerkin Method, aiming at realizing the uncertainty analysis in EMC simulations without changing the original solver. By using the Feature Selective Validation, it is clearly demonstrated that the proposed method is as accurate as the Stochastic Galerkin Method in a published example. Furthermore, it is proved that the proposed method can be generalized into the EMC software like the Monte Carlo method, but Stochastic Galerkin Method can't. It means that the proposed method is much better than the Stochastic Galerkin Method in the scope of application.

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