Boundary Element Computation of Line Parameters of On-chip Interconnects on Lossy Silicon Substrate

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Abstract **─** A BEM formulation is applied to the extraction of series parameters of interconnects on lossy silicon substrate. The numerical formulation can take into account both a semi-infinite homogeneous conductive substrate and a homogeneous conductive substrate of finite thickness with backside metallization and needs the discretization of only the contours of the traces.

Index Terms **─** Boundary element method, per-unit-length parameters, transmission lines, interconnects, silicon substrate.

I. INTRODUCTION

The broad-band transmission line behavior of interconnects on a lossy silicon substrate has been studied extensively by full-wave electromagnetic analysis [1-3] and, more recently, by quasistatic electromagnetic approaches [4-6]. The former, generally, is too time-consuming to be viable. In order to apply the latter, an accurate knowledge of the frequency-dependent per-unit-length (p.u.l.) parameters is needed and as far as the series parameters are concerned, different approaches have been proposed.

In [4], a general quasi-magnetostatic solver based on an integral formulation is adopted to compute the frequency-dependent distributed resistance and inductance parameters.

Alternatively, closed-form expressions for the series impedance parameters were derived using a complex image approach to approximate the effects of the complicated eddy-current loss mechanism in the silicon substrate [7, 8]. In the complex image approach, image conductors are placed at a frequency-dependent complex depth below the interconnects.

In the present paper, the same boundary integral formulation proposed in [9] for a different application is validated for the extraction of the series parameters of interconnects on a lossy silicon substrate, largely extending in this way the range of applications covered by this integral formulation. The adopted Green functions are those derived in [7, 8] and can take into account a semi-infinite homogeneous conductive substrate or a homogeneous conductive substrate of finite thickness with backside metallization. For the considered application, the proposed integral formulation has many advantages: compared to FEM or other volume approaches like that in [4], it does not need to discretize the substrate or to define a large enough computational domain and it only needs to discretize the surface of the conductors. Furthermore, compared to the existing approximate expressions (like those in [7, 8]) it can take into account any shape of the cross sections of the conductors, and not only rectangles. In Sec. III, some more details of the memory requirements will be given.

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II. THE BEM FORMULATION

The BEM formulation proposed in [9] for the modeling of railway systems is adapted here to interconnect structures. For validation purposes, we apply it to a typical on-chip interconnect structure on silicon substrate (Fig. 1).

Under the hypothesis of a time-harmonic regime with angular frequency ω, vector fields are represented using phasors. Displacement currents are neglected.

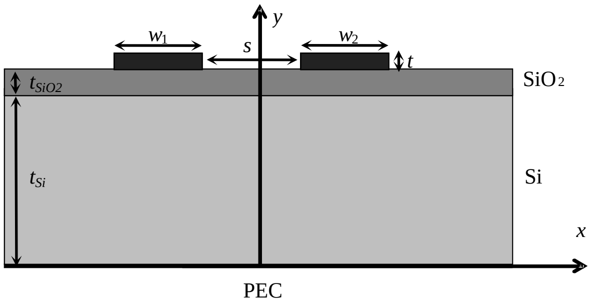

Fig. 1. The typical on-chip interconnect structure on silicon substrate.

Introducing the magnetic vector potential (MVP) as

$$\vec{B} = \mu\vec{H} = \nabla\times\vec{A}, \tag{1}$$

the electric field can be expressed as

$$\vec{E} = -j\omega\vec{A} - \nabla V, \tag{2}$$

where V is the electric scalar potential.

We decompose the MVP in a “source” and an “eddy” component

$$\vec{A}(\vec{r},t) = \vec{A}^s(\vec{r},t) + \vec{A}^e(\vec{r},t), \tag{3}$$

where the “source” component $\vec{A}^s$ satisfies the following condition

$$j\omega\vec{A}^s = -\nabla V, \tag{4}$$

so that the electric field is obtained in terms of the “eddy” component $\vec{A}^e$ only, i.e.

$$\vec{E} = -j\omega\vec{A}^e. \tag{5}$$

If the z-axis parallel of the cartesian coordinate system is parallel to the conductor’s axis, the z-component of the magnetic field is zero ($\vec{H} = H_x\vec{e}_x + H_y\vec{e}_y$) and all the other fields are z-directed ($\vec{E} = E\vec{e}_z$, $\vec{A} = A\vec{e}_z$, $\vec{A}^s = A^s\vec{e}_z$, and $\vec{A}^e = A^e\vec{e}_z$).

Furthermore, $\nabla V = (\partial V/\partial x)\vec{e}_x$ and A^s are only functions of frequency and do not depend on coordinates y and z, being constant values for each conductor.

Application of the Coulomb’s gauge to MVP leads to the Laplace equation for the MVP A_0 in air ($\Delta A_0 = 0$), so that the following boundary integral equation holds

$$\frac{1}{2}A_0 = \int_{c_1\cup c_2}\left(A_0\frac{\partial g}{\partial n} - g\frac{\partial A_0}{\partial n}\right)dl, \tag{6}$$

where g is the Green’s function of the Laplace equation out of the conductors. Taking into account the following boundary conditions

$$A_0 = A^e = A - A^s, \tag{7}$$

$$\frac{\partial A_0}{\partial n} = \mu_r\frac{\partial A}{\partial n} = \mu_r\frac{\partial A^e}{\partial n}, \tag{8}$$

where μ_r is the permeability of the conductor, we obtain

$$\frac{1}{2}A - \int_{c_1\cup c_2}\left(A\frac{\partial g}{\partial n} - g\mu_r\frac{\partial A}{\partial n}\right)dl - \left(\frac{1}{2}A_k^s - A_1^s\int_{c_1}\frac{\partial g}{\partial n}dl - A_2^s\int_{c_2}\frac{\partial g}{\partial n}dl\right) = 0, \tag{9}$$

$$\frac{1}{2}A + \int_{c_k}\left(A\frac{\partial g_k}{\partial n} - g_k\frac{\partial A}{\partial n}\right)dl = 0, \tag{10}$$

that can be solved together with Ampere’s law

$$\int_{c_k}\frac{\partial A}{\partial n}dl = -\mu_k I_k. \tag{11}$$

In (10), g_k is the fundamental solution of the Helmholtz equation. Note that the terms A_k^s in (9) are out of the sign of the integrals since they are constant over the cross-sections of the corresponding conductors.

In order to take into account a lossy substrate with ground plane, a complex image approximation of the Green's function can be used [7, 8]

$$g = \frac{1}{2\pi} log\sqrt{(x-x')^2+(y+y'+d)^2} + g_0, \quad (12)$$

$$g_0 = \frac{1}{2\pi} \log\left(\frac{1}{|r|}\right), \quad (13)$$

with d = (1-j)·δ in the case of a semi-infinite homogeneous conductive substrate and d = (1-j)·δ ·tanh[(1+j)t_{Si}] in the case of finite thickness with backside metallization, where δ is the skin depth in the substrate and t_{Si} is the substrate thickness. If a multilayer model for the substrate is to be considered, d can be computed from the surface impedance of the substrate as in [8].

The entries of the p.u.l. impedance matrix can be finally computed by means of the following formulas

$$Z_{hh} = \left.\frac{j\omega A_h^S}{I_h}\right|_{I_k=0}; \text{ (14a) } Z_{hk} = \left.\frac{j\omega A_k^S}{I_h}\right|_{I_k=0}. \quad (14)$$

In this work, the surface boundary integral equations (1-2) are discretized by means of constant boundary elements [10-11]. In order to check if the presence of corners introduces inaccuracies, the case of the two identical rectangular conductors without the presence of the lossy substrate is considered. Our BEM results are compared with FEM results obtained using a commercial software [12]. A good agreement is obtained in the frequency range 1-10 GHz (errors in resistance lower than 0.5 % and errors in inductance lower than 0.1 %).

III. NUMERICAL SIMULATIONS

For validation purposes, first a single copper microstrip line of a rectangular cross-section (Fig. 2) is considered. Our results are compared with those given by the commercial FEM software [12] (Figs. 3-4). FEM simulations are obtained using a mesh of 154987 triangles and requires 1.4 GByte of RAM, while BEM simulations use 100 nodes and 538 MByte of RAM.

In order to show the validity of the proposed method for any shape of the conductor cross-sections, the case of a single mictrostrip line of trapezoidal cross-section is also considered (Figs. 5-7). A good agreement with FEM results is obtained.

The self and mutual resistance and inductance parameters for the coupled on-chip interconnect structure of Fig. 1 on a high conductivity (10^4 S/m) and a medium conductivity (16.66 S/m) silicon substrate with t_{Si} = 300 μm thickness analysed in [6] have been computed by means of the proposed formulation (Figs. 8-11). The dimensions of the cross section of each conductor are w_1 = w_2 = 2 μm, t = 1 μm, and the separation between the conductors is s = 2 μm. Results are compared with those reported in [6] and those obtained by means of the FEM commercial software [12]. As it can be noted, BEM results are in good agreement with the FEM results, while the self resistance computed as proposed in [6] is not accurate (Fig. 8).

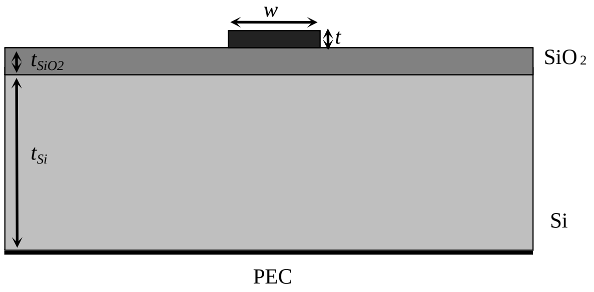

Fig. 2. A single copper (σ = 5.8·10^7 S/m) microstrip line with w = 4 μm, t = 1 μm, t_{SiO2} = 2 μm and t_{Si} = 300 μm.

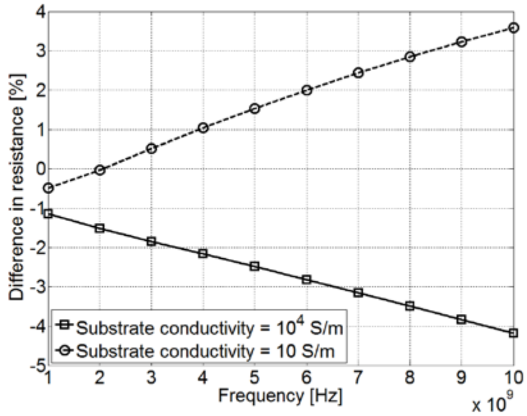

Fig. 3. Relative difference in resistance (FEM-BEM).

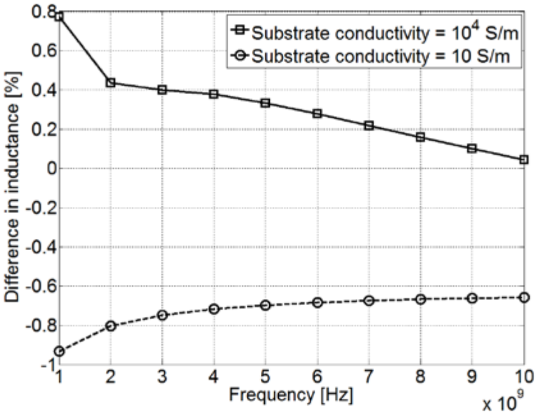

Fig. 4. Relative difference in inductance (FEM-BEM).

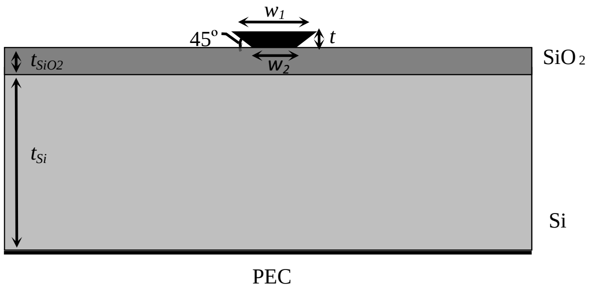

Fig. 5. A single copper microstrip line of trapezoidal cross-section with w_1 = 4 µm, w_2 = 2 µm, t = 1 µm, t_{SiO2} = 2 µm and t_{Si} = 300 µm.

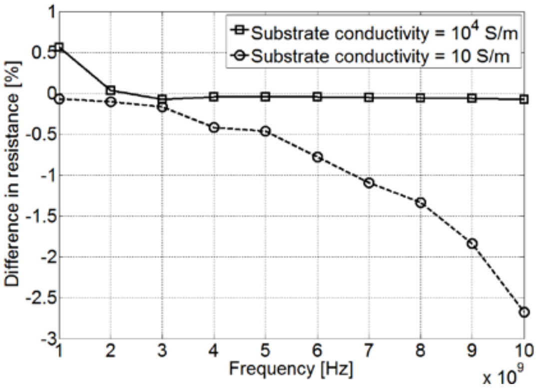

Fig. 6. Relative difference in resistance (FEM-BEM).

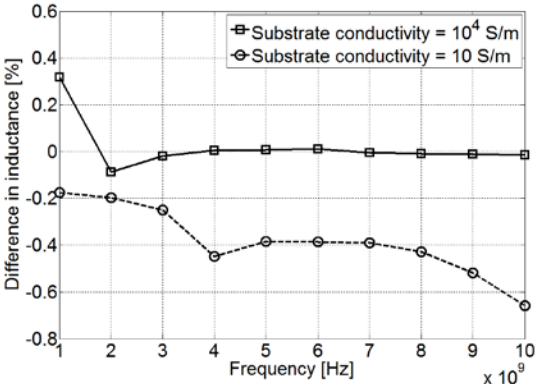

Fig. 7. Relative difference in inductance (FEM-BEM).

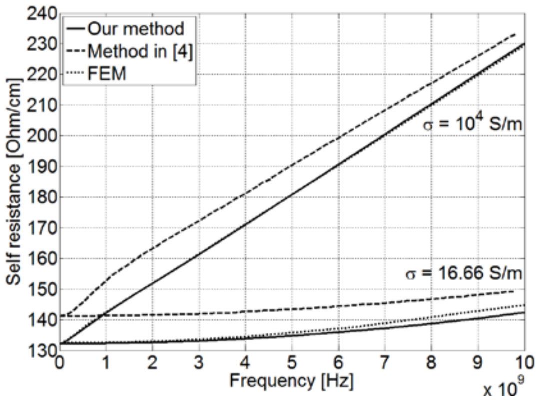

Fig. 8. Self resistance of symmetric coupled interconnects of Fig. 1.

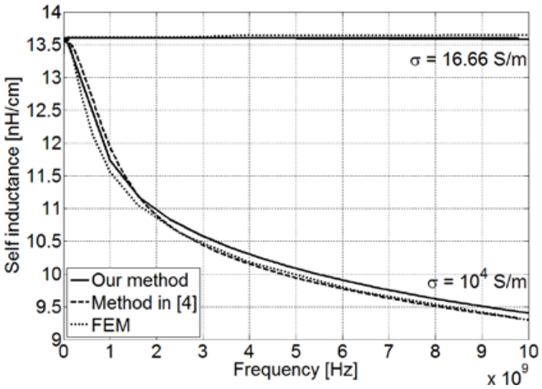

Fig. 9. Self inductance of symmetric coupled interconnects of Fig. 1.

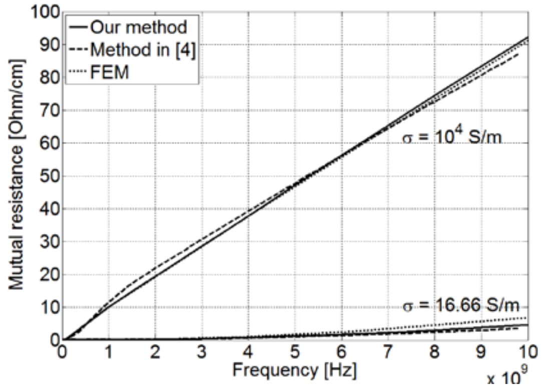

Fig. 10. Mutual resistance of symmetric coupled interconnects of Fig. 1.

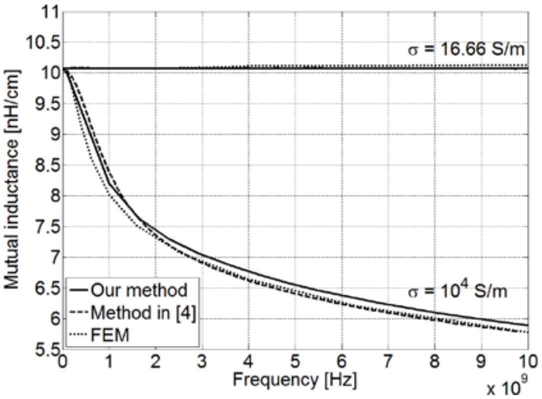

Fig. 11. Mutual inductance of symmetric coupled interconnects of Fig. 1.

BEM results are also compared with FEM results for self and mutual resistances of asymmetric interconnects, obtaining good agreement (Figs. 12-17). The following numerical values are considered (Fig. 1): $w_1 = 2$ µm, $w_2 = 1$ µm, $t = 1$ µm, $s = 2$ µm, $t_{SiO2} = 3$ µm, $t_{Si} = 300$ µm, $\sigma = 3.5\text{x}10^7$ S/m, and $\sigma_{Si} = 10^4$ S/m.

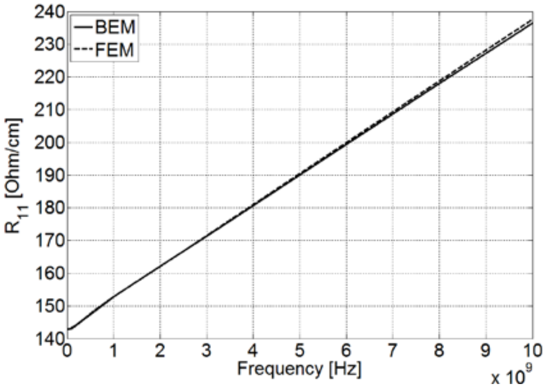

Fig. 12. Self resistance R_{11} for asymmetric coupled interconnects.

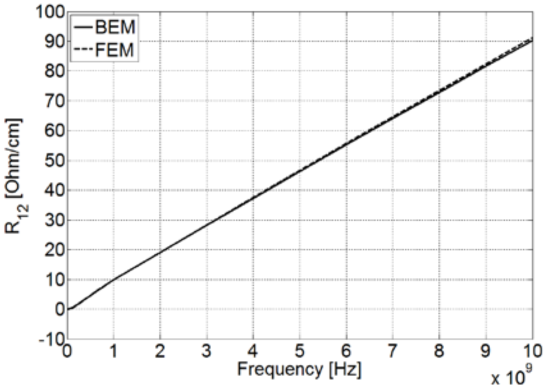

Fig. 13. Mutual resistance R_{12} for asymmetric coupled interconnects.

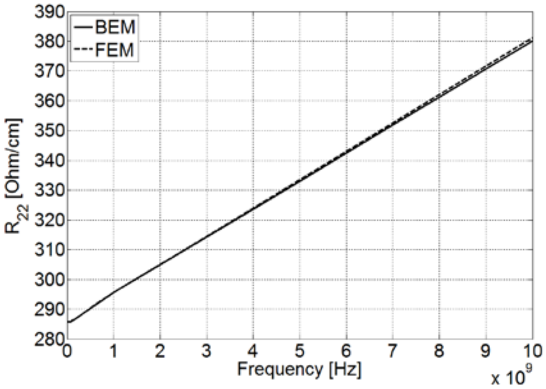

Fig. 14. Self resistance R_{22} for asymmetric coupled interconnects.

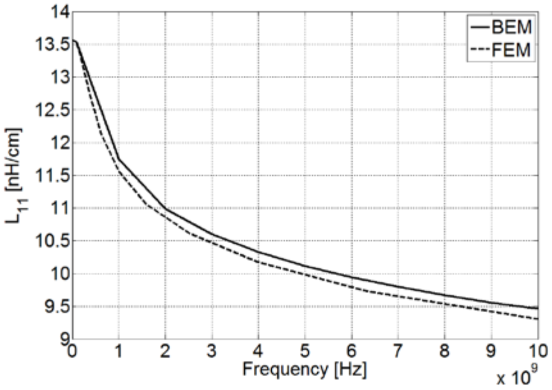

Fig. 15. Self inductance L_{11} for asymmetric coupled interconnects.

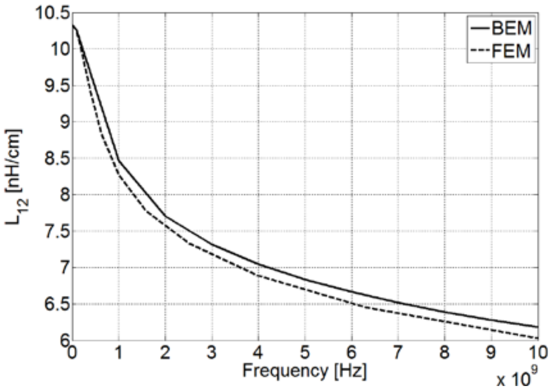

Fig. 16. Mutual inductance L_{12} for asymmetric coupled interconnects.

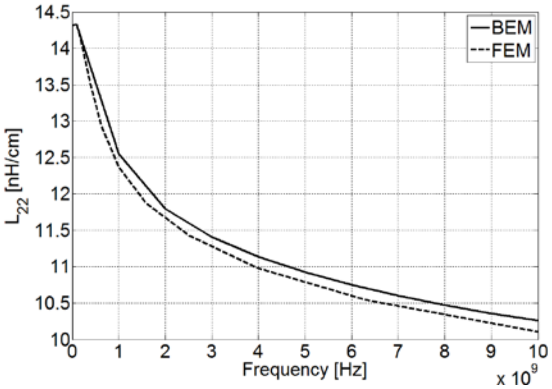

Fig. 17. Self inductance L_{22} for asymmetric coupled interconnects.

IV. CONCLUSION

The proposed formulation is proven to be a good compromise between general quasi-magnetostatic numerical solvers and approximate analytical expressions. The BEM formulation has been validated by comparison with other techniques for some typical test problems and the advantages of the proposed approach in terms of memory requirement have been shown. Future work will deal with the extension of the formulation to multilayer silicon substrates, 3D geometries, the modeling of roughness of the surfaces of the traces, and the implementation of surface impedance boundary conditions in order to further reduce the computational cost.

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