The Joy of Computing with Volume Integrals: Validation and Performance Comparison

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Abstract **─** This is the second paper in a series dealing with the application of computational electromagnetics to nondestructive evaluation, using the vehicle of volume-integral equations, as developed in the preceding paper in this volume. A challenge-problem in the field of nondestructive evaluation is the inspection of fastener sites for fatigue cracks in multilayer structures. Using the volume-integral equation approach, problems comprising multilayer structures, ferromagnetic fastener sites, gaps between materials interfaces, and the presence of fatigue cracks are accurately modeled. Simulated studies are presented with VIC-3D©, a proprietary volume-integral code, and comparisons are made with FEM highlighting performance advantages of the VIE approach. Further, the role that volume-integral equations play in the context of other well-known computational-electromagnetic algorithms is discussed.

Index Terms **─** Aircraft structures, computational electromagnetics, electromagnetic nondestructive evaluation, volume-integral equations.

I. INTRODUCTION

A problem of particular interest in the field of nondestructive evaluation is the inspection of fastener sites in aircraft structures for fatigue cracks. An important class of structures comprises plane-parallel layered media representing joints in both fuselage and wing locations. The layer stackup typically consists of two to four panels and often includes thin layers of a non-conducting sealant or adhesive between the panels. Additional complexity for computational electromagnetics concerns the fastener shape (countersunk or buttonhead fastener) and material type (where ferrous materials are prevalent). Also, fatigue cracks of complex morphology provide a particular challenge for representation using numerical methods. Prior work has investigated the problem of modeling an eddy current inspection of fastener sites in multilayer structures for fatigue cracks using a volume-integral equation approach [1]. Although good agreement was achieved with experimental results, simplications in the model representation were used. In this paper, the capability of the volume-integral equation approach with spatial decomposition algorithms is demonstrated to fully address the problem of ferromagnetic fastener sites in multilayer structures with gaps between the layers and cracks emanating from the holes.

II. VALIDATION STUDIES

A. The Cessna sandwich

To demonstrate the capability of the volume-integral equation method, the 'Cessna sandwich' example is presented [2]. Figure 1 illustrates the geometry of the Cessna-sandwich, which consists of three layers of aluminum comprising two different aluminum alloys (conductivities), two 'air gaps' representing the non-conducting sealant layers, and a titanium rivet-insert with a countersunk head and a shank connecting all layers [1]. In the model, the layers and gaps make up a uniform background or host (in x and y directions) with the fastener being an 'anomaly' requiring a local volume element mesh. Because the anomaly extends through several layers of material with different electromagnetic properties, we must use VIC-3D© that has been augmented with the spatial-decomposition algorithm (SDA) of Section II E of the preceding paper to solve the problem.

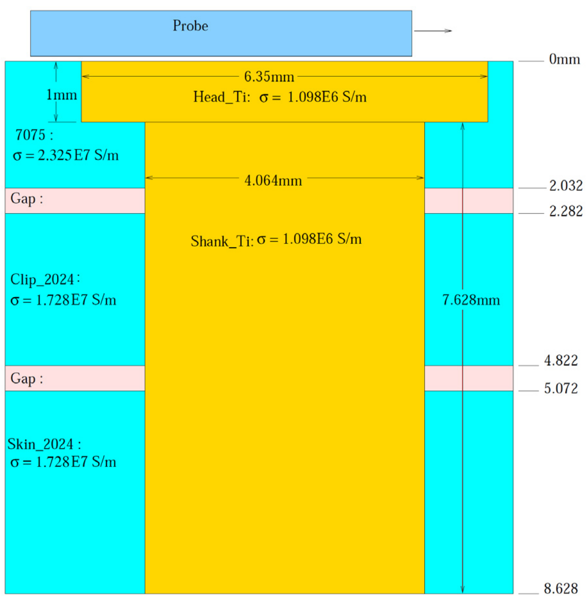

Fig. 1. The Cessna-sandwich problem.

Two model calculations were run based upon Fig. 1, the first with both gaps filled with air, and the second with the top gap filled with 7075-aluminum ($\sigma = 2.325 \times 10^7$), and the other gap filled with 2024-aluminum ($\sigma = 1.728 \times 10^7$). The probe was a simple air-core coil with inner and outer radii of 3.02 mm and 5.14 mm respectively and a height of 2.48 mm. Both model calculations were run with a coil lift-off of 1.0 mm, and a frequency of 2500 Hz. Five spatial-decomposition grids were used to represent the 'anomaly' regions, each with 8 × 8 × 2 cells, yielding a total of 1920 unknowns. The plot labeled 'Gap' corresponds to the air-filled gaps of Fig. 1, and the plot labeled 'No Gap' corresponds to the case in which the gaps are filled with aluminum. The solution time for the 'Gap' run with twenty-six probe scan points is about 6 minutes on an AMD/Athlon machine, whereas the 'No Gap' run took about 2 minutes.

For the VIC-3D© calculations shown in Fig. 2, it is clear that the effect of the air-filled gaps is to increase somewhat the peak magnitude of the impedance response when compared to the system with aluminum-filled gaps. The results in Fig. 2 are changes in the driving-point impedance of the coil due to the presence of the anomaly, in this case the rivet, compared to the 'host-only' impedance in the absence of the anomaly. As such, it is not unusual to have negative values of resistance. Consider, for example, the results at a probe position of -4.0 mm, for which $\delta Z_{\text{gap}} = -0.22 + j0.41\ \Omega$ and $\delta Z_{\text{no gap}} = -0.195 + j0.35\ \Omega$. In the former, the presence of the rivet insert reduces the resistance of the host structure with the gap, compared to the situation with aluminum filling the original gaps. On the other hand, the presence of the gaps increases the stored magnetic energy, which is manifested in the slightly larger value of reactance. Future work is planned to explore the physics of this model-calculation by showing the anomalous currents within the rivet, as well as the scattered field produced by those currents.

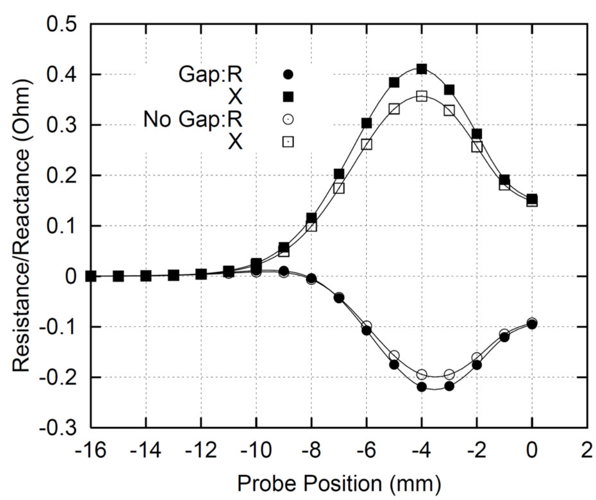

Fig. 2. Model results for the Cessna double sandwich of Fig. 1.

B. Comparison with finite-element method results

To provide a baseline for this challenge-problem concerning the performance of the volume-integral equation method, an FEM model for the Cessna sandwich problem was implemented in the Opera-3D© software package (V9.0)[3]. The numerical formulation of FEM is well established in the literature [4]. A diagram of the model is shown in Fig. 3. Irregular meshes of tetrahedral elements were used with the finite-element formulation to generally represent the complex geometries. The mesh for the model required at least two elements per skin depth in the fastener site region. The lateral dimensions of the plates in the model were extended well beyond the field generated by the eddy current coil to minimize the effect of the part or model domain edges on the measurement response. Continuity conditions were maintained between all part/air interfaces in the model. Far from the coil and fastener site, the boundary conditions at the FEM model boundary were defined with the tangential magnetic field equal to zero. The output of this model is the electric and magnetic field intensities. Change in impedance can then be calculated using the change in resistance associated with dissipated energy in the region of the conductor and the change in inductance related to the stored energy in the whole solution domain. More information on constructing eddy current models in Opera-3D© can be found in [3].

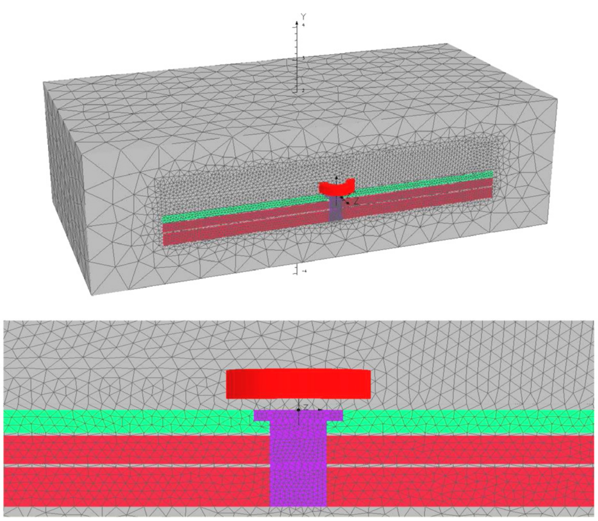

Fig. 3. Diagrams of fastener site FEM model with two gap layers between three aluminum panels.

Figure 4 displays the model results for the Cessna sandwich problem with both airgaps, and compares these results with the VIC-3D© results of Fig. 2 labeled 'Gap'. Clearly, the FEM response shows the same characteristics as does the VIC-3D© generated solution. Some differences found in the magnitude of the calculations are likely due to mesh-related error present in the models. Prior work has shown very good agreement between FEM and VIE for this class of problem [1]. Computation of the FEM model required 60 hours on a 3.02 GHz Pentium 4 with 1 GB of memory. For this complex problem, the advantage of the compact formulation with volume-integrals (1920 unknowns) over FEM is obvious.

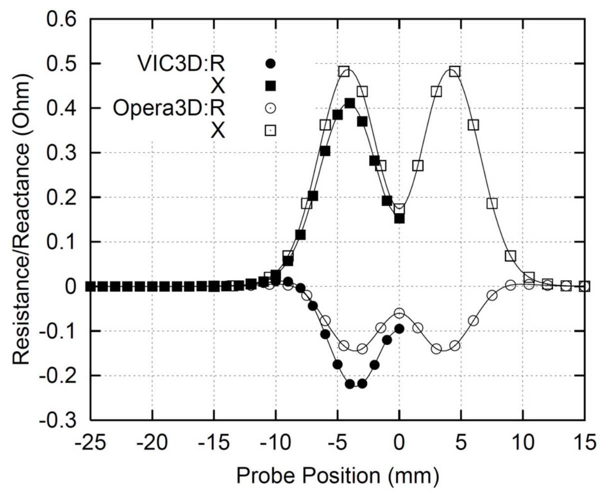

Fig. 4. FEM and VIC-3D© model results for the Cessna sandwich with two airgaps.

C. Simulated studies on crack characteristics

From the perspective of nondestructive evaluation, the critical requirement of a NDE model is an accurate representation of the eddy current measurement associated with crack detection. Thus, the model must accurately represent the measurement sensitivity to the crack condition with respect to features such as the fastener site and gaps between layers. To explore the sensitivity of the numerical model to varying crack conditions such as crack length and location, simulated studies were performed using VIC-3D©. Again, a class of problems based on setup standards for Cessna (the 'Cessna setups') was simulated in this study. (Future work will thoroughly explore validation of the simulated results with experimental measurements.) The Cessna setups comprise the complex ring probe shown in Fig. 5, the 'hi-lock' pin rivet of Fig. 6, and the various test setups shown in Fig. 7(a) to

Fig. 7(e). Test setup 0 is the root of the other setups, with each of the others differing by the placement of the crack (shown in brown). It should be observed that the setups contain both electrically conducting (lossy) media and ferromagnetic steels. The cracks are modeled as empty slots.

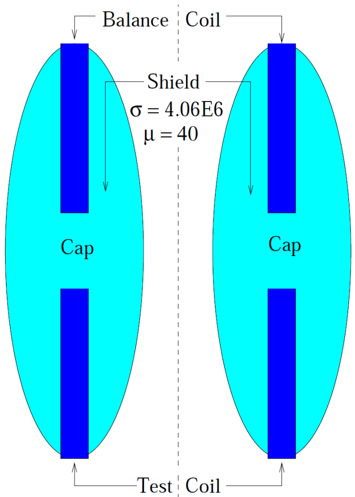

Fig. 5. The ring probe.

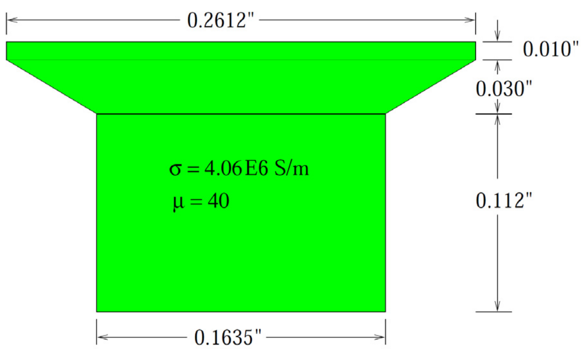

Fig. 6. The hi-lock pin rivet.

In modeling the setups, we use the spatial-decomposition algorithm with five grids, two above the air gap, and two below, with one within the gap. Each grid has 16 × 16 × 2 cells, yielding a total of 15,360 electric and magnetic currents to be determined. The ring probe core was modeled with 12,288 electric and magnetic currents, so that the entire package of unknowns for each setup is 27,648. The models were run at a frequency of 1.0 kHz. Data for determining the presence of a flaw are obtained by varying the lift-off of the ring probe over the range 0, 0.25, 0.7493, and 0.9398 mm. By 'lift-off' we mean the position of the bottom of the probe above the workpiece. The solution of the problem with 27,648 unknowns takes about 35 minutes for each lift-off, giving a total solution time of about 2.3 hours for each setup. About 1 Gb of storage for auxiliary files was required.

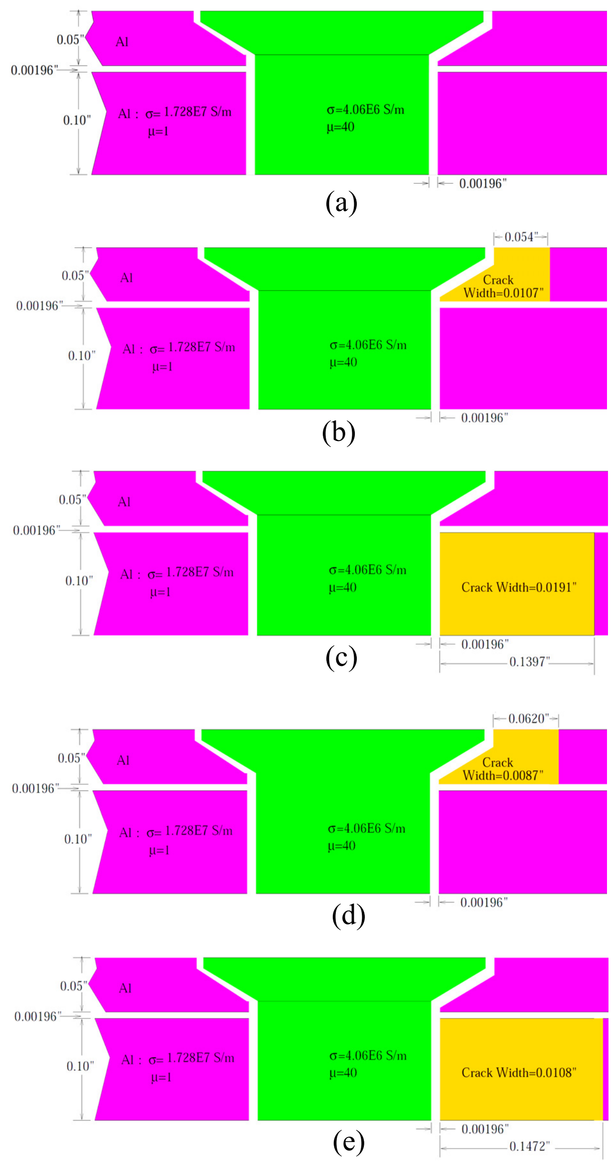

Fig. 7. Test setup (a) 0, (b) 1, (c) 2, (d) 3, (e) 4.

VIC-3D© computes impedances, but the industry typically uses analog instruments to measure data, so the final results of the computation are transformed into 'instrument volts,' as shown in Fig. 8. Prior work demonstrates how impedance calculations can be transformed into voltage measurements using calibration data with known samples [1]. Setup 0 is the reference, so it is zero for each lift-off along the negative real axis. The remaining curves indicate the presence of a flaw, and yield information that can be used in an inversion procedure. That will be the subject of another paper in this series.

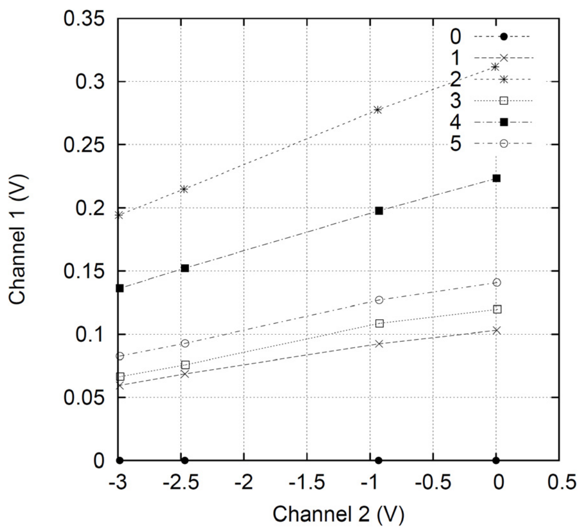

Fig. 8. Final scaled model results for setups 0-5, using setup 0 as the reference. The horizontal and vertical axes are both measured in instrument volts, V. The parametric points on the curves are lift-off values of 0, 0.25, 0.7493, and 0.9398 mm (right to left).

It is clear that setup 2 produces the largest signal, even though the flaw is in the second layer. This is due to the fact that the flaw in this setup has a larger volume than any of the other flaws. Furthermore, it is clear that the flaws in setups 1 and 3, even though they are in the upper layer, are more difficult to distinguish than the other two flaws. This is due to the fact that these two flaws have a smaller volume than those in setups 2 and 4. Setup 5 is the same as setup 4, except that the length of the crack is 0.0986 inch, instead of 0.1472 inch. The response of setup 5 is about 0.67 times the response of setup 4, which is consistent with the fact that the response is roughly proportional to the volume of the defect.

III. COMMENTS AND CONCLUSIONS

We have shown that the volume-integral approach has significant advantages over other numerical methods, such as the finite-element method, in that the formulation of the numerical model is much simpler in the former than in the latter. Furthermore, the solution time for VIC-3D© is extremely short for many problems in NDE, because the formulation in terms of the Galerkin variant of the method of moments on a regular grid results in operators that have very special structures; they are either three-dimensional convolutions, or two-dimensional convolutions and one-dimensional correlations, which means that we can use three-dimensional FFT's to evaluate them in a conjugate-gradient search algorithm. The use of a highly irregular mesh in the finite-element technique does not allow a similar advantage in the solution process.

This advantage accrues from the very different nature of the physics that goes into the formulation of the mathematical models. In volume-integral equations, as well as boundary-integral equations, the unknowns are anomalous currents that are supported in a compact domain, namely the domain of the anomaly; in the example of the Cessna series, the anomaly is the rivet or rivet and crack. In finite-element or finite-difference methods, the unknowns are the electric and magnetic fields, which extend to infinity. This has two disadvantages; it increases the number of unknowns, and requires some method of approximating the 'boundary-at-infinity' in order to truncate the problem domain. This increases the complexity in simply defining the model, and presents an extreme challenge to prospective users who are not skilled in computational electromagnetics.

Furthermore, the FEM method is not particularly well-suited to solve typical problems in NDE, because the anomalies, such as rivets or cracks, require a very complicated mesh, with a large number of very irregular cells. Finally, we conclude from the model calculations of the Cessna series that the spatial decomposition algorithm is a very efficient method of solving problems in which an anomaly extends through layers with different electric and magnetic properties. The spatial decomposition algorithm formulation was, also, demonstrated to be valid for cases where a conducting anomaly is present in a non-conducting layer, such as air. This approach greatly expands the capability of volume-integral equation methods for complex problems in computational electromagnetics, and in particular nondestructive evaluation. Lastly, rapid solution of the forward problem for computational electromagnetics will, also, be beneficial for the practical application of advanced inverse method techniques for quantitative nondestructive evaluation of material discontinuities.

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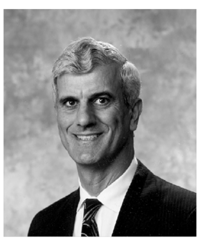

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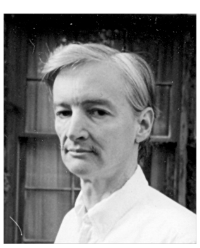

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