Shape Design Optimization of Power Frequency Electromagnetic Devices Using Numerical Methods

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Abstract - **This paper presents the mathematical formulation concerning the solution of inverse electromagnetic problems, i.e. the shape optimization of power frequency electromagnetic devices, based on a combination of numerical methods. The optimization problem is solved using deterministic methods in which the electromagnetic field problem is treated as a subproblem of the optimization process. The field problem is calculated using the finite element (FE) method. Three deterministic approaches are studied in detail, the quadratic extended penalty method (QUA), the augmented Lagrange multiplier (ALM) method and the constrained quasi-Newton method (PLBA-CR). The work highlights the advantages and drawbacks of each approach. The search direction for the optimization is found by two distinct methods, the direct differentiation of the FE matrices and the finite difference (FD) method. In total, three problems are discussed in order to show the power and applicability of the theory presented. The PLBA-CR, when combined with the direct differentiation of the FE matrices, appears to offer important advantages over the other methods.**

I. INTRODUCTION

The design of electromagnetic devices, such as electromagnets, electrical machines etc., has always been a challenge for electrical engineers. This process normally involves the determination of the shapes, dimensions, position of the core, permanent magnets and windings of the device, amongst other factors, which may produce prescribed electromagnetic quantities such as flux distributions, forces and torques.

In the past, such designs were a task based very much on the engineer's experience and intuition. After the advent of the computer in the fifties and its subsequent widespread use in the eighties, the whole process of design has changed. It is now possible to analyse electromagnetic devices using computer aided design (CAD) techniques, see for instance [1]-[3]. However, a pure field analysis package may in many situations leave the engineer in an uncomfortable situation of having to change some key parameters in the design and then rerunning the program until an acceptable result is obtained.

In recent years there has been an enormous amount of work concerning the solution of inverse electromagnetic problems by means of numerical methods, see for instance the proceedings of the last Compumags and CEFCs. In addition to the solution of particular problems there have been advancements towards the automatization of some specific tasks, the shape optimization of electromagnetic devices is one example.

Mathematically, inverse problems such as the shape optimization of power frequency electromagnetic devices may be stated as a constrained optimization problem [4]. In general one may write

Minimize

$$F = F\left(\{p\}, \varphi\left(\{p\}\right)\right) \tag{1}$$

Subject to

$$g_j\left(\{p\}, \varphi\left(\{p\}\right)\right) \le 0 \quad j = 1,\ldots,l \tag{2}$$

$$h_k\left(\{p\}, \varphi\left(\{p\}\right)\right) = 0 \quad k = 1,\ldots,m \tag{3}$$

$$p_i^L \le p_i \le p_i^U \quad i = 1,\ldots,n \tag{4}$$

where F represents the objective function, g_j are the inequality constraints, h_k are the equality constraints, p_i stands for the design variables and φ for the field variable. It has been assumed that the field variable φ is also a function of the design variable p. It is also important to note that the lower and upper limits given in (4) define the region of search in the n-dimensional space.

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Classically (1) to (4) may be solved using either stochastic or deterministic methods. What differentiates these two sets is the way their numerical methods iterate.

Stochastic methods are based on probabilistic rules, i.e. they attempt to achieve the solution by exhaustive evaluations of the objective function (1). This feature is often seen as an advantage of these methods since a global optimum may be achieved theoretically. However, in many cases that same feature is regarded as a major drawback since the computational cost may become prohibitive. Recently special attention has been paid to the method of simulated annealing and the method of genetic algorithms for solving inverse problems, see for instance [5]-[7].

Deterministic methods, on the other hand, are based on an iterative line search where the design parameters p are varied systematically until an optimum value is found. Mathematically this is expressed by

$$\{p\}^{q+1} = \{p\}^{q} + \alpha^{q} S^{q} \qquad (5)$$

where α stands for the step size and S for the search direction which is calculated by

$$\{S\}^{q} = -\nabla F^{q} + \beta^{q} \{S\}^{q-1} \qquad (6)$$

with

$$\beta^{q} = \left|\nabla F^{q}\right|^{2} \Big/ \left|\nabla F^{q-1}\right|^{2} \qquad (7)$$

Thus, it becomes clear that the calculation of S requires the total differentiation of F with respect to p which in turn requires the differentiation of φ with respect to p. Indeed, deterministic methods have the advantage of using the information from one iteration to move to another. In many cases this feature makes the solver converge rapidly to an acceptable solution. This can represent substantial savings in computational costs. However, due to their nature deterministic approaches suffer from the potential limitation of getting stuck in a local minimum. For some works concerning the use of deterministic methods to inverse problems in electromagnetics refer to [8]-[10].

The aim of this paper is to present the mathematical formulation for the solution of shape optimization of power frequency electromagnetic devices. This is achieved by combining deterministic methods, which are used for the optimization process, with the FE method, which is used for solving the field problem. Three deterministic approaches are investigated: the quadratic extended penalty method (QUA), the augmented Lagrange multiplier (ALM) method and the constrained quasi-Newton method (PLBA-CR). The advantages and drawbacks of each approach are highlighted. Concerning the calculation of the search direction, the work describes in detail how (6) may be obtained by direct differentiation of the FE matrices for time-harmonic, magnetostatic and electrostatic systems. Finally three problems are solved in order to show the applicability of the theory.

II. Sensitivity Analysis

We shall describe in this part a methodology to obtain quantitative information on how the performance of the device is affected by changes in the design variables p_i. It is this information that provides the essential guideline for the search direction S given in (5). It will be assumed that the field analysis will be made using the FE method. Calculating the total derivative of (1) with respect to p_i gives

$$\frac{dF\left(\{p\}, \varphi\left(\{p\}\right)\right)}{dp_i} = \frac{\partial F}{\partial p_i} + \left\{\frac{\partial F}{\partial \varphi}\right\}^{\mathrm{T}} \left\{\frac{\partial \varphi}{\partial p_i}\right\} \quad i = 1, \ldots, n \qquad (8)$$

The first term of (8), that is $\partial F / \partial p_i$, may be found from F which is normally given. The second term of (8) involves the calculation of $\left\{\partial \varphi / \partial p_i\right\}$ which can be evaluated from the FE formulation.

Time Harmonic System

Two dimensional time harmonic problems are governed by the following equation

$$\nabla \cdot \left(\nu \nabla A^{*}\right) - \sigma j \omega A^{*} = -J_e \qquad (9)$$

where ν is the magnetic reluctivity (linear), σ is the conductivity, $j = \sqrt{-1}$, ω is the frequency, J_e is the source current (assumed to be flowing only in the axial direction) and A^{*} is the complex vector potential (entirely oriented in the axial direction). It is important to note that in this case the field variable φ is A^{*}. Equation (9) may be solved term by term using the finite element method together with the Galerkin's approach to yield

$$\sum_{j=1}^{n} \left[\left(K_{ij} + \sigma j \omega T_{ij}\right) A_j^{*} - R_i\right] = 0 \quad i = 1, \ldots, n \qquad (10)$$

where n is the number of nodes. The three terms K_{ij}, T_{ij} and R_i may be found in [2], [3]. Thus, the

derivative $\left\{\partial A^*/\partial p_i\right\}$ may be obtained by differentiating (10), that is

$$\sum_{j=1}^{n}\left[\left(K_{ij}+\sigma j\omega T_{ij}\right)\right]\frac{\partial A_j^*}{\partial p}=\sum_{j=1}^{n}\left\{\left\{\frac{\partial R_i}{\partial p}\right\}-\frac{\partial\left[\left(K_{ij}+\sigma j\omega T_{ij}\right)\right]}{\partial p}\left\{A_j^*\right\}\right\} \tag{11}$$

The right hand side of (11) may be obtained by considering the derivative of K_{ij}, T_{ij} and R_i with respect to p, that is

$$\frac{\partial}{\partial p}K_{ij}A_j^*=\sum_e\int_{\Omega_e}\nu\frac{\partial\nabla N_{ij}}{\partial p}A_j\left|\mathrm{G}^e\right|dudv+\sum_e\int_{\Omega_e}\nu\nabla N_{ij}A_j^*\frac{\partial\left|\mathrm{G}^e\right|}{\partial p}dudv \tag{12}$$

$$\frac{\partial}{\partial p}T_{ij}A_j^*=\sum_e\int_{\Omega_e}\frac{\partial}{\partial p}\left(-\sigma j\omega N_{ij}A_j^*\right)\left|\mathrm{G}^e\right|dudv-\sum_e\int_{\Omega_e}-\sigma j\omega N_{ij}A_j^*\frac{\partial\left|\mathrm{G}^e\right|}{\partial p}dudv \tag{13}$$

$$\frac{\partial}{\partial p}R_i=\sum_e\int_{\Omega_e}\frac{\partial}{\partial p}\left(N_iJ_e\right)\left|\mathrm{G}^e\right|dudv+\sum_e\int_{\Omega_e}N_iJ_e\frac{\partial\left|\mathrm{G}^e\right|}{\partial p}dudv \tag{14}$$

in which $\left|\mathrm{G}^e\right|$ is the Jacobian of the element of reference and N is the trial function. After the calculation of $\left\{\partial A^*/\partial p_i\right\}$, the expression for $\left\{\partial B^*/\partial p_i\right\}$ may be obtained using $B^*=\nabla\times A^*$.

Magnetostatic System

Two dimensional magnetostatic systems are governed by the Poisson equation

$$\nabla^2A=-\mu J \tag{15}$$

in which A represents the magnetic vector potential (the state variable φ in this case), μ the magnetic permeability and J the current density (oriented in the axis direction). Similar to the previous system, equation (15) may be solved using the FE method together with the Galerkin's approach to yield

$$\sum_{j=1}^{n}K_{ij}A_j-R_i=0\quad i=1,\ldots,n \tag{16}$$

where n is the number of nodes. The two terms K_{ij} and R_i are identical to those from the previous system. In this case the derivative $\left\{\partial A/\partial p_i\right\}$ may be obtained from (16), that is

$$\sum_{j=1}^{n}\left[K_{ij}\right]\frac{\partial A_j}{\partial p}=\sum_{j=1}^{n}\left\{\left\{\frac{\partial R_i}{\partial p}\right\}-\frac{\partial\left[K_{ij}\right]}{\partial p}\left\{A_j\right\}\right\} \tag{17}$$

which can be calculated using (12) and (14). Likewise, after the calculation of $\left\{\partial A/\partial p_i\right\}$, the expression for $\left\{\partial B/\partial p_i\right\}$ may be obtained using $B=\nabla\times A$.

Electrostatic System

Similar to the magnetostatic system two dimensional electrostatic problems are also described by the Poisson equation

$$\nabla^2\mathrm{V}=-\frac{\rho}{\varepsilon} \tag{18}$$

in which V represents the electric scalar potential (the state variable φ in this case), ε the electric permittivity and ρ the charge density. In this case an axisymmetric system will be considered. Equation (18) can be solved using the FE method together with Galerkin's approach to yield

$$\sum_{j=1}^{n}K_{ij}\mathrm{V}_j-R_i=0\quad i=1,\ldots,n \tag{19}$$

in which n is the number of nodes. In this case the two terms K_{ij} and R_i are slightly different to those from a system with Cartesian symmetry, see [3] for instance. The derivative $\left\{\partial\mathrm{V}/\partial p_i\right\}$ may be achieved by differentiating (19), that is

$$\sum_{j=1}^{n}\left[K_{ij}\right]\frac{\partial\mathrm{V}_j}{\partial p}=\sum_{j=1}^{n}\left\{\left\{\frac{\partial R_\mathrm{i}}{\partial p}\right\}-\frac{\partial\left[K_{ij}\right]}{\partial p}\left\{\mathrm{V}_j\right\}\right\} \tag{20}$$

The right hand side of (20) may be calculated by considering

$$\frac{\partial}{\partial p}K_{ij}\mathrm{V}_j=\sum_e\int_{\Omega_e}\pi r_0\varepsilon\frac{\partial\nabla N_{ij}}{\partial p}\mathrm{V}_j\left|\mathrm{G}^e\right|dudv+\sum_e\int_{\Omega_e}\pi r_0\varepsilon\nabla N_{ij}\mathrm{V}_j\frac{\partial\left|\mathrm{G}^e\right|}{\partial p}dudv \tag{21}$$

and

$$\frac{\partial}{\partial p}R_i = \sum_e \int_{\Omega_e} \pi r_0 \frac{\partial}{\partial p}\left(N_i \rho_e\right)\left|G^e\right| du dv + \sum_e \int_{\Omega_e} \pi r_0 N_i \rho_e \frac{\partial \left|G^e\right|}{\partial p} du dv \qquad (22)$$

where r_0 represents the mean value of the distances of the vertices of a generic element to the 0z axis. For a linear element $r_0 = (r_1 + r_2 + r_3)/3$. The expression for $\{\partial E/\partial p_i\}$ can then be obtained using $E = -\nabla V$.

It is important to note that for the three systems described previously the global system of equations, i.e. equations (10), (16) and (19), and their derivative with respect to p_i, i.e. equations (11), (17) and (20) respectively, are characterized by the matrix $[K_{ij}]$. Therefore, the application of an appropriate method for solving (10), (16) or (19) could represent a major saving in computational time. Indeed this is achieved if the Cholesky-decomposition is used as discussed by [9]. In this case the computational effort for the calculation of a single additional gradient vector $\{\partial\varphi/dp_i\}$ is reduced to forward and backward substitutions using the already decomposed matrix $[K_{ij}]$ from the corresponding global system.

An alternative way to calculate $\{\partial\varphi/\partial p_i\}$ would be to use the finite difference method rather than using the direct differentiation of the FE matrices. In general one may write

$$\frac{\partial\varphi}{\partial p_i} = \frac{\varphi\left(p + h_i e_i\right) - \varphi\left(p - h_i e_i\right)}{2h_i} \qquad (23)$$

where h_i, $h_i > 0$, is a small perturbation that is made in the global system of equations of the respective system. Indeed, equation (23) is much simpler and easier to implement into an existing FE code. However, there are two drawbacks associated in calculation of (23). The first is concerned with the introduction of round off errors. The second is related with the high number of field calculations required. A simple comparison to illustrate the latter will be made in the analysis of the results.

III. Numerical Optimization Methods

We shall consider in this part the mathematical formulation of three deterministic methods which are capable of solving the general optimization problem posed by (1) to (4). The advantages and limitations of each approach will be highlighted.

Deterministic methods may be divided into two sets: indirect and direct methods. Indirect approaches are also known as SUMT (sequential unconstrained minimization techniques). The concept of these techniques is to create a pseudo function Ψ using the original objective and constraint functions and then minimize this pseudo function as an unconstrained function. In general form one may write

Minimize

$$\Psi\left(\{p\}, r_p\right) = F\left(\{p\}, \varphi\left(\{p\}\right)\right) + r_p P\left(\{p\}, \varphi\left(\{p\}\right)\right) \qquad (24)$$

where r_p is a penalty multiplier and P is an imposed penalty function whose form depends on the SUMT being employed. Normally, the penalty multiplier is updated according to $r_p^q = r_p^{q-1}\gamma$. In this paper two SUMT are investigated, the quadratic extended penalty function method (QUA) and the augmented Lagrange multiplier method (ALM). In both cases the unconstrained function is minimized using the BFGS method [4].

Direct methods are those approaches that tackle the original objective and constraint functions directly rather than creating a pseudo function. In this work only the constrained quasi-Newton method PLBA-CR will be investigated.

Quadratic Extended Penalty Function Method (QUA)

The quadratic extended penalty function method (QUA) was proposed in 1976 by [11]. The pseudo function is defined everywhere and also has a continuous first and second derivative. This important feature allows the application of second order methods, if needed, for the unconstrained minimization of the pseudo function. The QUA method also provides a sequence of improving feasible designs. In theory, this approach tends to be numerically better conditioned when compared to its predecessors [4].

Mathematically the penalty function is defined by

$$P(p) = \sum_{j=1}^{l} \tilde{g}_j(p) \qquad (25)$$

where

$$\tilde{g}_j(p) = \begin{cases} \dfrac{-1}{g_j(p)} & \text{if} \quad g_j(p) \le -\varepsilon \\ \dfrac{1}{\varepsilon}\left[\left[\dfrac{g_j(p)}{\varepsilon}\right]^2 + \dfrac{3g_j(p)}{\varepsilon} + 3\right] & \text{if} \quad g_j(p) > -\varepsilon \end{cases} \qquad (26)$$

in which ε is in this case the transition parameter

defined by $\varepsilon = -C(r_p)^a$; $1/3 \le a \le 1/2$ and C is a constant. It is clear that due to its quadratic form the penalty function (26) may become highly nonlinear. This is often seen as the disadvantage of this approach. Highly nonlinear functions may converge slowly or not converge at all. In addition, it has also been noticed [4] that the QUA method is sensitive to the value of the penalty multiplier r_p, a feature that may lead to overflow. This may be avoided by selecting a small r_p, say 1 and a soft γ, say 0.7.

Augmented Lagrange Multiplier Method (ALM)

The augmented Lagrange multiplier method (ALM) is an approach which incorporates the advantages of the penalty methods. In particular the ALM method includes information concerning the constraint functions in the process which updates the Lagrange multipliers. This feature enhances the efficiency and reliability of this approach. In fact, it has been argued by [12] that the use of SUMT which do not include Lagrange multipliers is obsolete as a practical optimization tool.

In mathematical form the ALM method may be defined by a pseudo function expressed as

$$L(p,\lambda,r_p) = F(p) + \sum_{j=1}^{l}\left[\lambda_j \psi_j + r_p \psi_j^2\right] + \sum_{k=1}^{m}\left\{\lambda_{k+m} h_k(p) + r_p\left[h_k(p)\right]^2\right\} \qquad (27)$$

where

$$\psi_j = \max\left[g_j(p), -\frac{\lambda_j}{2r_p}\right] \qquad (28)$$

in which λ stands for the Lagrange multipliers. The update formulas for the Lagrange multipliers are

$$\lambda_j^{q+1} = \lambda_j^q + 2r_p\left\{\max\left[g_j(p), \frac{-\lambda_j^q}{2r_p}\right]\right\} \quad j = 1,l \qquad (29)$$

$$\lambda_{k+l}^{q+1} = \lambda_{k+l}^q + 2r_p h_k\left(p^q\right) \quad k = 1,m \qquad (30)$$

In summary, the following important advantages can be highlighted concerning the ALM approach. First, the method is relatively insensitive to the values of r_p. Second, it is not necessary to increase $r_p \to \infty$ to get the optimum solution since the process of updating the Lagrange multipliers requires information concerning the constraint functions, a feature which speeds up the convergence process. Third, in theory it is possible to obtain precise $g_j(p) \le 0$ and $h_k(p) = 0$. Fourth, the starting point may be either feasible or infeasible.

Constrained Quasi-Newton Method (PLBA-CR)

In the PLBA-CR approach the search direction is found by solving a subproblem with quadratic objective and linear constraint. The objective function is augmented using Lagrange multipliers and an exterior penalty so that the resulting one dimensional search is unconstrained. In mathematical terms the subproblem to be solved, i.e. the search direction vector S, may be expressed by

$$\text{Minimize} \qquad f = c^T S + 0.5 S^T H S \qquad (31)$$

$$\text{Subject to} \quad A^T S \le b\ ,\ N^T S = e\ ,\ S \le 0 \qquad (32)$$

where

$$c^T = \left[\frac{\partial F\left[p,\varphi(p)\right]}{\partial p_1} \ \cdots \ \frac{\partial F\left[p,\varphi(p)\right]}{\partial p_i}\right] \qquad (33)$$

A is an $n \times l$ matrix of the gradient of the inequality constraint ($a_{ij} = \partial g_j(p)/\partial p_i$), N is an $n \times m$ matrix of the gradient of the equality constraint ($n_{ik} = \partial h_k(p)/\partial p_i$), H is an $n \times n$ approximate Hessian matrix of the Lagrange function, $b_j = -\left[g_j(p)\right]$ and $e_k = -\left[h_k(p)\right]$.

The QP subproblem that gives constraint correction can be developed by neglecting the first term of (31) and subject to the same constraints as (32). The solution to this subproblem gives a direction with the shortest distance to the constraint boundary from infeasible point. The subproblem for the objective reduction algorithm can be defined by setting the right-hand side vector e in (32) to be zero. The step size (α) can be calculated with the required reduction in objective which is based on a fractional reduction (γ) as $\alpha = |\gamma\ f|/|c \cdot S|$ [4].

IV. RESULTS

Magnetostatic Problem

The problem consists in determining the optimum shape of the poles of an electromagnet in order to maintain the magnetic flux density B constant in its air gap. Half of the electromagnet is shown in Fig.2a. The 2D model used in the analysis is given in Fig.2b. The 2D domain is made up of three main regions defined by $\Omega = \Omega_F + \Omega_C + \Omega_A$, where Ω_F, Ω_C and Ω_A represent the ferromagnetic, coil and air region, respectively. Dirichlet and Neumann boundary

conditions are imposed on Γ_0 and on Γ_1 respectively. The mesh used in the simulations consisting of 156 nodes and 263 elements is illustrated in Fig.3a. A zoom in the pole face showing the moving nodes is given in Fig.3b. The data used in the simulations are given as follows: $\mu = 1000\mu_0$ in Ω_F and $J = 1000\ \mathrm{A/cm^2}$ in Ω_C.

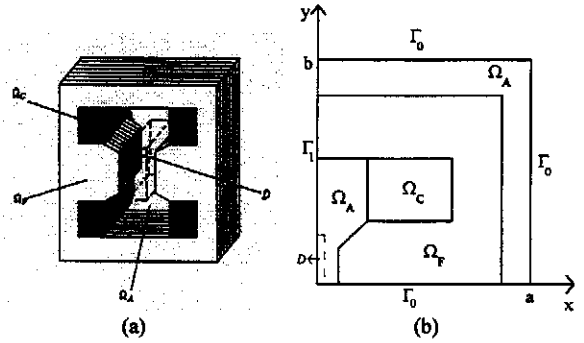

Fig. 2. Electromagnet

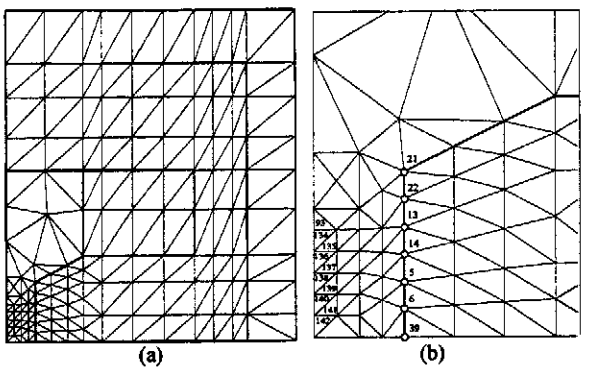

Fig.3 FE mesh used in the simulations

The aim of this problem is to find the optimum position of the moving nodes indicated in Fig.3b, the design variables p_i, which set up a shape in the pole of the electromagnet that insure a given constant flux density B_d over D (which includes 9 elements, also indicated in Fig.3b). Mathematically the problem can be defined as

$$\text{Minimize} \qquad F = \sum_{q=1}^{9} \left| B_{c_q} - B_d \right|^2 \tag{34}$$

Subject to

$$g_j(p) \equiv \frac{p_j^2 - \left(p_j^U + p_j^L\right)p_j + p_j^U p_j^L}{\left(p_j^U - p_j^L\right)^2} \le 0 \tag{35}$$

$$0.012 \le p_j \le 0.017 \ \text{m} \ ; \quad j = 1,\ldots,6 \tag{36}$$

This problem was also used to validate the sensitivity analysis. The comparison for the calculation of $\{\partial B/\partial p\}$ using the differentiation of the FE matrices and the FD method for node 5 is given in Table I.

Each component of the gradient vector ∇F is obtained as a summation of terms of the form

$$\frac{dF}{dp_j} = \frac{\partial F}{\partial p_j} + \frac{\partial F}{\partial |B_c|}\frac{\partial |B_c|}{\partial p_j} = 2\left(|B_c| - B_d\right)\frac{\partial |B_c|}{\partial p_j} \tag{37}$$

The term $\partial |B_c| / \partial p_j$ in (37) is calculated using the formulation presented in section II.

TABLE I

RESULTS CONCERNING THE CALCULATION OF $\{\partial B/\partial p\}$

	Node 5 $\{\partial B/\partial p\}$	
Element	FE	FD
93	-3.1506	-3.1437
134	-7.4744	-7.4641
135	-5.0445	-5.0147
136	-6.6736	-6.6680
137	-5.1984	-5.1649
138	-7.5811	-7.5815
139	-7.2837	-7.2755
140	-7.6979	-7.6616
141	-8.2650	-8.2737
142	-7.9274	-7.8926

The final result for $B_d = 0.35\text{T}$ is given in Table II. The number of function evaluations corresponds to the number of field solutions required in which FE stands for the direct differentiation of the FE matrices and FD for the finite difference method. The final shape obtained for the pole of the electromagnet is shown in Fig.4.

TABLE II

FINAL RESULT FOR $B_d = 0.35\text{T}$

Element	QUA $\lvert B_c \rvert$ (T)	ALM $\lvert B_c \rvert$ (T)	PLBA-CR $\lvert B_c \rvert$ (T)
134	0.3498	0.3510	0.3493
135	0.3479	0.3472	0.3483
136	0.3499	0.3492	0.3496
137	0.3518	0.3505	0.3522
138	0.3502	0.3501	0.3495
139	0.3514	0.3501	0.3507
140	0.3499	0.3507	0.3493
141	0.3507	0.3503	0.3495
142	0.3499	0.3509	0.3492
No.of function evaluations (FE)	742	642	151
No. of function evaluations (FD)	2474	2388	755

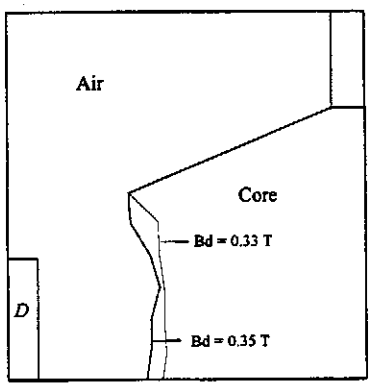

Fig.4 - Final shape obtained using the PLBA-CR method

Electrostatic Problem

The objective in this idealised problem is to reduce the E tangential at the top surface of an insulator to values lower than 16 KV/m (this is just an example). The two dimensional model investigated is given in Fig.5. The mesh used in the simulations consisting of 261 nodes and 494 elements is given in Fig.6. The insulator is made of porcelain ($\varepsilon_r = 7$) and is surrounded by air. Neumann boundary condition was imposed on Γ_1. The problem was reduced to find the optimum values of L_1 and the three arcs defined by R_1, R_2 and R_3 and their respective angles, θ_1,θ_2 and θ_3. The initial values for these variables are $R_1 = 4.0\text{mm}$, $R_2 = R_3 = 5.0\text{mm}$ and $\theta_1 = \theta_2 = \theta_3 = 90$.

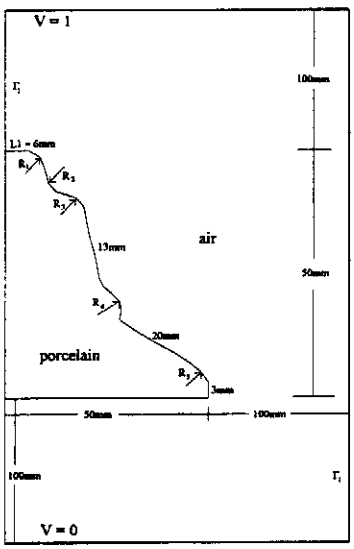

Fig.5 - Idealised insulator

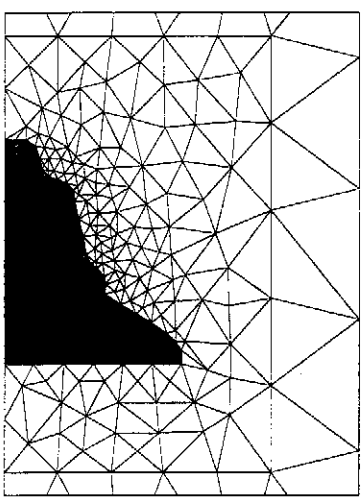

Fig.6 - Mesh used in the simulations

The objective function in this case is given by

$$\text{Minimize} \qquad F = \sum_{i=1}^{n} \left| E_{c_i} - E_{d_i} \right|^2 \qquad (38)$$

in which n is the number of test points where E will be calculated. Four points were considered, one in the middle of L_1 and the other three situated in the middle of the surface of the arcs 1,2 and 3. The design variables were subject to the following constraints $3 \le p \le 10mm$ for L_1; $3 \le p \le 10mm$ for R_1, R_2 and R_3; $0 \le p \le 90$ degrees for θ_1,θ_2 and θ_3. The initial and final shape of the insulator for the target defined are given in Fig.7. Table III gives the information concerning the optimization process.

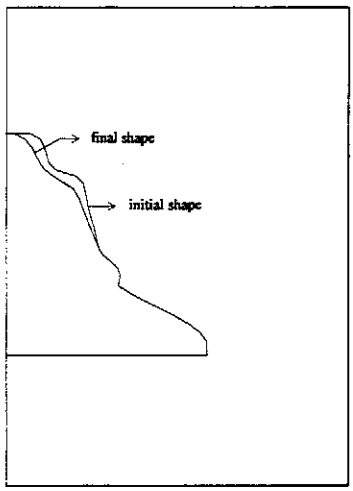

Fig.7 - Initial and final shape for the insulator

TABLE III
RESULTS CONCERNING THE ELECTROSTATIC PROBLEM

Variables	ALM	PLBA-CR
L_1 (mm)	3.0	3.25
R_1 (mm)	8.25	8.0
θ_1 (degrees)	61.94	64.12
R_2 (mm)	6.32	6.14
θ_2 (degrees)	53.10	55.10
R_3 (mm)	7.62	7.48
θ_3 (degrees)	43.63	45.74
No. of function evaluations (FE)	496	164

Time Harmonic Problem

The idealised problem considered here consisted in the determination of the optimum shape of a conducting cylinder of copper placed under the influence of a transverse time harmonic field. The objective is to obtain the magnetic flux density in the interior region of the cylinder at certain prescribed values, whilst keeping its area constant. The cylinder is assumed to be infinitely long in the z direction. One quarter of the 2D model is shown in Fig.8 (not to scale). The detail of the mesh used consisting of 264 nodes and 472 elements is shown in Fig.9.