Investigation on the Electric Field Inverse Problem of HV Transmission Lines and Discussion on Its Application

Fan Yang [1], Hao Wu[1], Wei He[1], Tao Chen [2] and Duan Nie [3]

[1] State Key Laboratory of Power Transmission Equipment & System Security and New Technology, College of Electrical Engineering, Chongqing University
Chongqing, 400044 China
yangfancqu@gmail.com

[2] Chongqing Electric Power Research Institute, Chongqing 401123, China
chentaocqep@yahoo.cn

[3] Chongqing Power Grid, Chongqing 400014, China
diduancq@yahoo.cn

***Abstract*—**The electric field inverse problem of high voltage transmission lines is investigated in this paper. Firstly the equations for the electric field forward problem of transmission lines were formulated, then equations for the inverse problem were formulated according to the forward problem and the least square theory. A method to solve the inverse problem is presented in the paper, in which the global regularization was used to process the ill-posed characteristic of the inverse problem. The Damped Gauss-Newton iterative method was used to search the optimum solution.

A 500kV double-circuit transmission line was taken as an example for the calculations of the inverse problem. The inverse problems for different situations were calculated, including the situation when the transmission lines were in normal operation, and the situation when the transmission lines were in faulty state. The solutions to the inverse problem with and without regularization were compared in the paper, and the results indicated that the global regularization can effectively eliminate the ill-pose of the inverse problem. The electric field inverse problem can be used to confirm the electrification state of the ultra high voltage transmission lines, and also can be used in the environmental assessment tests to reduce the measurement workload.

***Index Terms*—**Transmission line, electric field, inverse problem, global regularization, Damped Gauss Newton.

I. INTRODUCTION

The electromagnetic field in the vicinity of high-voltage (HV) transmission lines (TLs) were investigated comprehensively in the past two decades, including the measurements on the electric field, magnetic field, audible noise, and the calculations of the electric field and magnetic field produced by TLs [1-3]. For the calculations of the electric field and magnetic field produced by the HV TLs, besides the sizes of the TLs, the electrical parameters of TLs, such as potential, phase angle, should be given, then the electric field at the measuring points can be computed.

A. Forward Problem and Inverse Problem of TL Electric Field

The process in which the electric field is calculated by the given parameters of the source, is usually called the "forward problem". And according to the definition of the "forward problem", the electric field "inverse problem" of TLs is the process to compute the parameters of the source according to the electric field at the measuring points. Using the forward problem of the TLs, we can assess the environmental influence caused by the TLs, and through the calculation of the inverse problem, the parameters of the source can be confirmed.

B. Application of the Electric Field Inverse Problem of TLs

Recently the application of the electric field inverse problem has arisen more frequently in engineering, such as with the detection of the electrification state of the ultra-high-voltage (UHV) TLs, and the environmental assessment of TLs.

With the increase of installed capacity in China, it's necessary to develop UHV power transmission, ranging from a 750kV voltage rating to 1200kV. According to the safety operation rules, it must be confirmed whether the transmission lines are charged during equipment maintenances, which needs electroscopes for different voltage rating [9, 10]. With the increase of voltage rating, distance between transmission lines and ground grows. For instance, when the height of transmission lines of 750kV voltage rating may exceed 30m, it's not convenient to adopt the traditional insulating stick or insulating rope to detect electrification state any more. Using the electric field inverse problem in this paper, the potential and phase angle of the transmission lines can be calculated according to the measured electric field strength at the measuring points, then the electrification state of the TL can be confirmed.

In addition, assessments on the electromagnetic environment around the HV TLs have been carried out around the world. In the environmental assessment on the TLs, the electric and magnetic strength of a large number of points should be measured, which means a huge workload. In order to reduce the workload in the environmental assessment tests, the electric field inverse problem of the TLs can be used. That is, according to the measured electric field strength at some measuring points near the ground, the parameters of the field source can be calculated by solving the inverse problem, then with the obtained parameters of the TLs, the real electric field distribution around the transmission lines can be calculated. Hence some measurement workload can be avoided by using the inverse problem.

Therefore the solution to the electric field inverse problem of TLs is significant for the application of the electric field inverse problem.

And in the remaining parts of the paper the method to solve the inverse problem will be presented, which are organized as follows: Section II presents the way to set up the model of the inverse problem for the 500kV double-circuit TLs. The method to solve the inverse problem is described in Section III. Due to the ill-posed characteristic and the ill-conditioning characteristic of the solution to the inverse problem, the global regularization is used to deal with the ill-posed characteristic caused by the interference and errors. Damped Gauss-Newton (DGN) method is used to search the optimum solution to the inverse problem. Section IV shows some computational examples, and results indicate that the global regularization can effectively eliminate the ill-posed characteristic. Applications of the inverse problem for the electric field of TL in the practical engineering are presented in Section IV. Finally concluding remarks will be given in Section V.

II. MODEL AND EQUATION FOR THE ELECTRIC FIELD INVERSE PROBLEM OF TRANSMISSION LINE

A. Model Setup

A 500kV double-circuit three-phase TL was taken as an example. The cross section and the distribution of the measuring points are shown in Fig. 1. The parameters in the model are as follows: D_1=4.5m, D_2=5.5m, D_3=11.5m, H_1=11m, H_2=18m. H_3 is the distance between the measuring points and the ground, and H_3=1.8m. The equivalent radius (R') of the four-bundle conductor is 0.323m, the radius (r) of the sub-conductor is 0.0148m, and the radius of lightning conductor (R'_L) is 0.0054m. In the computation, the influence from the conductor sag was ignored.

B. Equation for the Electric Field Forward Problem

The forward problem is the process to compute the electric field at the measuring points according to the potential and phase angle of the TL. The charge simulation method (CSM) is the most frequently used method according to the characteristics of the electric field around the TL [1-3].

According to the principle of CSM, line simulation charges were used to simulate the free charge in the TLs and lightning lines. And the simulation charge was located at the center of the conductor and lightning conductor, and each conductor was simulated with one line simulation charge. Set the count of the simulation charges as M, for the model shown in Fig. 1 M = 8. Set the count of matching points in the model as N, hence M=N. According to the Maxwell equations, the

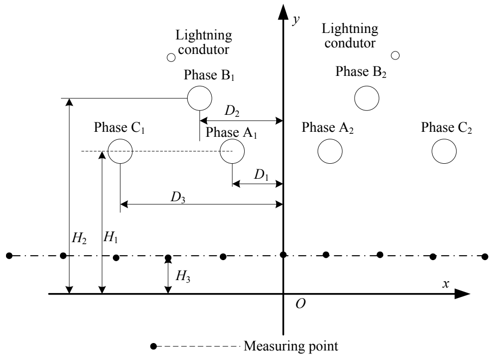


Fig. 1. Schematic diagram of cross section of the 500KV double-circuit three-phase transmission line.

size of the simulation charges could be obtained according to (1), then the electric field at any measuring point could be computed according to (2).

$$[p][Q]=[\varphi] \tag{1}$$

$$\begin{aligned}[f_x][Q]&=[E_x]\\ [f_y][Q]&=[E_y]\\ [f_z][Q]&=[E_z]\\ [E]&=\sqrt{E_x^2+E_y^2+E_z^2}\end{aligned} \tag{2}$$

where [p] is the potential coefficient between the simulation charges and matching points. [Q] is the size of the simulation charges, and [φ] is the potential at the matching points. [f_x], [f_y], [f_z] are the electric field strength coefficient between the simulation charges and measuring points along x, y, z axis respectively. [E_x], [E_y], [E_z] are the corresponding electric field strength, and [E] is the total electric field strength at the measuring points.

C. Equation for the Electric Field Inverse Problem

The inverse problem is the process to calculate the potential and phase angle of the TLs according to the electric field at the measuring points.

Similar to the calculation of the electric field forward problem of the TLs, the CSM was used and the size of the simulation charges was computed first. However, different from the calculation of the forward problem, the size of the simulation charges was computed according to (2) in the inverse problem, instead of (1) used in the forward problem.

Hence anew problem rises up. For equation (2), we can not find its solution directly, so the equation described by (2) must be converted. The least squares theory has been proved to be an effective way to solve question similar with (2) [4, 5], then the solution to the inverse problem can be converted into an extreme value problem. Suppose the nonlinear operator of the forward problem is $F(q)$, here q is the size of the simulation charges, and then the nonlinear inverse problem described by (2) can be rewritten as follows:

$$E=F(q)\,. \tag{3}$$

The corresponding least square equation is as follows:

$$\min_q \left\|E-F(q)\right\|^2 \tag{4}$$

where E is the measured electric field strength at the measuring points. For the model shown in Fig. 1, the count of the simulation charges is 8 (M=8), and the count of the measuring points is usually confirmed according to the engineering requirements.

To compute the real potential and phase angle of the TL using the electric field inverse problem, the more measuring points are set, the more real information of the TL can be obtained. Therefore the count of measuring points N is often larger than M for the model shown in Fig. 1. So (1) is a nonlinear over-determined equation.

III. SOLUTION TO THE ELECTRIC FIELD INVERSE PROBLEM

A. Least Square Solution to the Nonlinear Inverse Problem

For the nonlinear least squares problem shown in (3), the piece-wise linearization iterative algorithm can be used [6, 7]. In order to realize the approximation of the nonlinear operator F, it is necessary to consider the Taylor expansion of $F(q+\delta q)$ at q:

$$F(q+\delta q)=F(q)+\frac{\partial F}{\partial q}\delta q+\frac{1}{2}(\frac{\partial^2 F}{\partial q^2})(\delta q)^2 + R(q,\delta q)\,, \tag{5}$$

where $R(q, \delta q)$ is a remainder term. When $\|\delta q\|$ is small enough, as to the right side of equation (5), the high-order terms can be ignored and the right side can be approximated with the first approximation, then

$$F(q+\delta q)\approx F(q)+\frac{\partial F}{\partial q}\delta q\,. \tag{6}$$

Therefore, the linear relationship between $\delta E = F(q+\delta q) - F(q)$ and δq can be obtained as follows:

$$\delta E \approx \frac{\partial F}{\partial q}\delta q \ . \tag{7}$$

Suppose $q^*=q+\delta q$ is the exact solution to equation (3). That is, $F(q^*)=E$, here E is the electric field strength at the measuring points. Then, at point q close to q^*, according to equation (6), the linear operator equation about δq can be obtained as follows:

$$E - F(q) \approx \frac{\partial F}{\partial q}\delta q \, . \tag{8}$$

The linear operator equation (6) constitutes the basis of the Newton-type iterative solution method of the nonlinear equations.

For the least squares problem shown as equation (4), if q is the optimum solution, the necessary condition is that the gradient of q ($g(q)$) at q is zero. Suppose the objective function for (1) is Φ, that is, $\Phi=\| E-F(q) \|^2$. Then, the following can be reformulated:

$$\begin{aligned} g(q) = \nabla \phi = [F(q) - E]\frac{\partial F(q)}{\partial q} \\ = J^T(q)[F(q)-E] = 0 \end{aligned} \ , \tag{9}$$

where J is the Jacobean matrix corresponding to $[F(q)\text{-}E]$.

Equation (9) is the normal equation of the nonlinear least square problem. Therefore, the solution to the nonlinear least square problem can be transferred into the solution to the normal equation. But the normal equation is still a nonlinear equation. In order to resolve this problem, the Hessian matrix can be also introduced here [7].

B. Global Regularization and Damped Gauss-Newton Method

The electric field inverse problem of TLs is the process to compute the potential and phase angle according to the electric field at the measuring points. Because the electric field at the measuring points is the measured value, interference maybe comes up, which will result in a wrong solution to the inverse problem; this is the so-called ill-posed characteristic of the solution to the inverse problem. To reduce the ill-posed of the inverse problem, regularization is frequently used in the solution to the inverse problem.

The linear equations (9) obtained from the linearization of the nonlinear equation will inherit the ill-conditioning characteristic of the original equation, so it is necessary to introduce the regularization. Regularizing the linear equation $E - F(q^{(k)}) \approx \frac{\partial F}{\partial q^{(k)}}\delta q$ corresponding to equation (8), then the following optimization problem can be obtained.

$$\min\left\{ \left\| E - F(q^{(k)}) - \frac{\partial F}{\partial q^{(k)}}\delta q \right\|^2 + \alpha \left\| W(\delta q) \right\|^2 \right\} \ , \tag{10}$$

where W is a linear operator. If $W=I$, the regularization will be imposed on δq. This is the "crawling method", and it has the following disadvantages: 1) it can not control the characteristics of all the solutions. Because the regularization is imposed on δq, rather than q. 2) The solution q^* depends on the initial solution $q(0)$ and the minimum path $\delta q(k)$, k=1, 2, …, N, where N is the iterative times. 3) If different methods are used to calculate the value of $\delta q(k)$, different q^* will be obtained.

In order to overcome the disadvantages of the "crawling method", in this paper, the following global regularization was adopted.

$$\min\left\{ \left\| E - F(q^{(k)}) - \frac{\partial F}{\partial q^{(k)}}\delta q \right\|^2 + \alpha \left\| W(q^{(k)} + \delta q) \right\|^2 \right\} . \tag{11}$$

In the so-called global regularization, the regularization is imposed on q, so the obtained step size may not be small. It is actually the linearization of the following optimization problem.

$$\min\left\{ \left\| E - F(q) \right\|^2 + \alpha \left\| W(q) \right\|^2 \right\} . \tag{12}$$

Equation (12) is the regularization of the original nonlinear inverse problem.

C. Damped Gauss-Newton Iterative Method

To ensure the objective function value for (12) diminish in the next iteration, the Damped Gauss-Newton iterative method was used in this paper. In the DGN, the δq is replaced with $\omega\delta q$ ($0.1<\omega<0.5$) to make the δq small enough, which make the objective function value for (12) diminish in the next iteration.

To ensure the objective function value for (12) diminish in the next iteration, the Damped Gauss-

Newton (DGN) iterative method was used in this paper. In the DGN, the δq is replaced with $\omega\delta q$ ($0.1<\omega<0.5$) to make the δq small enough, which make the objective function value for (12) diminish in the next iteration.

IV. CALCULATION EXAMPLES

To verify the validation of the method presented in the method, model shown in Fig. 1 was taken as an example. To get more real information of the TL, 11 points are laid out along a horizontal line with a height of 1.8m from the ground as shown in Fig.1. The distance between two adjacent measuring points is 5m.

For the solution to the electric field inverse problem of the TL, the measured electric field strength at the 11 measuring points, which may contain interference, should be given first. To verify the validation of the method proposed in this paper, a set of simulated measured electric field strength at the measuring points was offered in the following way. Firstly according to the computation method of the forward problem, the electric field strength at all the measuring points was calculated with the parameters when the TLs were in normal condition, which could be taken as the standard value of the electric field strength at the measuring points. Add a random error data with ±10% (Note: the errors of the symmetric points are equal) to the standard values of the electric field strength at all the measuring points, and the sum could be taken as the simulated measured values of the electric field strength.
Calculations on the electric field inverse problem of the TL at different situation were carried out in the paper, such as the TLs were in normal operation, or in faulty state respectively.

Table 1. Solution without regularization.

Conductor	A_1	B_1	C_1	A_2	B_2	C_2
Potential (10^5V/m)	2.516	2.449	1.593	1.121	2.494	3.024
Phase angle	116.3	186.1	242.1	31.9	83.0	237.6

Table 2. Solution with regularization.

Conductor	A_1	B_1	C_1	A_2	B_2	C_2
Potential (10^5V/m)	3.016	3.037	3.031	2.997	3.042	3.023
Phase angle	0.7	121.0	240.0	0	120.2	239.7

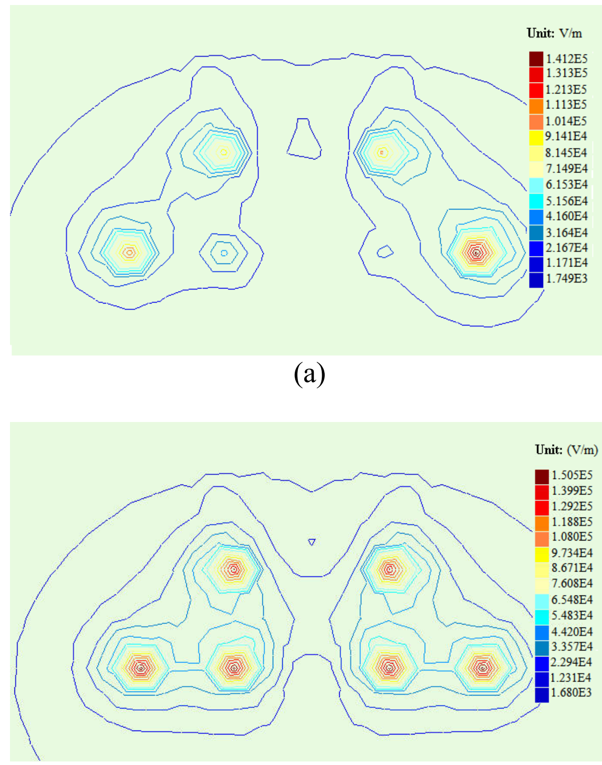
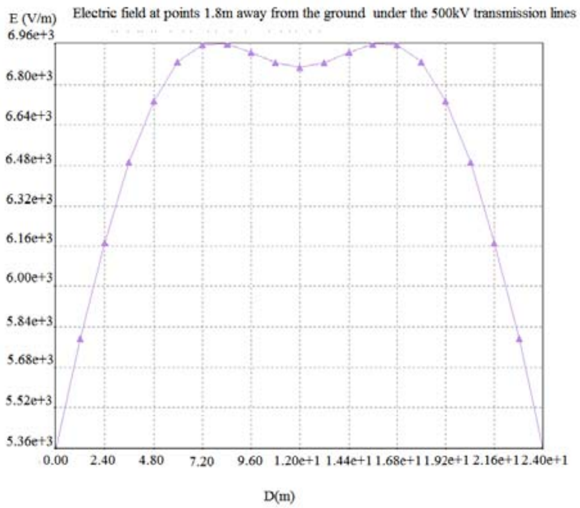


Fig. 2. Electric field distribution in the vicinity of 500kV double-circuit TLs for normal operation situation, (a) Without regularization, (b) With regularization.

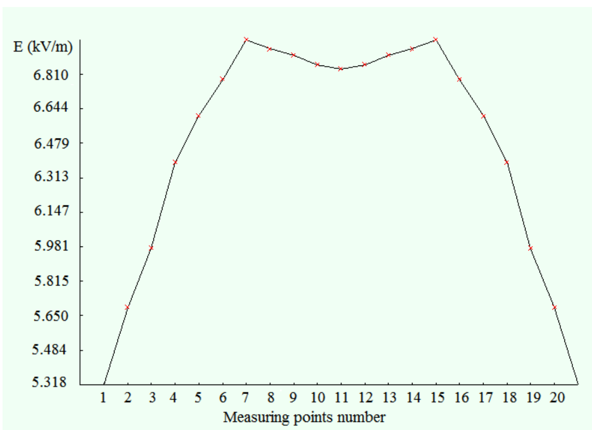
A. Calculation for Normal Operation

When the TL shown in Fig. 1 was in normal operation, the parameters of the TL were as follows: phase potential was 303kV, and the phase angle from A1 to C1 was 0º, 120º, 240º. Hence the simulated electric field strengths at the 11 measuring points were as follows: 2.680 kV/m, 4.718 kV/m, 7.214 kV/m, 7.250 kV/m, 8.954 kV/m, 10.706 kV/m, 8.954 kV/m, 7.250 kV/m, 7.214 kV/m , 4.718 kV/m, 2.680 kV/m. As to the lightning conductor, both the boundary potential and phase were zero.

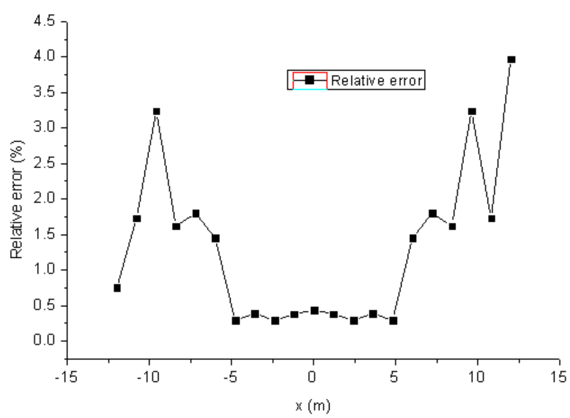
Table 1 and Fig. 2 (a) are the computational results when the global regularization was not adopted. Table 2 and Fig. 2 (b) are the computational results when the global regularization was adopted. It can be seen from Table 1 and 2 that, when the regularization was not used, the deviations of the computational results are relatively large, which indicate that the



(a)

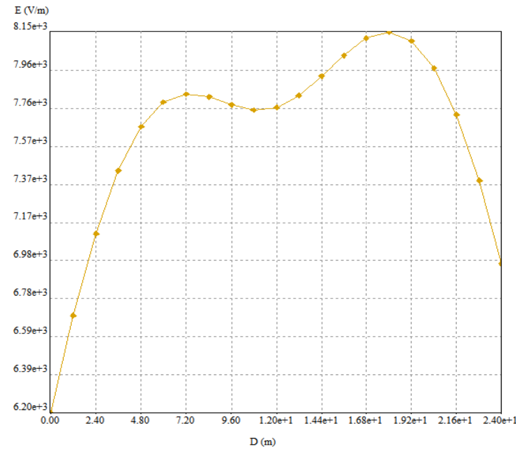


(b)

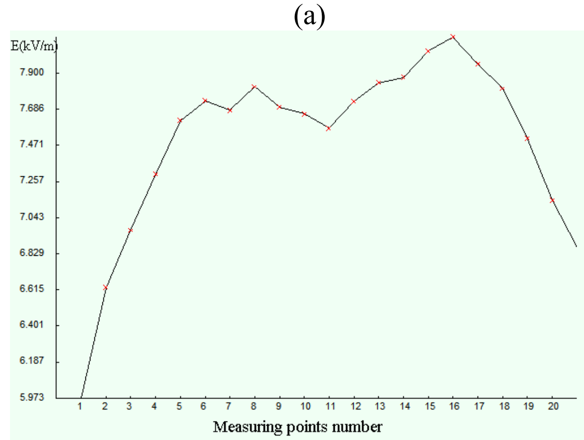


(c)

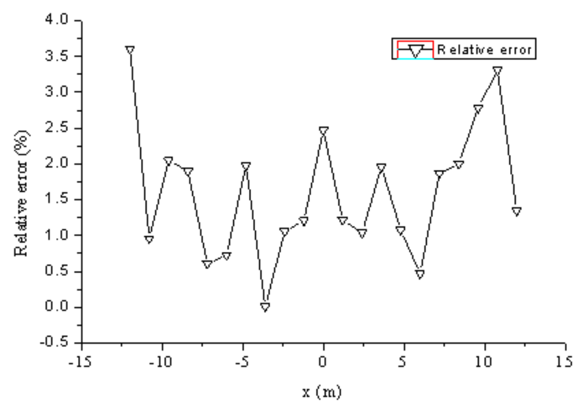
Fig. 3. Comparison between the results obtained by (a) the IES-Electro, (b) the inverse problem, and (c) the relative error.



(a)



(b)



(c)

Fig. 4. Electric field at measuring points obtained by (a) IES-Electro, (b) the inverse problem, and (c) the relative error.

global regularization can effectively eliminate the influence caused by the interference in the measurement.

In order to verify the accuracy of the method used to solve the inverse problem, further calculations were carried out. In the model shown in Fig. 1, along the horizontal line with a height of 1.8m from the ground, 21 points from left to right are selected for computation. The distance between two adjacent measuring points is 1.2m. The electric field strength at the 21 measuring points was computed respectively with the IES-Electro software and the inverse problem method proposed in this paper, the IES-Electro software is a BEM based software [8]. Fig. 3 shows the comparison between the results obtained by method in this paper and IES-Electro. It can be seen from the computation results that the maximum relative error is 3.97%, which is within the allowed error range. Hence the accuracy of the method to solve the electric field inverse problem of the TL was verified.

B. Calculation for Faulty State

Suppose that line of phase C1 was in phase anomaly state, of which the phase angle was 120º, and the line of phase B1 was short-circuit. Hence the simulated measured electric field strength at the 11 measuring points was computed, and the following was its value: 3.724 kV/m, 6.004 kV/m, 8.726 kV/m, 9.182 kV/m,10.475 kV/m, 11.556 kV/m, 9.313 kV/m, 7.263 kV/m, 7.186 kV/m, 4.684 kV/m, and 2.62 kV/m.

In order to better verify the accuracy of the solution to the inverse problem, the electric field strength at the 21 measuring points were computed with the IES-Electro and method in this paper respectively, and the computation results are shown in Fig. 4, including the actual electric field strength and the relative error. It can be seen from the computational results that the maximum relative error is 3.61%, which is within the allowed error range. Hence when the TLs were not in normal operation state, the method in the paper is still can solve the inverse problem correctly.

V. CONCLUSIONS

The electric field inverse problem of transmission lines was investigated in this paper. First the equations for the electric field forward problem were formulated, and equations for the inverse problem were formulated according to the forward problem and the least square theory. A method to solve the inverse problem was presented in the paper, in which the global regularization was used to process the ill-posed characteristic of solution to the inverse problem. And the DGN iterative method was used to search the optimum solution.

A 500kV double-circuit transmission line was taken as an example for the calculation of the inverse problem. The inverse problems for different situations were calculated, including the situation when the transmission lines were in normal operation, and the situation when the transmission lines were in faulty state. Solutions to the inverse problem with regularization and without regularization were compared, and results indicate that the global regularization can effectively eliminate the ill-pose of the inverse problem.

Application of the electric field inverse problem was discussed in the paper. It could be used to confirm the electrification state of the UHV TLs, and also could be used in the environmental assessment tests to reduce the measurement workload.

ACKNOWLEDGEMENT

This study is supported by National ‘111’ project (B08036), the State Key Laboratory project (2007DA10512709102) and the National College Students Innovation project (091061106). The authors wish to acknowledge the support provided by the Chinese NSF and Chongqing Science & Technology Commission.

REFERENCES

[1] B. Florkowska, A. Jackowicz-Korczynski, and M. Timler, “Analysis of electric field distribution around the high-voltage overhead transmission lines with an ADSS fiber-optic cable”, *IEEE Transactions on Power Delivery*, vol.19, no.3, pp. 1183-1189, 2004.

[2] Y. Liu, L.E. Zaffanella, “Calculation of electric field and audible noise from transmission lines with non-parallel conductors”, *IEEE Transactions on Power Delivery*, vol.11, no.3, pp. 1492-1497, 1996.

[3] D. Lee, K.-Y. Shin, and S.-D. Lee, “Technique to decrease the electric field intensity on conductor surface using the asymmetrical-sized conductor bundle”, *IEEE/PES Transmission and Distribution Conference*, vol. 4, pp. 1-6, 2008.

[4] E. Coccorese, R. Martone, and F.C. Morabito, “A neural network approach for the solution of electric and magnetic inverse problems”,

IEEE Transactions on Magnetics, vol. 30, no. 5, pp. 2829-2839, 1994.

[5] M.A. Hussain, B. Noble, and B. Becker, “Computer simulation of an inverse problem for electric current computed tomography using a uniform triangular discretization”, *The Annual International Conference of the IEEE Engineering in Engineering in Medicine and Biology Society*, pp. 448-450, 1989.

[6] M. Trlep, A. Hamler, B. Hribernik, "The use of DRM for inverse problems of Poisson's equation", *IEEE Translations On Magnetics*, vol.36, no.4, pp. 1649-1652, 2000.

[7] H. Kama, and Z. Xiang, *The inverse problem in Electromagnetic field and its application,* The Science Press, 2005.

[8] http://www.integratedsoft.com/

[9] IEEE Standard 1127-1990, “IEEE guide for the design, construction, and operation of safe and reliable substations for environmental acceptance”.

Fan Yang is with the College of Electrical Engineering, Chongqing University. His main research field is regarding the numerical calc-ulation of electromagnetic fields and applications in power systems. He received his Ph.D degree of electrical engineering from Chongqing University in 2008.

Hao Wu is studying for her Bachelor’s degree in the College of Electrical Engineering, Chongqing University. Her main interests include the design of algorithms and programming.

Wei He is with the College of Electrical Engineering, Chongqing University. His main research field is regarding the electromagnetic field compatibility and fault detection of electric apparatuses.

Tao Chen is with the Chongqing Electric Power Research Institute. Her main research field is regarding the electromagnetic field compatibility of power systems. She received her Ph.D degree in 2006 at Chongqing University.

Duan Nie is with the Chongqing Power Grid. Her interests include the fault detection of high-voltage equipments.