Adaptable Nonstandard FDTD Schemes for the Precise Evaluation of Electrostatic Fields

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***Abstract*—The reduction of the total computational overhead in the design of complex geometries, such as modern aircrafts, is a very challenging problem, particularly when electrostatic fields (ESF) for lightning protection, are considered. To this aim, an efficient ESF evaluation scheme, based on the nonstandard finite-difference time-domain (NS-FDTD) method, is proposed. Combining the total-field/scattered-field (TF/SF) concept with a distinct sine-wave form, the novel technique cancels the accumulative errors caused by the static field component. Numerical results reveal that the featured method enables the use of high-frequency discretization models to ESF problems, with notable accuracy and seriously decreased design costs.**

***Keywords*—FDTD methods, nonstandard (NS)-FDTD techniques, numerical analysis, radar cross section.**

I. INTRODUCTION

There are many electric evaluation issues when it comes to the design of an aircraft. Amid them, lightning protection is a crucial safety factor [1]-[3]. So, to avoid the risk of lightning strikes, the electrostatic field (ESF) strength around airframe must be analyzed. Typically, the finite difference technique or the finite element method are employed for the static analysis [4], while, physical optics and the geometrical theory of diffraction [5] treat high frequency scenarios. On the other hand, for medium frequencies, the finite-difference time domain (FDTD) algorithm could be a solid choice [6], [7]. Thus, for wideband designs, several of the prior options should be combined, leading to rather inefficient implementations, explicitly for 3-D real-world applications. Not to mention that such combinations usually lead to excessively large numerical calculation models, i.e. discretized lattices, that can hardly be handled, even by contemporary computational systems.

In this paper, we present a rigorous ESF calculation technique via the 3-D NS-FDTD method, that is a high frequency analysis tool, to treat all the electrical design requirements of an aircraft. However, the NS-FDTD method is applicable to only one preset wave (and not a dc) [8]. Hence, we introduce an adaptable procedure utilizing the total-field/scattered-field (TF/SF) separation model [7] as a source of the ESF and a tuned input wave form which cancels the accumulative errors generated by the dc component. Our algorithm is validated via the analysis of a perfectly electric conductor (PEC) sphere [9]. It is shown that the proposed method can successfully extend the application of high-frequency discretized grids to ESF computations, reducing significantly the system resource costs in the electrical design of realistic problems.

II. THE ESF EVALUATION METHODOLOGY

A. Incorporation of the Input Wave Form

To embody the NS-FDTD method in the ESF analysis, we adopt a modified sine-wave form with a dc component as:

$$\begin{cases} E^{inc}(t,\boldsymbol{r}) = 0 , & \omega t - \boldsymbol{k}_{\text{num}}\cdot\boldsymbol{r} \le 0 \\ E^{inc}(t,\boldsymbol{r}) = -1+\cos(\omega t - \boldsymbol{k}_{\text{num}}\cdot\boldsymbol{r}), & 0 < \omega t - \boldsymbol{k}_{\text{num}}\cdot\boldsymbol{r} \le \omega T_{\text{w}} \\ E^{inc}(t,\boldsymbol{r}) = 1-\cos(\omega t - \boldsymbol{k}_{\text{num}}\cdot\boldsymbol{r}), & \omega T_{\text{w}} < \omega t - \boldsymbol{k}_{\text{num}}\cdot\boldsymbol{r} \end{cases} \quad (1)$$

where t is the time, ω the angular frequency, $\boldsymbol{r}$ the TF/SF separation position vector, and $\boldsymbol{k}_{\text{num}}$ the numerical wavenumber. Furthermore, $T_w = m(2\pi/\omega)$ is set long enough for the incident wave E^{inc} to fully cover the TF/SF separation area, with m a natural number. The second and third branch in (1) correspond to $E_{\text{dc}} = -1$ and $E_{\text{dc}} = 1$ in the time average, respectively. In fact, the second E^{inc} term is the prior calculation part for the accumulative error owing to the dc-component in $0 < t \le 2T_w$. Its error is used to cancel the corresponding error in the third E^{inc} term of (1). The time diagram of E^{inc}, during the aforementioned procedure, is described in Fig. 1, which indicates that the ESF observation is performed after the period of $2T_w$, via:

$$E^{\text{static}} = \frac{1}{T}\int_0^T E(t)\,dt \quad \text{with} \quad T = 2\pi/\omega . \quad (2)$$

So, the total amount of calculation time-steps in our scheme is $(2T_w + T)/\Delta t$, unlike in the usual pulse response analysis where end time-steps can not be set beforehand. Additionally, the use of the TF/SF separation model, as a field source, permits the generation of along arbitrary directions.

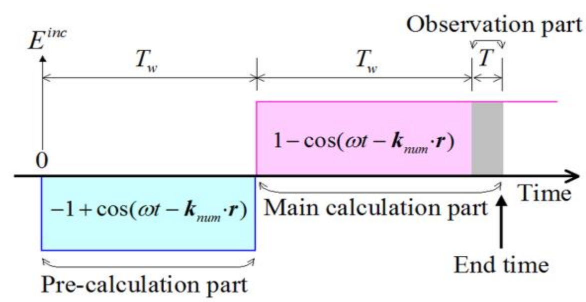

Fig. 1. Time diagram of the E^{inc} field for the proposed ESF evaluation.

B. Verification of the ESF Calculation Scheme

The theoretical solution for a PEC sphere, of radius a and a smooth surface, placed in a uniform ESF, is given by [9]:

$$E(r,\theta) = E_0\left(1+\frac{2a^3}{r^3}\right)\cos\theta \quad \text{for} \quad r \ge a , \quad (3)$$

where E_0 is the applied ESF, r the distance from the center of the sphere, and θ the angle in spherical coordinates. Note that the electric potential of the sphere is 0. Using (3), we examine the ability of our scheme to treat the specific problem. The sphere is discretized into cubic cells of width Δ and is set at the center of TF/SF separation area. Moreover,

Submitted On: September 1, 2020
Accepted On: September 5, 2020
https://doi.org/10.47037/2020.ACES.J.351104
1054-4887 © ACES

the incident field of (1) propagates along the x axis with $\theta = 90^o$, $\varphi = 0^o$, $T_w = 3600\Delta t$, and $\Delta = \lambda/10$, with $\lambda = 1$ m and $\Delta t = (\omega/2\pi)/36$. In this context, Fig. 2 illustrates the behavior of the E_z component, indicating that our ESF evaluation scheme, combined with the NS-FDTD method, is in excellent agreement with both the FDTD technique and the theoretical solution of (3), for $E(a,0) = 3$ V/m. Moreover, the NS-FDTD approach is more accurate than the FDTD one in the vicinity of the sphere surface. This fact proves the superiority of the new formulation over existing methods, leading to the conclusion that the featured ESF evaluation scheme can be deemed a powerful tool for such problems.

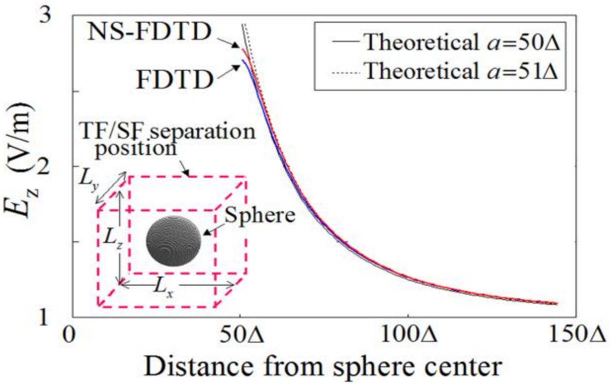

Fig. 2. Variation of the ESF (E_z) component vs. distance (along the z-axis) from the sphere center, derived by various methods. The applied field, with $E_0 = 1$ V/m, is parallel to z axis. The theoretical values via (3) are shown for the cases of a and $a + \Delta$, since a discretized cell model has an ambiguity of one Δ in the FDTD algorithm. The TF/SF separation area size is $L_{x,y,z} = 290\Delta$.

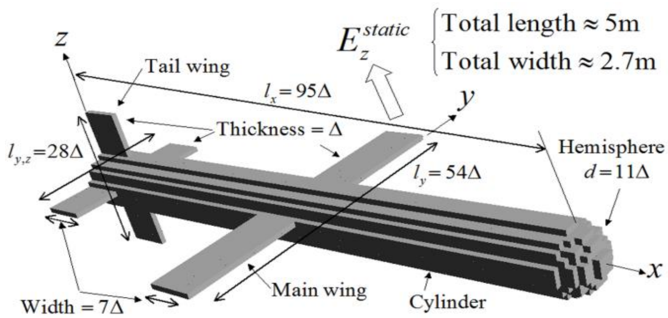

Fig. 3. A realistic aircraft model discretized by 9449 cubic PEC cells. The tail wing is placed at the center of the cylinder, while the main wing position is one Δ beyond the tail wing towards the z direction. The cylinder lies along the x axis and the main wing along the y axis.

III. Application to a Realistic Aircraft Design

Next, we apply our method to the ESF analysis of a real-world aircraft model, as depicted in Fig. 3. All lattice cells are PEC cubes with a width of $\Delta = \lambda/20$ and $\lambda = 1$ m, whereas the applied ESF magnitude is $E_z^{\text{static}} = 1$ V/m. In this problem, we consider a TF/SF separation area with $L_x = 210\Delta$ and $L_{y,z} = 160\Delta$, illuminated by an incident field traveling along the x direction through (1) for $T_w = 7200\Delta t$ and $\Delta t = (\omega/2\pi)/72$. Numerical results are given in Fig. 4, together with the FDTD ones, for the sake of comparison. A very satisfactory coincidence may be promptly detected, while the demanding ESF concentration on the wingtips is simulated very well [10]. Also, as noted in Fig. 2, the NS-FDTD technique is more accurate than FDTD one, since $E_z^{\text{NS-FDTD}} > E_z^{\text{FDTD}}$. From this fact, it is deduced that we can successfully obtain the ESF solution by means of our evaluation method using the high-frequency discretized cell model directly in the 3-D NS-FDTD method. Therefore, the proposed algorithm can drastically reduce the total computational cost of the design process, even for rather complex applications, such as the static field analysis of various aircrafts, with electrically large size, arbitrary shapes, ad diverse material properties.

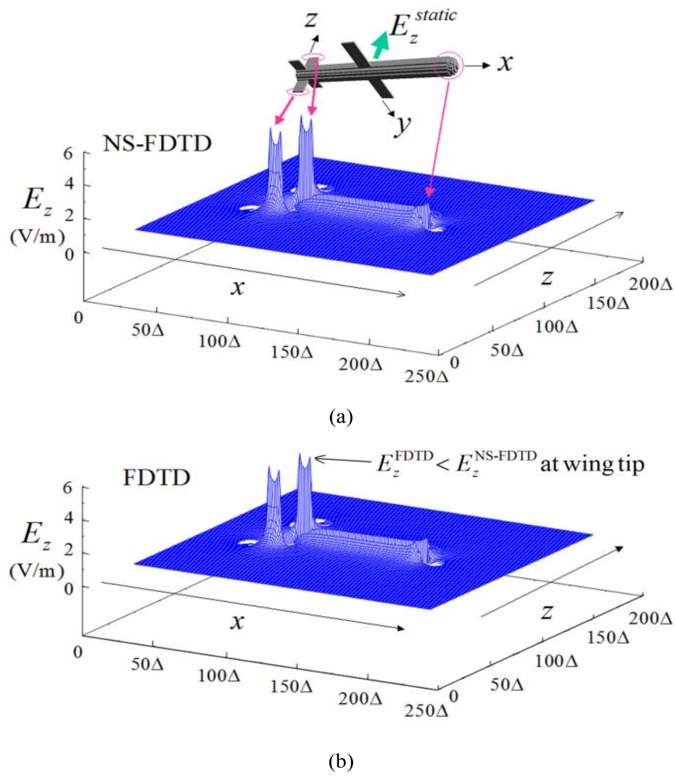

Fig. 4. Distribution of the ESF (E_z) magnitude along the central vertical cross-section of the aircraft, given in Fig. 3, calculated in terms of (a) the NS-FDTD and (b) the FDTD method.

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