The Diffraction by the Half-plane with the Fractional Boundary Condition

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***Abstract* —** In this article, there is considered the electromagnetic plane wave diffraction by the half-plane with fractional boundary conditions. As a mathematical tool, the fractional calculus is used. The theoretical part is given based on which the near field, Poynting vector and energy density distribution are calculated. Interesting results are obtained for the fractional order between marginal values, which describes a new type of material with new properties. The results are analyzed.

***Index Terms* —** Electromagnetic waves, half-plane, integral boundary conditions.

I. INTRODUCTION

The problem investigated in the study is a new approach to the diffraction problem including a half-plane surface and the plane wave as an incidence wave. The new method is called as the fractional derivative method (FDM). The method yields to explain the continuous intermediate stages of the two canonical states of the electromagnetic field. The first studies related to the fractional approach for the electromagnetic theory and its applications are investigated by Engheta [1-3]. Then, Veliyev developed the idea for the boundary condition which is called the fractional boundary condition (FBC) [4-6].

Fractional boundary condition describes the intermediate boundary condition between Dirichlet and Neumann boundary conditions. When the fractional order $\nu = 0$, this boundary condition corresponds to the perfect electric conductor (PEC). When $\nu = 1$, it stands for the perfect magnetic conductor (PMC). For fractional order between $0 < \nu < 1$, FBC corresponds to the intermediate case between the PEC and PMC [5, 6]. The solution to this problem for the marginal values of the fractional order is given in the book [7].

In this article, the fractional boundary condition is applied to the half-plane surface which describes the new type of material with interesting properties.

II. THE FORMULATION OF THE PROBLEM

In the formulation of the problem section, the main theoretical background is highlighted. In Fig. 1, the geometry of the problem is given.

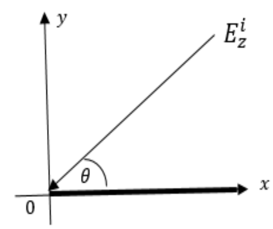

Fig. 1. The geometry of the problem.

The incidence E-polarized electromagnetic wave can be denoted as $\vec{E}_z^i = e^{-ik(x\cos\theta + y\sin\theta)}\hat{e}_z$. Note that, the time dependency is $e^{-i\omega t}$, θ is the angle of incidence and $\hat{e}_z$ is the unit vector in the z-direction. The incidence wave is scattered by a half-plane located at $y = 0$, $x \epsilon [0, \infty)$. The total electric field $\vec{E}_z = \vec{E}_z^i + \vec{E}_z^s$ must satisfy the fractional boundary condition [6, 8]:

$$D_{ky}^{\nu} E_z(x, y) = 0, \quad y \to \pm 0, \quad x > 0. \tag{1}$$

Here, D_{ky}^{ν} denotes the operator of the fractional derivative. $\nu \in [0,1]$. The value $\nu = 0$ corresponds to the perfect electric conductor and the value $\nu = 1$ corresponds to the perfect magnetic conductor. The expression for the scattered field is:

$$E_z^s(x, y) = -i\frac{e^{\pm i\pi\frac{\nu}{2}}}{4\pi}\int_{-\infty}^{\infty} F^{1-\nu}(q)e^{ik\left(xq+|y|\sqrt{1-q^2}\right)}(1-q^2)^{\frac{\nu-1}{2}}dq, \tag{2}$$

where, $F^{1-\nu}(q)$ is the Fourier transform of the fractional current density $\tilde{f}^{1-\nu}(\xi)$:

$$F^{1-\nu}(q) = \int_{-\infty}^{\infty} \tilde{f}^{1-\nu}(\xi)e^{-ikq\xi}d\xi. \tag{3}$$

The fractional current density is expanded as orthogonal series by Laguerre polynomials with unknown coefficients f_n^{ν}:

$$\tilde{f}^{1-\nu}\left(\frac{\varsigma}{k}\right) = e^{-\varsigma}\varsigma^{\nu-\frac{1}{2}}\sum_{n=0}^{\infty} f_n^{\nu} L_n^{\nu-\frac{1}{2}}(2\varsigma), \varsigma = kx. \tag{4}$$

After some mathematical operations the problem is reduced to the solution of the linear algebraic equation system (SLAE):

$$\sum_{n=0}^{\infty} f_n^{\nu} C_{nm}^{\nu} = B_m^{\nu}, \tag{5}$$

where,

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$$C_{nm}^{\nu} = \frac{1}{k}\gamma_n^{\nu}(-1)^{n+m}\int_{-\infty}^{\infty}\frac{(1-iq)^{n-m-\nu-\frac{1}{2}}}{(1+iq)^{n-m+\nu+\frac{1}{2}}}(1-q^2)^{\nu-\frac{1}{2}}dq,$$

$$B_m^{\nu} = -4\pi e^{i\frac{\pi}{2}(1-3\nu)}sin^{\nu}(\theta)\frac{(icos(\theta)-1)^m}{(icos(\theta)+1)^{\nu+m+\frac{1}{2}}}.$$

After solving this SLAE, unknown coefficients f_n^{ν} is determined. This gives the ability to find the fractional current density $\tilde{f}^{1-\nu}$ with (4) and its Fourier transform with the (3) [8]. Then, the near scattered electric field distribution can be found with (2). Before ending the theoretical part, it is needed to mention that there exists the relationship between the fractional order and the impedance value for normal incidence (η) which is $\eta = -i \times tan\left(\frac{\pi}{2}\nu\right)$ [4].

III. RESULTS OF THE NUMERICAL SIMULATION

Based on the above mentioned mathematical algorithm the program package was created in MatLab which gives an ability to calculate near field distribution, Poynting vector distribution and Energy density distribution. Below there are given the obtained results and their analysis.

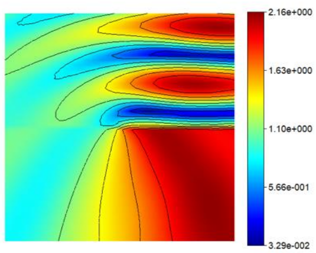
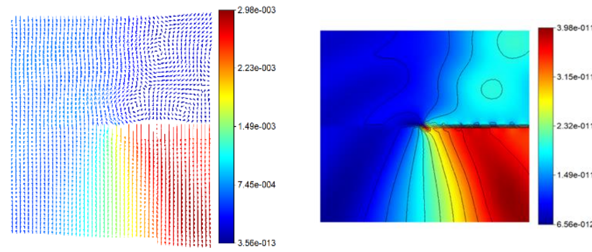


Fig. 2. Near E_z field distribution calculated with FDM at $\nu = 0.5, \theta = \frac{\pi}{2}$.

Figure 2 shows the near field distribution when fractional order, $\nu = 0.5$ as we see under the half-plane, the high field values are observed instead of shadow. This is not an ordinary behavior of the material. It seems that such material works like a capacitor for electromagnetic waves. It accumulates the energy and then radiates it below (this is also clearly seen in Fig. 3 and Fig. 4. Such behavior is known for resonators but usual resonators need a more complex structure. Also, antennas have such behavior when they direct energy in a certain direction. Figure 3 (a) shows the Poynting vector distribution. Here, we see that, in reality, the energy flow is in the lower part of the half-plane. This proves the idea given in the description of Fig. 3. Figure 3 (b) shows the energy density distribution and here also most of the energy is given in the lower part of the half-plane.

Figure 4 (a) gives the near field distribution obtained with our method and Fig. 4 (b) gives the same distribution obtained with the analytical formula [7-8].

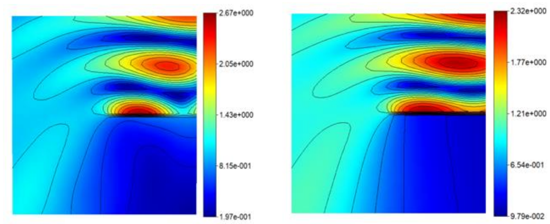


(a) (b)

Fig. 3. Poynting vector (a) and energy density distribution (b) at $\nu = 0.5, \theta = \frac{\pi}{2}$.

As it can be seen in figures, the results are quite similar to each other.

Fractional order value $\nu = 0.5$ corresponds to the material for which the corresponding impedance is $\eta = -i$. Such material does not exist in nature but it has very interesting properties. If such a material can be made artificially, it would have a lot of useful applications such as high-quality resonators, waveguides, and antennas.



(a) (b)

Fig. 4. The near E_z field distribution calculated with FDM at $\nu = 1, \theta = \frac{\pi}{2}$ and analytical solution for PMC

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