Quantifying Sub-gridding Errors in Standard and Hybrid Higher Order 2D FDTD Simulations

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***Abstract*—Sub-gridding errors for a 2D Finite-Difference Time-Domain (FDTD) simulation are compared for both the standard FDTD and Hybrid higher order FDTD cases. Sub-gridding contrast ratios of 1:3, 1:9, 1:15, and 1:27 are considered and analyzed. A correlation is seen between the increase of contrast ratio with the increase of sub-gridding errors for both standard and hybrid cases. However, a trend of errors reduction when using hybrid formulations over standard formulations is apparent for each contrast ratio.**

***Keywords*—Finite-difference time-domain method, high order FDTD, numerical error analysis, sub-gridding.**

I. Introduction

Due to increased development in 5G and IoT technologies, FDTD sub-gridding methods are necessary for these electrically large simulation domains. A standard FDTD [1] requires the minimum number of Yee cells to be at least 10 within the minimum wavelength [2]. Sub-gridding methods prove useful in accurately and efficiently analyzing electrically large domains with relatively low allocations of resources and memory. Sub-gridding consequently can lead to the appearance of errors caused by dispersion and stability [3-5]. Electrically large subgrid regions can also lead to errors [6], an example is when conducting full wave simulation of a large antenna array of multiple wavelengths. The relative error that arises with increased electrical sizes of sub-gridded regions, was previously discussed independently from the contrast ratio for 1D and 2D FDTD simulations [7].

This paper extends the work presented in [7] and will investigate the errors of 2D simulations with higher contrast ratios of 1:9, 1:15, and 1:27 using traditional 2nd order formulations as well as higher order FDTD methods.

II. Two-Dimensional FDTD Domain

The 2D FDTD setup, as shown in Fig. 1, involves a Gaussian pulse propagating through a domain of 308 by 243 cells in the x and y direction, respectively. Additionally, there is a subgrid region of 143 by 30 cells surrounding the source corresponding to about 7λ by 1.5λ. The considered Perfectly Matched Layer (PML) boundary consists of 10 coarse cells in all four directions.

The outer dimensions of the domain in Fig. 1 is static for all considered contrast ratios with the size of the course grid remains constant. As the contrast ratio increases, the fine cell size (dx_{fine}) decreases and thus creates a denser subgrid region. The parameters for each contrast ratio are outlined in Table I.

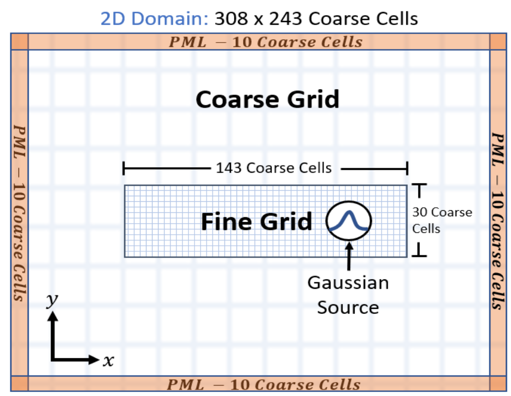


Fig. 1. Example from [1, Section 7.5.2]: Line source simulated with a 2D FDTD code. Striped box respresents physical location of fine grid region.

All previously mentioned parameters hold true for both traditional 2nd order formulations (S22) as well as for the hybrid cases of 4th and 2nd order FDTD formulations (HS24). Specifically, for the hybrid case, a 2nd order approximation will be used in the subgrid region (fine grid) and a 4th order approximation will be utilized in the remaining outer domain, including the PML (course grid). This hybrid case will aim to reduce errors when comparing to a reference domain. This reference domain will consist of a uniform mesh with cell sizes corresponding to cell size of the sub grid region, i.e., dx_{fine} and dy_{fine}. This reference domain will be calculated with the traditional 2nd order approximation throughout based on the formulation in [1].

TABLE I. 2D Domain Parameters

Contrast Ratio	Coarse Cell Size ($dx = dy$)	Fine Cell Size ($dx_{fine} = dy_{fine}$)	Time Step Size (dt)	# of Time Steps
1:3	3 *mm*	1 *mm*	2.1 *ps*	3,000
1:9	3 *mm*	0.33 *mm*	0.7 *ps*	9,000
1:15	3 *mm*	0.2 *mm*	0.42 *ps*	15,000
1:27	3 *mm*	0.11 *mm*	0.24 *ps*	27,000

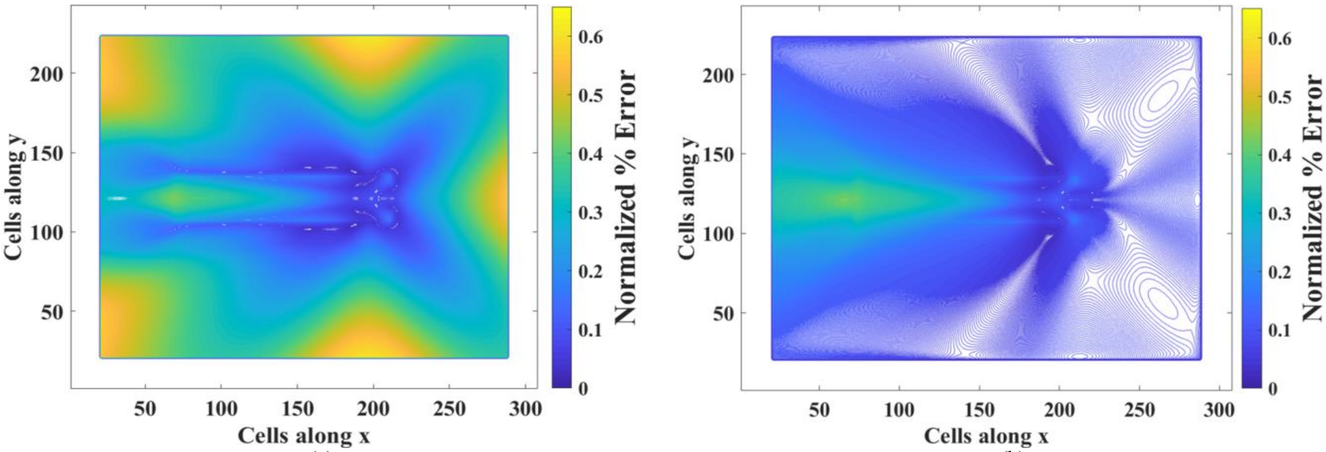
III. Error Analysis

The normalized error for each S22 and HS24 case will be calculated by comparing to the reference domain described in Section II using the expression:

$$\frac{max(|E_{z,reference}(i,j,t)-E_{z,subgrid}(i,j,t)|)}{max(|E_{z,reference}(i,j,t)|)} \times 100\% , \qquad (1)$$

Submitted On: December 8, 2020
Accepted On: January 4, 2021

https://doi.org/10.47037/2020.ACES.J.351183



(a) (b)

Fig. 2. Normalized percentage errors for the 2D FDTD as described in Fig. 1 and Table I for 1:3 contrast ratio: (a) S22 errors and (b) HS24 errors.

where the maximum absolute difference between the reference and the subgridded domain are compared for every time step at every location within the domain.

IV. RESULTS & ANALYSIS

The S22 and HS24 errors are depicted in Fig. 2 for a 1:3 contrast ratio. As shown, the errors for S22 are heavily concentrated outside of the fine grid region where the errors are increasing as they propagate outside of the fine grid region. In contrast, the error from the HS24 case is significantly lower outside of the fine grid region. The results for other contrast ratios are summarized in Table II.

TABLE II. 2D DOMAIN PARAMETERS

Contrast Ratio	S22 Errors	Hybrid Errors	Hybrid Improvement $\frac{\lvert Error_{S22} - Error_{Hybrid} \rvert}{\lvert Error_{S22} \rvert} \times 100\%$
1:3	0.6168%	0.4202%	32%
1:9	0.6971%	0.1803%	74%
1:15	0.7036%	0.1614%	77%
1:27	0.7061%	0.1550%	78%

The purpose of using higher contract ratio is to be able to conduct simulations where certain areas of the domain have fine geometrical details. It is apparent that there is a trend of increasing errors for the S22 case as the contrast ratio increases. However, the use of proposed hybrid formulation reduces maximum errors to an acceptable level due to better matching of the numerical phase velocities between the subgrid region and the free space region while allowing for higher contrast ratio.

V. CONCLUSION

The presented numerical results show strong improvement with the use of the hybrid formulations for 2D domains when fine discretization in sub areas, are required. This hybrid formulation becomes increasingly necessary when the contrast ratios increase as the sub-gridding using the standard S22 formulation yields increasingly worse errors. Further investigations will involve larger contrast ratios and extension to 3D simulation domains.

ACKNOWLEDGMENTS

This project is partially supported by Futurewei Technology Inc., New Jersey Research Center, Bridgewater, NJ, USA.

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