# Time-modulated Coupled-cavity System for Optical Switching

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***Abstract*—The scattering parameters of a time-modulated two-coupled-cavity system are obtained using coupled mode theory and verified using the finite-difference time-domain method. We find that a suitably tailored time-modulation of the cavities allows for switching the output on and off. The proposed functionality can be used in direct amplitude modulation of an optical carrier from a frequency-modulated RF message.**

***Index Terms*—cavity resonators, finite-difference time-domain method, switch, time-varying circuits.**

## I. INTRODUCTION

Linear time-invariant photonic systems form the backbone of many modern optical signal processing systems. However, material nonlinearities [1] and time-variation [2], [3] can be exploited to extend the design space of photonic device functionality. This presentation introduces a linear time-varying photonic switching device. We show how RF modulation of the refractive index of two coupled cavities can switch the output from on to off. This device functionality can be used for direct amplitude modulation of an optical signal using a phase or frequency modulated RF signal.

## II. COUPLED CAVITIES

Fig. 1 is a schematic depiction of two coupled resonant cavities each of which is coupled to a waveguide. The resonance frequency and loss rate of each cavity in isolation is $\omega_0$ and $\gamma$, respectively. Temporal coupled mode theory (CMT) [4] can be used to obtain the resonances of the two-cavity coupled system by diagonalizing:

$$\frac{d}{dt}\begin{bmatrix}a_1(t)\\a_2(t)\end{bmatrix} = -i\begin{bmatrix}\omega_0 & \kappa\\ \kappa & \omega_0\end{bmatrix}\begin{bmatrix}a_1(t)\\a_2(t)\end{bmatrix} - \begin{bmatrix}\gamma & 0\\ 0 & \gamma\end{bmatrix}\begin{bmatrix}a_1(t)\\a_2(t)\end{bmatrix}, \quad (1)$$

where $a_j(t)$ is the field amplitude in the $j$th resonator and $\kappa$ is the coupling rate between the cavities. The coupled eigenstates are given by $a_\pm(t) = [a_1(t) \pm a_2(t)]/\sqrt{2}$ with frequencies $\omega_\pm = \omega_0 \pm \kappa$. Fig. 1 depicts the qualitative nature of the modes of the two-cavity system. Essentially $a_+(t)$ is an even combination of the isolated cavity modes, and $a_-(t)$ is an odd combination of the isolated cavity modes.

In this work, the effect of time-modulation of the refractive index of the cavities is investigated. The time-modulation is modeled as a perturbation to the cavity resonance frequency in the CMT analysis. Including the time-modulation term as

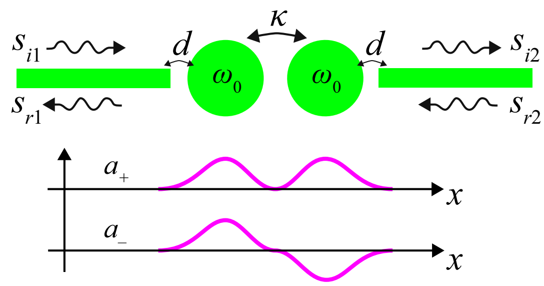


Fig. 1. Top: schematic depiction of two coupled resonant cavities each of which is coupled to a waveguide. Bottom: example depiction of the spatial part of the coupled-cavity fields.

well as coupling to the input and output waveguides, the CMT equations can be written as:

$$\dot{\mathbf{a}} = [-i\mathbf{\Omega} - i\delta\mathbf{\Omega}(t) - \mathbf{\Gamma}]\mathbf{a} + \sqrt{2}\mathbf{D}^{\mathrm{T}}\mathbf{s}_{\mathrm{inc}}, \quad (2)$$

where $\mathbf{a} = \begin{bmatrix}a_+(t) & a_-(t)\end{bmatrix}^T$, $\mathbf{\Gamma}$ is a diagonal matrix with $\gamma$ on the diagonal, $\mathbf{\Omega}$ is given by:

$$\mathbf{\Omega} = \begin{bmatrix}\omega_0+\kappa & 0\\ 0 & \omega_0-\kappa\end{bmatrix}, \quad (3)$$

and

$$\mathbf{D} = \frac{1}{\sqrt{2}}\begin{bmatrix}d & d\\ d & -d\end{bmatrix}. \quad (4)$$

$d$ is the coupling rate between the cavity and the waveguide. $\mathbf{s}_{\mathrm{inc}} = \begin{bmatrix}s_{i1} & s_{i2}\end{bmatrix}^{\mathrm{T}}$ is a vector describing the incident field amplitudes in ports 1 and 2. In the presence of waveguide coupling, the loss rate decomposes into $\gamma = \gamma_i + \gamma_c$ where $\gamma_i$ is the intrinsic cavity loss rate, and $\gamma_c$ is the loss rate due to cavity coupling. The loss rate is related to the waveguide coupling parameter $d$ via $d^2 = 2\gamma_c$ [4].

In the isolated cavity basis we choose the time-modulation term to take the form:

$$\delta\mathbf{\Omega}_{iso}(t) = \begin{bmatrix}\delta\omega\cos(\omega_m t) & 0\\ 0 & -\delta\omega\cos(\omega_m t)\end{bmatrix}, \quad (5)$$

where the refractive indices of the two cavities are modulated at a frequency $\omega_m$ and $\pi$ out of phase. This time-modulation term introduces a spatial perturbation with odd parity. The result is coupling between the even and odd coupled-cavity

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modes. This can be seen by inspecting $\delta\mathbf{\Omega}_{iso}(t)$ in the coupled-cavity basis [3]:

$$\delta\mathbf{\Omega}(t) = \begin{bmatrix} 0 & \delta\omega\cos(\omega_m t) \\ \delta\omega\cos(\omega_m t) & 0 \end{bmatrix}. \tag{6}$$

Fig. 2 (a) shows how one could implement the schematic system of Fig. 1. We start with a one-dimensional photonic crystal consisting of air holes etched into an InP semiconductor waveguide. The cavities are formed by removing two sets of three holes. The coupling between the cavities ($\kappa$) is controlled by the number of holes between the cavities, and the waveguide coupling ($\gamma_c$) is controlled by the number of holes on either side of the cavities. The index modulation is performed by applying electrodes to either side of the cavity and applying a voltage. The applied voltage is sinusoidal with frequency $\omega_m$. The applied signal is inverted when applied to cavity 2.

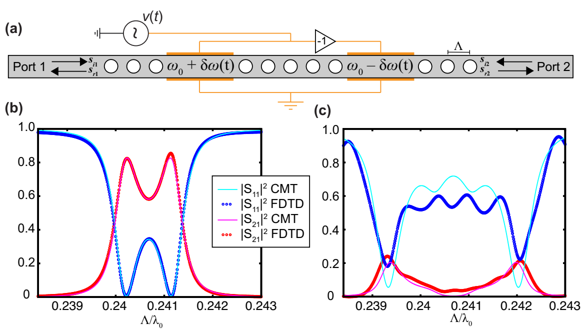


Fig. 2. (a) Schematic depiction of the two-coupled-cavity system modeled using two-dimensional FDTD. (b) The transmission ($|S_{21}|^2$) and reflection ($|S_{11}|^2$) coefficients of the cavity system in (a) when no modulation is applied. (c) The transmission and reflection coefficients of the cavity system in (a) when modulation is applied and chosen according to Eqs. 10 and 11.

The scattered wave amplitude into ports 1 and 2 is given by:

$$\mathbf{s}_{\text{ref}} = -\mathbf{s}_{\text{inc}} + \mathbf{D}\mathbf{a}, \tag{7}$$

where $\mathbf{s}_{\text{ref}} = \begin{bmatrix} s_{r1} & s_{r2} \end{bmatrix}^{\mathrm{T}}$ represents the amplitudes of the outgoing waves in ports 1 and 2 [4]. Fig. 2 (b) displays the reflection and transmission spectra for incidence from port 1 calculated using CMT and simulated using the two-dimensional finite-difference time-domain (FDTD) method. The two reflection dips and transmission peaks resulting from the coupled-cavity resonances are clear. We chose the coupling rate so that the power throughput would be high ($> 80\%$) at the resonance frequencies $\omega_0 \pm \kappa$. The throughput power approaches 100% as $\gamma_i/\gamma_c \to 0$. There is good qualitative agreement between the CMT and FDTD simulation results.

## III. Modulated Cavities

To solve the CMT equations when the modulation is turned on, one must introduce the ansatz $a_\pm(t) = \sum_n a_{\pm,n} e^{-i(\omega+n\omega_m)t}$ and solve for the Fourier series coefficients $a_{\pm,n}$. The equations governing $a_{\pm,n}$ are:

$$\begin{aligned} &[-i(\omega + n\omega_m - w_0 - \kappa) + \gamma]a_{+,n} + \\ &\frac{\delta\omega}{2}(a_{-,n+1} + a_{-,n-1}) = ds_{i1}\delta_{n,0}, \end{aligned} \tag{8}$$

and

$$\begin{aligned} &[-i(\omega + n\omega_m - w_0 + \kappa) + \gamma]a_{-,n} + \\ &\frac{\delta\omega}{2}(a_{+,n+1} + a_{+,n-1}) = ds_{i1}\delta_{n,0}, \end{aligned} \tag{9}$$

assuming incidence from port 1. To solve these equations we keep only the $n = -1, 0, 1$ harmonics. More harmonics can be kept in the solution which will improve agreement between CMT and simulation and experimental results, but it becomes more difficult to obtain analytic design rules. With only zeroth and first order harmonics, we find that if $\omega_m$ and $\delta\omega$ are chosen as:

$$\omega_m = \sqrt{\frac{3\gamma^2 + 4\kappa^2}{5}}, \tag{10}$$

and

$$\delta\omega = \sqrt{8(\omega_m^2 + \gamma^2)}, \tag{11}$$

then the throughput power at $\omega_0 \pm \kappa$ (which was nominally above 80%) goes to zero. Fig. 2 (c) shows the output spectra when the modulation is applied and set according to Eqs. 10 and 11 which corresponds to $\omega_m = 0.0005$ and $\delta\omega = 0.0017$. Qualitatively, the transmitted and reflected spectra switch roles when the modulation is applied. In these simulation results, the output swings from above 0.8 without modulation to below 0.06 using the proposed modulation scheme. This represents a switching ratio of 11.2 dB. Because the throughput is proportional to $\omega_m$ around the set point Eq. 10, this device functionality would allow one-step amplitude modulation of an optical carrier wave from a frequency or phase modulated RF signal.

## IV. Conclusion

This work shows how temporal CMT can be used to obtain design rules that leverage time-modulated coupled cavities to perform novel optical signal processing. We show how a truncated Fourier series provides acceptable comparison to FDTD simulation results. The presentation will also discuss a method by which we use Fourier analysis to obtain scattering spectra using FDTD simulations with time-varying refractive indices.

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