Constitutive Parameter Optimization Method of Obliquely Incident Reflectivity for Conformal PML

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Abstract **─** The conformal perfectly matched layer (PML), i.e., an efficient absorbing boundary condition, is commonly employed to address the open-field scattering problem of electromagnetic wave. To develope a conformal PML exhibiting a significant absorption effect and small reflection error, the present study proposes the constitutive parameter optimization method of obliquely incident reflectivity in terms of the conformal PML. First, the recurrence formula of obliquely incident reflectivity is derived. Subsequently, by the sensitivity analysis of constitutive parameters, the major optimal design variables are determined for the conformal PML. Lastly, with the reflectivity of the conformal PML as the optimization target, this study adopts the genetic algorithm (GA), simulated annealing algorithm (SA) and particle swarm optimization algorithm (PSO) to optimize the constitutive parameters of the conformal PML. As revealed from the results, the optimization method is capable of significantly reducing the reflection error and applying to the parameter design of conformal PML.

Index Terms **─** Conformal PML, obliquely incident reflectivity, parameter optimization, sensitivity analysis.

I. INTRODUCTION

The concept of the perfectly matched layer (PML) was initially proposed by Berenger [1] in 1994. The PML refers to an artificial truncation boundary condition, applying to electromagnetic scattering computation as a local boundary condition that exhibit excellent performance [2-7]. Subsequently, the conformal PML was introduced by Kuzuoglu and Mittra, which can be developed as the similar shell to the geometry of cylindrical and spherical scatterer to save spatial scattering elements [8]. Figure 1 illustrates the construction of a conformal PML.

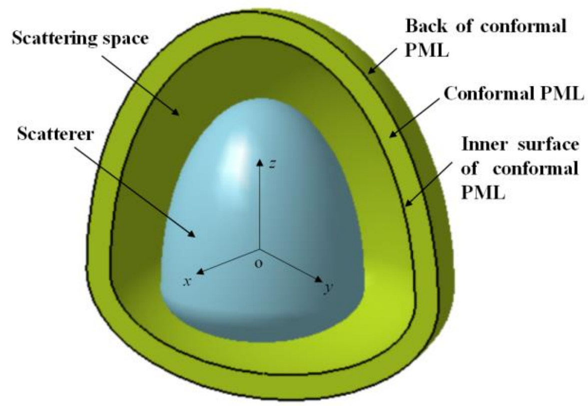

Fig. 1. The construction of a conformal PML.

Theoretically, the absorbing efficiency of conformal PML [9-13] is not determined by the incident angle and frequency of the incident electromagnetic wave, while it is only dependent of the thickness of the absorbing boundary layer. Compared with the conventional boundary conditions [14-16], the conformal PML can more easily fit the complex shape scatterers since it can maintain the consistent geometry with the scatterers. Furthermore, the conformal PML [17,18] can be formed in the space close to the surface of scatterers. Thus, the number of elements meshed in scattering space can be down-regulated maximally, thereby considerably saving the computational memory and time. Thus, the conformal PML is considered one of the most excellent local boundary conditions over the recent few years. To design a conformal PML that exhibits a significant absorption effect and small reflection error, the present study proposes a constitutive parameter optimization method given the derivation of obliquely incident reflectivity. The optimization method is expected as a basic method to design high-performance conformal PML.

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II. FUNDAMENTAL PARAMETERS OF CONFORMAL PML

The three-dimensional electromagnetic scattering field with a conformal PML is expressed as:

$$\nabla\times\left(\frac{1}{\mu_r}\overline{\overline{\Lambda}}^{-1}\bullet\nabla\times\boldsymbol{E}^s\right)-\omega^2\varepsilon_r\overline{\overline{\Lambda}}\boldsymbol{E}^s=0\,, \tag{1}$$

where $\boldsymbol{E}^s$ denotes the scattering electric field. μ_r and ε_r respectively represent the relative permeability and relative dielectric constant. $\overline{\overline{\Lambda}}$ indicates the constitutive parameter of conformal PML in tensor.

Given the literature [11,19,20], the components of relative permeability $\overline{\overline{\mu}}=\mu_r\overline{\overline{\Lambda}}$ and relative dielectric constant $\overline{\overline{\varepsilon}}=\varepsilon_r\overline{\overline{\Lambda}}$ for conformal PML are respectively written as follows:

$$\begin{cases}\overline{\overline{\mu}}_1=\dfrac{s_2s_3}{s_1}\mu_r & \overline{\overline{\varepsilon}}_1=\dfrac{s_2s_3}{s_1}\varepsilon_r\\ \overline{\overline{\mu}}_2=\dfrac{s_1s_3}{s_2}\mu_r & \overline{\overline{\varepsilon}}_2=\dfrac{s_1s_3}{s_2}\varepsilon_r\,,\\ \overline{\overline{\mu}}_3=\dfrac{s_1s_2}{s_3}\mu_r & \overline{\overline{\varepsilon}}_3=\dfrac{s_1s_2}{s_3}\varepsilon_r\end{cases} \tag{2}$$

where subscripts 1 and 2 denote two tangential components; subscript 3 represents the normal component in local coordinate system of conformal PML; *s* indicates the complex extension variable following a certain direction.

When the electromagnetic wave is obliquely incident from infinity, it can be approximately considered that the local coordinate system complies with the global coordinate system. If an arbitrarily polarized planar electromagnetic wave is obliquely incident to the interface (Fig. 2), the *xoy* plane acts as the interface, and the *xoz* plane refers to the reflective surface. The incident plane is defined as the incident wave propagation direction $\boldsymbol{k}_i$ and normal direction $\hat{n}$ of the interface. In terms of the incident wave, the electric field $\boldsymbol{E}_i$ and magnetic field $\boldsymbol{H}_i$ are in a plane perpendicular to the propagation direction $\boldsymbol{k}_i$, and the orientation of $\boldsymbol{E}_i$ in this plane may be arbitrary. On the whole, the propagation direction of the uniform plane wave is not constantly perpendicular to the incident surface, and the arbitrary wave incident can fall into two components [20].

The electromagnetic wave is assumed to be decomposed into the vertical polarization and horizontal polarization plane wave, defined as $\boldsymbol{E}_i^{\perp}$ and $\boldsymbol{E}_i^{\parallel}$ respectively, the uniform plane wave is expressed as $\boldsymbol{E}_i=\boldsymbol{E}_i^{\perp}+\boldsymbol{E}_i^{\parallel}$. Under the available reflectivity $R^{\perp}$ and $R^{\parallel}$ of the vertical polarization and horizontal polarization plane wave, the reflected wave is expressed as $\boldsymbol{E}_r=R^{\perp}\boldsymbol{E}_i^{\perp}+R^{\parallel}\boldsymbol{E}_i^{\parallel}$. Thus, the reflectivity of the plane wave is defined as:

$$R=\sqrt{\left(R^{\perp}\cos\alpha\right)^2+\left(R^{\parallel}\sin\alpha\right)^2}\,, \tag{3}$$

where α denotes the polarization angle of the incident electromagnetic wave.

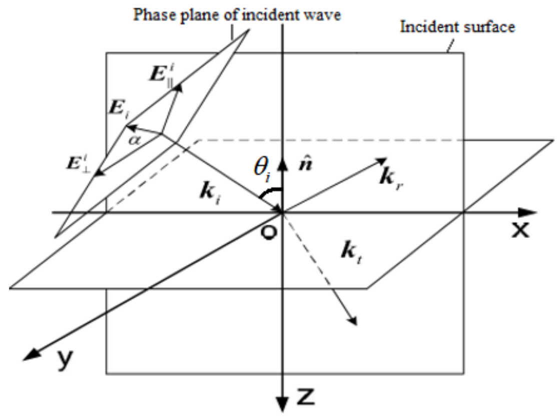

Fig. 2. Obliquely incident electromagnetic wave at arbitrarily polarized direction.

Overall, the conformal PML is utilized in multiple layers. Suppose that the physical space located by the conformal PML is vacuum or air, the relative permeability and relative dielectric constant of conformal PML are $\mu_r=\varepsilon_r=1$. According to the expressions of relative permittivity and permeability in (1) and (2), the wave number k_i and impedance η_i of the *i*th layer of conformal PML can be written as:

$$\begin{cases}k_i=\omega\left(\dfrac{s_2s_3}{s_1}\right)\sqrt{\mu_0\mu_{ir}\varepsilon_0\varepsilon_{ir}}=\left(\dfrac{s_2s_3}{s_1}\right)k_0\\ \eta_i=\sqrt{\dfrac{\mu_r}{\varepsilon_r}}=\sqrt{\dfrac{\mu_0}{\varepsilon_0}}\bullet\sqrt{\dfrac{\mu_{ir}}{\varepsilon_{ir}}}=\eta_0\sqrt{\dfrac{\mu_{ir}}{\varepsilon_{ir}}}=\eta_0\end{cases}\,, \tag{4}$$

where $\mu_{ir}=1$ and $\varepsilon_{ir}=1$ respectively denote the relative permeability and relative dielectric constant of the *i*th layer for conformal PML; $k_0=\omega\sqrt{\mu_0\varepsilon_0}$ and $\eta_0=\sqrt{\mu_0/\varepsilon_0}$ respectively represent the wave number and impedance of vacuum or air.

III. OBLIQUELY INCIDENT REFLECTIVITY OF MULTILAYER CONFORMAL PML

Under the sufficiently far source, the incident electromagnetic wave may be approximated as a uniform plane wave. For the multiple shell of the conformal PML, the case of vertical polarization at the incident point or local area exhibiting small curvature is illustrated in Fig. 3. The angle between the incident wave and interface of the ith layer refers to the incident angle θ_i. If xoz indicates the reflective surface, and the electric field is vertical polarization, the propagation of electric and magnetic fields in the ith and the $(i+1)$th layers is presented in Fig. 3.

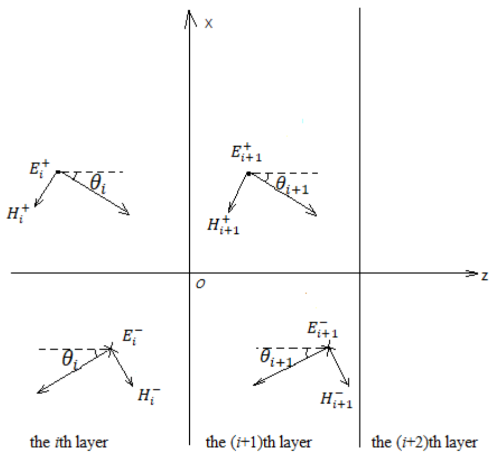

Fig. 3. Propagation of obliquely incident vertical polarization wave in multilayer conformal PML.

If the oblique incidence is vertical polarization, the electric field only covers the z-component. Hence, the electric and magnetic fields in the ith layer of the conformal PML are expressed as:

$$\begin{cases} \boldsymbol{E_i} = \left[E_i^i e^{-jk_i(x\sin\theta_i + z\cos\theta_i)} + E_i^r e^{-jk_i(x\sin\theta_i - z\cos\theta_i)} \right] \hat{\boldsymbol{e}}_y \\ \boldsymbol{H_i} = \dfrac{1}{\eta_i} \Big[E_i^i \left(-\cos\theta_i \hat{\boldsymbol{e}}_x + \sin\theta_i \hat{\boldsymbol{e}}_z \right) e^{-jk_i z\cos\theta_i} \\ \qquad + E_i^r \left(\cos\theta_i \hat{\boldsymbol{e}}_x + \sin\theta_i \hat{\boldsymbol{e}}_z \right) e^{-jk_i z\cos\theta_i} \Big] e^{-jk_i x\sin\theta_i} \end{cases}, \quad (5)$$

where θ_i respectively denotes the incident angle of the ith layer. E_i^i and E_i^r respectively represent the electric field amplitude of incident and reflected waves of the ith layer.

Likewise, the electric and magnetic fields in the (i+1) th layer of conformal PML can be expressed as:

$$\begin{cases} \boldsymbol{E}_{i+1} = \left[E_{i+1}^i e^{-jk_{i+1}(x\sin\theta_{i+1} + z\cos\theta_{i+1})} + E_{i+1}^r e^{-jk_{i+1}(x\sin\theta_{i+1} - z\cos\theta_{i+1})} \right] \hat{\boldsymbol{e}}_y \\ \boldsymbol{H}_{i+1} = \dfrac{1}{\eta_{i+1}} \Big[E_{i+1}^i \left(-\cos\theta_{i+1} \hat{\boldsymbol{e}}_x + \sin\theta_{i+1} \hat{\boldsymbol{e}}_z \right) e^{-jk_{i+1} z\cos\theta_{i+1}} \\ \qquad + E_{i+1}^r \left(\cos\theta_{i+1} \hat{\boldsymbol{e}}_x + \sin\theta_{i+1} \hat{\boldsymbol{e}}_z \right) e^{-jk_{i+1} z\cos\theta_{i+1}} \Big] e^{-jk_{i+1} x\sin\theta_{i+1}} \end{cases}. \quad (6)$$

The continuity condition of tangential components of electric and magnetic field at the interface (z=0) is written as:

$$\begin{cases} E_{iy} = E_{(i+1)y} \\ H_{ix} = H_{(i+1)x} \end{cases}. \quad (7)$$

By combining (6) and (7), it yields:

$$\begin{cases} E_{iy} = E_i^i e^{-jk_i(x\sin\theta_i + z\cos\theta_i)} + E_i^r e^{-jk_i(x\sin\theta_i - z\cos\theta_i)} \\ H_{ix} = \dfrac{\cos\theta_i}{\eta_i} \left(-E_i^i e^{-jk_i z\cos\theta_i} + E_i^r e^{-jk_i z\cos\theta_i} \right) e^{-jk_i x\sin\theta_i} \end{cases}. \quad (8)$$

Likewise, $E_{(i+1)y}$ and $H_{(i+1)x}$ can be written as:

$$\begin{cases} E_{(i+1)y} = E_{i+1}^i e^{-jk_{i+1}(x\sin\theta_{i+1} + z\cos\theta_{i+1})} + E_{i+1}^r e^{-jk_{i+1}(x\sin\theta_{i+1} - z\cos\theta_{i+1})} \\ H_{(i+1)x} = \dfrac{\cos\theta_{i+1}}{\eta_{i+1}} \left(-E_{i+1}^i e^{-jk_{i+1} z\cos\theta_{i+1}} + E_{i+1}^r e^{-jk_{i+1} z\cos\theta_{i+1}} \right) e^{-jk_{i+1} x\sin\theta_{i+1}} \end{cases}. \quad (9)$$

Suppose that the reflectivity of the interface between the ith and $(i+1)$th layers is defined as:

$$R_i = E_i^r / E_i^i . \quad (10)$$

According to (6)-(10), the reflectivity of the interface between the ith and $(i+1)$th layers of the conformal PML for the vertical polarization plane wave can be written as:

$$R_i^{\perp} = \frac{R_a + R_b R_{i+1}^{\perp} e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}{R_b + R_a R_{i+1}^{\perp} e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}, \quad (11)$$

where R_a and R_b are:

$$\begin{cases} R_a = \left(\eta_{i+1}\cos\theta_i - \eta_i \cos\theta_{i+1} \right) \\ R_b = \left(\eta_{i+1}\cos\theta_i + \eta_i \cos\theta_{i+1} \right) \end{cases}. \quad (12)$$

A similar derivation can be applied under horizontal polarization as illustrated in Fig. 4. The reflectivity of the interface between the ith and $(i+1)$th layer of the conformal PML is inferred as:

$$R_i^{\parallel} = \frac{R_c + R_d R_{i+1}^{\parallel} e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}{R_d + R_c R_{i+1}^{\parallel} e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}, \quad (13)$$

where R_c and R_d are expressed as follows:

$$\begin{cases} R_c = \left(\eta_{i+1}\cos\theta_{i+1} - \eta_i \cos\theta_i \right) \\ R_d = \left(\eta_{i+1}\cos\theta_{i+1} + \eta_i \cos\theta_i \right) \end{cases}. \quad (14)$$

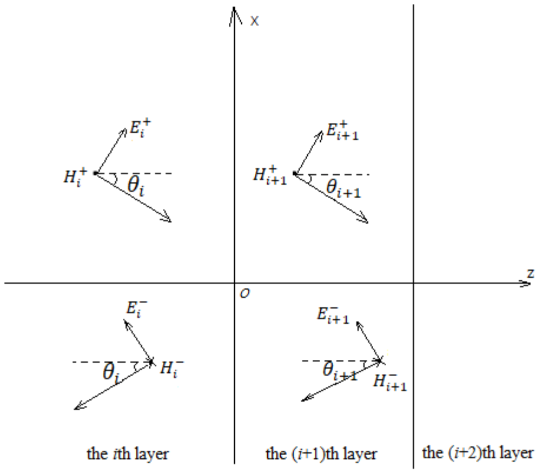

Fig. 4. Propagation of obliquely incident horizontal polarization wave in multilayer conformal PML.

If the extension order of complex extension variable is set to *m*, the complex extension variable [11] is defined as:

$$s(\xi_3) = 1 - j\delta\left(\frac{\xi_3}{t}\right)^m \tag{15}$$

where ξ_3 denotes the normal distance of the isometric surface along the *z*-axis; *t* represents the total thickness of the conformal PML; δ indicates the loss factor of the conformal PML.

For the case of two-dimensional, we can deduce that $s_1 = s_2$. Given Eq. (5), the impedance of the conformal PML is identical to that of the original scattering space; thus, the wavenumber of the conformal PML is rewritten as:

$$k_i = \left(\frac{s_2 s_3}{s_1}\right)k_0 = s_3 k_0 = \left\{1 - j\delta\left(\frac{\xi_3}{t}\right)^m\right\}k_0 . \tag{16}$$

Thus, (12) and (14) are simplified as:

$$\begin{cases} R_a = \cos\theta_i - \cos\theta_{i+1} \\ R_b = \cos\theta_i + \cos\theta_{i+1} \\ R_c = \cos\theta_{i+1} - \cos\theta_i \\ R_d = \cos\theta_{i+1} + \cos\theta_i \end{cases} . \tag{17}$$

Introducing (11) and (13) into (3), the reflectivity of the *i*th layer of the conformal PML under arbitrary polarization can be written as:

$$R_i = \sqrt{\left(R_i^{\perp}\cos\alpha\right)^2 + \left(R_i^{\parallel}\sin\alpha\right)^2} . \tag{18}$$

If the back surface of the *n*th layer of the conformal PML is total reflective, i.e., R_n=-1, the recurrence formula of reflectivity for the multilayer conformal PML can be written as:

$$\begin{cases} R_i^{\perp} = \dfrac{\left(\cos\theta_i - \cos\theta_{i+1}\right) + \left(\cos\theta_i + \cos\theta_{i+1}\right)R_{i+1}^{\perp}e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}{\left(\cos\theta_i + \cos\theta_{i+1}\right) + \left(\cos\theta_i - \cos\theta_{i+1}\right)R_{i+1}^{\perp}e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}} \\ R_i^{\parallel} = \dfrac{\left(\cos\theta_{i+1} - \cos\theta_i\right) + \left(\cos\theta_{i+1} + \cos\theta\right)R_{i+1}^{\parallel}e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}{\left(\cos\theta_{i+1} + \cos\theta\right) + \left(\cos\theta_{i+1} - \cos\theta_i\right)R_{i+1}^{\parallel}e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}} \\ R_i = \sqrt{\left(R_i^{\perp}\cos\alpha\right)^2 + \left(R_i^{\parallel}\sin\alpha\right)^2} \\ R_n = -1 \end{cases} . \tag{19}$$

While under vertical polarization, the recurrence formula of reflectivity for the multilayer conformal PML is simplified as:

$$\begin{cases} R_i = \dfrac{\left(\cos\theta_i - \cos\theta_{i+1}\right) + \left(\cos\theta_i + \cos\theta_{i+1}\right)R_{i+1}e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}}{\left(\cos\theta_i + \cos\theta_{i+1}\right) + \left(\cos\theta_i - \cos\theta_{i+1}\right)R_{i+1}e^{-2jk_{i+1}d_{i+1}\cos\theta_{i+1}}} \\ R_n = -1 \end{cases} . \tag{20}$$

Overall, the reflectivity of the conformal PML is expressed in logarithmic coordinate system, e.g., (21):

$$\begin{cases} \sigma = 20\lg|R| = 20\lg|R_0| \\ \sigma \sim \left(\delta, m, d_i, n, \theta_i, \lambda\right) \end{cases} . \tag{21}$$

Accordingly, the obliquely incident reflectivity of conformal PML depends only on the loss factor δ, extension order m, layer thickness di, overall layer number n, incident angle θ_i, and wavelength λ. Except for θ_i and λ, the other parameters, pertaining to the basic design parameters of the conformal PML, are to be discussed and optimized in the following sections.

IV. OPTIMIZATION ANALYSIS OF OBLIQUELY INCIDENT REFLECTIVITY

According to (21), the constitutive parameters affecting the obliquely incident reflectivity of the conformal PML include the layer thickness, loss factor, extension order and overall layer number. To delve into the impact of the mentioned constitutive parameters on the reflectivity, the central difference method undergoes a sensitivity analysis. The basic formula is expressed as:

$$\begin{aligned} f'(x_i) &= \frac{f(x_i+h) - f(x_i-h)}{2h} + O(h^2) \\ &\approx \frac{f(x_i+h) - f(x_i-h)}{2h}, \end{aligned} \tag{22}$$

where the step size h is set to 1% of the corresponding parameter. The sensitivity function $f(x_i)$ can be considered the obliquely incident reflectivity σ in (21), while the variables x_i can be considered one of the layer thickness, loss factor, extension order, and overall layer number in (21).

It is assumed that the frequency and wavelength of incident electromagnetic wave are set to f=30GHz and λ=0.01m, the initial parameters of conformal PML are set to δ=10, m=2, $d_i = 0.05\lambda$, n=5, and the disturbance step size is set to h=1% for the initial parameters of conformal PML, the sensitivity function $f(x_i)$ can be computed at the different incident angles θ_i (Table 1). Obviously, for the reflectivity impact of the conformal PML, the first refers to the layer thickness, the second is the extension order, while the overall layer number and loss factor are relatively small. Accordingly, the layer thickness and extension order should be adopted as the major design variables for reflectivity optimization.

Table 1: Sensitivity analysis results of constitutive parameters for conformal PML

Incident Angle/(°)	Layer Thickness	Loss Factor	Extension Order	Overall Layer Number
5	-52.8360	0.6605	-10.6282	-1.3027
10	-52.3634	0.6546	-10.5332	-1.2940
15	-51.5483	0.6444	-10.3692	-1.2788
20	-50.3495	0.6294	-10.1281	-1.2562
25	-48.7120	0.6089	-9.7987	-1.2247
30	-46.5717	0.5822	-9.3683	-1.1824
35	-43.8678	0.5484	-8.8245	-1.1273
40	-40.5626	0.5071	-8.1598	-1.0571
50	-36.6704	0.4584	-7.3770	-0.9704
55	-32.2839	0.4036	-6.4948	-0.8674
60	-27.5813	0.3448	-5.5489	-0.7510
65	-22.7980	0.2850	-4.5867	-0.6268
70	-18.1664	0.2271	-3.6549	-0.5021
75	-13.8553	0.1732	-2.7876	-0.3834
80	-9.9363	0.1242	-1.9991	-0.2745
85	-6.3879	0.0799	-1.2852	-0.1760

Given the requirements of actual electromagnetic scattering computation, this study adopts a five-layer conformal PML for reflectivity optimization. The cases of the identical value of layer thickness and different values of layer thickness are presented in the following. To optimize the obliquely incident reflectivity of conformal PML, the genetic algorithm (GA) [21], simulated annealing algorithm (SA) [22] and particle swarm optimization (PSO) [23] are employed.

(1) Case of the identical value of layer thickness

If the thickness of each layer is the same d, the layer thickness of the conformal PML in (21) can be rewritten as:

$$d_i = d = l * \lambda , \tag{23}$$

where the layer thickness can be expressed as l times the wavelength for the convenient calculation.

Given the precision of electromagnetic scattering computation and the computational cost of optimization iteration, the ranges of the layer thickness and extension order should be limited. Furthermore, to expand the search range of optimization algorithm, the value range of extension order is expanded from the integer to the real number. Thus, the optimization model under the identical value of layer thickness is defined as:

$$\begin{cases} \min \quad R = f\left(d, m\right) \\ s.t. \quad \dfrac{\lambda}{20} \le d \le \dfrac{\lambda}{2} \\ \qquad 1 \le m \le 10 \end{cases} . \tag{24}$$

The optimization programs utilizing algorithms of GA, SA and PSO are developed and conducted in MATLAB. The optimization of obliquely incident reflectivity for the conformal PML is shown in Fig. 5. The optimization results are listed in Tables 2 and 3. It is suggested that the decrease in the reflectivity is obvious after optimization. Compared with the three algorithms, PSO has the shortest running time and the maximal number of generations. GA exhibits the minimum number of generations, the longest running time and the maximal decrease. The number of generations and running time of SA are between GA and PSO.

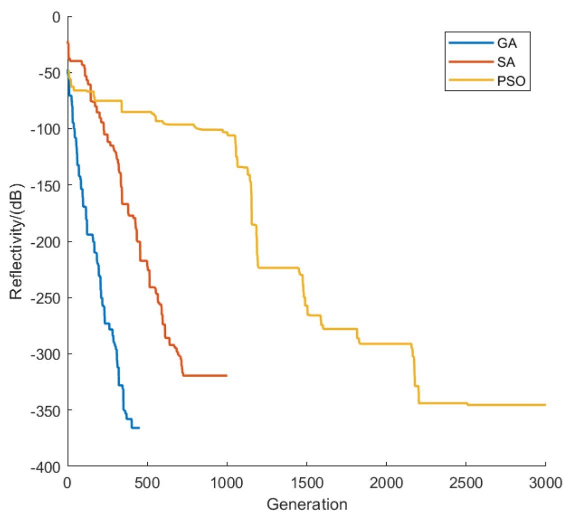

Fig. 5. Optimization of the obliquely incident reflectivity for conformal PML under the identical value of layer thickness.

Table 2: Parameter optimization results of conformal PML under the identical value of layer thickness

Opti.	Layer Thickness/(*λ)	Extension Order
Initial	0.05	2
GA	0.2818	4.0228
SA	0.3992	4.4515
PSO	0.3931	2.5613

Table 3: Reflectivity optimization results of conformal PML under the identical value of layer thickness

Opti.	Reflectivity (dB)	Decrease in Reflectivity (%)	Running Time (s)
Initial value	-120.1470	—	—
GA	-346.9674	188.7857	27.9730
SA	-336.1821	179.8090	16.2063
PSO	-335.8055	179.4955	10.1944

(2) Case of the different values of layer thickness

If the thickness of each layer d_i is different, the layer thickness of the conformal PML in (21) is rewritten as:

$$d_i = l_i * \lambda , \tag{25}$$

where the layer thickness is expressed as l_i times the wavelength for the convenient calculation.

Likewise, the optimization model under the different values of layer thickness is expressed as:

$$\begin{cases} \min & R = f\left(d_1, d_2, \cdots, d_n, m\right) \\ s.t. & \dfrac{\lambda}{20} \le d_i \le \dfrac{\lambda}{2} \\ & 1 \le m \le 10 \end{cases} . \tag{26}$$

The optimization of obliquely incident reflectivity for the conformal PML is shown in Fig. 6. The optimization results are listed in Tables 4 and 5. It is suggested that the decrease in the reflectivity is obvious after optimization. Compared with the three algorithms, PSO exhibits the minimum number of generations and the least running time. SA has the largest number of generations. GA has the longest running time and the largest decrease. Compared with the case of the identical value of layer thickness, the reflectivity after optimization under different values of layer thickness is reduced for GA and PSO, while the running time of optimization is extended for SA and PSO.

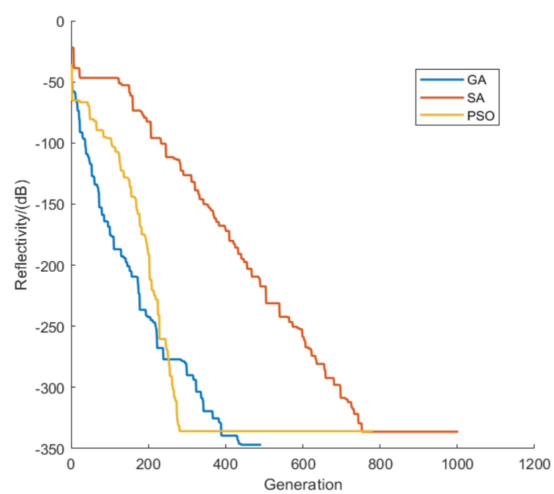

Fig. 6. Optimization of obliquely incident reflectivity for conformal PML under the different values of layer thickness.

Table 4: Parameter optimization results of conformal PML under the different values of layer thickness

Opti.	**Layer Thickness/(*λ)**					**Extension Order**
	The 1st Layer	**The 2st Layer**	**The 3st Layer**	**The 4st Layer**	**The 5st Layer**	
Initial	0.05	0.05	0.05	0.05	0.05	2
GA	0.4249	0.3588	0.2395	0.4576	0.3231	3.2311
SA	0.4159	0.4894	0.1113	0.4008	0.2789	2.9122
PSO	0.3898	0.3190	0.1401	0.3785	0.4022	2.8880

Table 5: Reflectivity optimization results of conformal PML under the different values of layer thickness

Opti.	**Reflectivity (dB)**	**Decrease in Reflectivity (%)**	**Running Time (s)**
Initial value	-120.1470	—	—
GA	-365.8395	204.4932	26.7663
SA	-319.3636	165.8107	19.3103
PSO	-361.1313	200.5745	11.1017

V. CONCLUSION

In the present study, the recurrence formula of oblique incidence reflectivity for the conformal PML is derived. The sensitivity analysis of reflectivity is conducted, covering the constitutive parameters for the conformal PML, i.e., the layer thickness, loss factor, extension order and overall layer number. The layer thickness and extension order are taken as the major optimization design parameters. Next, with the oblique incidence reflectivity as the optimization target, the optimization iteration and analysis on the conformal PML are conducted by the GA, SA and PSO algorithms. As revealed from the results, all the three optimization algorithms can determine the optimal reflectivity efficiently, which can distinctly enhance the absorbing effect of the conformal PML. Thus, the constitutive parameter optimization method in the present study can act as the basic parameter design method for conformal PML, as well as potential technical approach to determine electromagnetic scattering.

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