Benchmark Electromagnetic Inverse Scattering by Using Differential Evolution – A Big Data Perspective

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Abstract **–** The benchmark electromagnetic inverse scattering problem is re-visited in this paper from a big data perspective. It serves as the benchmark application problem in systematic parametric study of differential evolution (DE). Representative strategies with a full sweeping of intrinsic control parameters are applied to draw a systematic picture of DE. Insights extracted from preliminary numerical results are presented to rebut the questionable statements and advise DE applicants.

Index Terms **–** Benchmark electromagnetic inverse scattering, differential evolution, parametric crime, parametric study, stochastic crime.

I. INTRODUCTION

A. Questionable statements in DE

Among the rapid expanding family of natural optimization algorithms, DE [1],[2] proposed by Price and Storn in 1995 is a very simple but very powerful evolutionary algorithm. It quickly earned its reputation as a prominent function optimizer through self-assessment and international showdowns. It has been applied to electromagnetic inverse scattering [3],[4], antennas [5], electromagnetic composite materials [6], frequency selective surfaces [12], microwave absorbers [8], and a lot of other mathematical and engineering optimization problems [9],[10],[11],[12],[13].

The standard notation DE/x/y/z of DE strategies [14] implies vast variants. As a matter of fact, only two operators, differential mutation and crossover, are involved in the standard notation that it is unable to cover all variants of DE strategies. Evolution mechanism, parental selection, and survival selection are missing.

There are two mechanisms to evolve the population in DE. The classic DE (CDE), also known as two-array method, applies static one while the dynamic DE (DDE) [15], or one-array method [16], evolves the population dynamically. A close analogy between the relationship between CDE and DDE and that between Jacobi and Gauss-Seidel method in linear algebra [17] can be made. Although it is well known that Gauss-Seidel method might converge faster more reliably than Jacobi method, it is claimed that there is "no dramatic difference in performance between the one- and two-array methods".

Differential mutation has been established as the crucial evolutionary operator leading to the success of DE. Its generic formulation to generate a mutant $\mathbf{v}^{n+1,i}$ for mother $\mathbf{p}^{n,i}$ reads:

$$\mathbf{v}^{n+1,i} = \mathbf{b}^{n,i} + \sum_{y\geq 1} F_y \left(\mathbf{d}^{n,i,y,1} - \mathbf{d}^{n,i,y,2}\right),$$

where $\mathbf{p}^{n,i} \in \mathbf{P}^n$, the population of the nth generation, $\mathbf{b}^{n,i}$ is the differential mutation base, $\mathbf{d}^{n,i,y,1}$ and $\mathbf{d}^{n,i,y,2}$ form the yth pair of donors, and F_y is the mutation intensity, also known as scale factor, for the yth vector difference. Please note that the notations of population individuals in this paper interchangeably represent the individuals as well as their N-dimensional vector of optimization parameters $\mathbf{x}$ whenever possible.

It has been a consensus in DE that different differential mutation bases balance the exploration and exploitation processes in DE differently. Accordingly, strategies with different differential mutation bases may have different performance. An anonymous reviewer comments that "in practice, DE/rand/1 is the most widely used strategy. Moreover, DE/best/1 is more prone to being trapped in a local optimum".

The successful innovation of differential mutation unfortunately shades other important ideas in DE as crossover does in genetic algorithms (GA). One of the victim operators in DE is crossover. It has been claimed that "The crossover method is not so important although Ken Price claims that binomial is never worse than exponential" [18].

The above highlighted statements have been well circulated in DE community. However, accumulating evidences pose stronger and stronger challenge against them. Serious measures have to be taken to examine these dubious statements to avoid potential damages to applicants' confidence in DE.

B. A big data practice

In 2004, after applying DE in electromagnetics for four years, this author started to get annoyed by the unsatisfactory performance of DE and inconsistent claims about DE strategies and intrinsic control parameters. An ambitious effort to reveal the relationship

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between DE strategies, their intrinsic control parameters and mathematical features of optimization problems was triggered.

One of the fundamental activities in this effort is a literature survey on DE [9],[10],[13]. Mining and reviewing ever-growing literatures on DE is undoubtedly a big data process. It has been going on until today although there is a short break in 2012 due to this author's transfer from National University of Singapore to University of Electronic Science and Technology of China and a burglary into this author's office. Full details of the literature survey will be given in this author's coming monography.

Another essential activity in the effort is a systematic parametric study on DE [9],[10],[19],[20]. DE strategies, their corresponding intrinsic control parameters, termination conditions, toy functions and benchmark application problems, their corresponding non-intrinsic control parameters, form a testing system. Three performance indicators, reliability, efficiency, and robustness are defined to quantify performance of tested DE strategies.

This effort fits perfectly into the framework of big data. During this process, existing data is collected and huge amount of new data is generated. Most importantly, all available data is mined for insights to have more pleasant experience in future applying DE and develop better DE strategies.

C. Benchmark electromagnetic inverse scattering problem

Electromagnetic inverse scattering [21] are of great interest to both scientific researchers and engineers. Locating multiple two-dimensional perfectly conducting objects illuminated by TM-z plane waves and reconstructing their shape is a benchmark electromagnetic inverse scattering problem [4]. Scattered electric fields of some representative objects measured in controlled laboratory environment are also available [22],[23].

The benchmark electromagnetic inverse scattering problem has been solved by using a variety of inversion algorithms such as Newton-Kontorovitch algorithm [24], binary genetic algorithm [25], real-coded genetic algorithm [21], DE [4], and differential evolution with individuals in groups (GDE) [26]. Under the persistent promotion of this author, it has been accepted by both electromagnetic and optimization communities as one of the benchmark electromagnetic optimization problems due to its practical value and affordable computational cost.

D. Contributions of this paper

In this paper, the benchmark electromagnetic inverse scattering problem is re-visited. It serves as the benchmark application problem in systematic parametric study of DE. Representative strategies related with the aforementioned questionable statements are applied to solve the benchmark electromagnetic inverse scattering problem. Full sweeping of representative intrinsic control parameters has been conducted to draw a systematic picture of DE. Insights extracted from preliminary numerical results are presented to rebut the questionable statements and advise DE applicants. Moreover, parametric and stochastic crimes are defined to promote appropriate practice of applying and comparing stochastic and/or intrinsic control parameters-dependent optimization algorithms.

II. BENCHMARK ELECTROMAGNETIC INVERSE SCATTERING PROBLEM

A. Configuration

For better readability and completeness of this paper, the problem geometry is re-depicted in Fig. 1, where O is the origin of the global coordinate system, Ω, a circle of radius R^{meas}, is the measuring (data) domain in which the scattered electric fields are measured, the black dots on Ω are receivers, D is the imaging (object) domain which is usually chosen to be circular or rectangular.

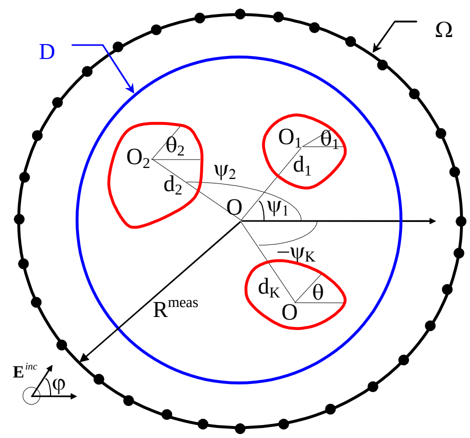

Fig. 1. Geometry of the electromagnetic inverse scattering problem.

The objects of interest are the K perfectly conducting cylinders in D, $O_i(d_i, \psi_i)$ is the local origin of the ith cylinder which can be any point within the cylinder contour C_i, d_i is the distance between O and O_i, ψ_i is the angle from horizontal or +x direction to vector $\overrightarrow{OO_i}$. C_i is represented by a local shape function $\rho_i = F_i(\theta_i)$ in the local polar coordinate system. Physically, $F_i(\theta_i) \geq 0\ \forall \theta_i \in [0,2\pi]$. Apparently, the same contour can be represented by infinite sets (O_i, F_i).

The local shape function $F_i(\theta_i)$ used to be approximated by a trigonometric series $F_i^T(\theta_i)$ of order N/2 [24],[25]:

$$F_i^T(\theta_i) = \sum_{j=0}^{N/2} A_{ij} cos(j\theta_i) + \sum_{j=1}^{N/2} A_{i(j+N/2)} sin(j\theta_i),$$

where A_{ij} are coefficients.

Five contours are generated as shown in Fig. 2 by setting the coefficients as a random number uniform in the suggested ranges [25]. The local shape functions are shown in the left figure while the contours are shown in the right one. Only the local shape function in red dash line is non-negative. The corresponding contour in red color is regular. Apparently, it is very hard for $F_i^T(\theta_i)$ to meet the physical requirement on local shape functions.

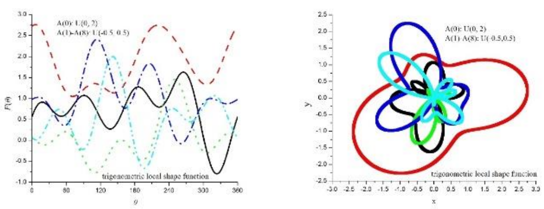

Fig. 2. Trigonometric local shape functions.

In 2000, Qing [27],[28] proposed the closed cubic B-splines local shape functions $F_i^B(\theta_i)$ with N control points C_{ij} to approximate $F_i(\theta_i)$:

$$F_i(\theta_i) \approx F_i^B(\theta_i) = \sum_{j=0}^{3} C_{i\,\mathrm{mod}(m-1+j,N)} Q_i(t),$$

where

$$Q_0(t) = \frac{1}{6}(1-t)^3,$$
$$Q_1(t) = \frac{1}{2}t^3 - t^2 + \frac{2}{3},$$
$$Q_2(t) = -\frac{1}{2}t^3 + \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{6},$$
$$Q_3(t) = \frac{1}{6}t^3,$$
$$t = \frac{N}{2\pi}\theta_i - m,$$
$$m = \left\lfloor \frac{N}{2\pi}\theta_i \right\rfloor.$$

It has been proven that:

$$\min_{0\le j\le N-1} C_{ij} \le F_i^B(\theta_i) \le \max_{0\le j\le N-1} C_{ij}.$$

Therefore, by setting $\min_{0\le j\le N-1} C_{ij} \ge 0$, non-negative definiteness of $F_i^B(\theta_i)$ can be guaranteed.

Similarly, five contours are generated as shown in Fig. 3 by setting the control points as a random number uniform in the specified ranges. The local shape functions are shown in the left figure. All of them are non-negative. In addition, the contours are shown in the right one. All contours are regular.

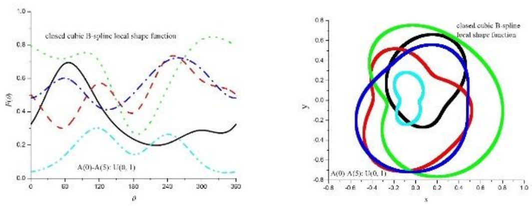

Fig. 3. Closed cubic B-spline local shape functions.

The objects of interest are illuminated by TM plane waves (time factor $e^{j\omega t}$ assumed and suppressed where $\omega = 2\pi f$ is the angular frequency) of unit amplitude:

$$\mathbf{E}^i(\mathbf{r}) = \hat{z} exp(-jk_0\hat{k}\cdot\mathbf{r}),$$

where $\mathbf{r} = x\hat{x} + y\hat{y}$, $k_0 = \omega/c$ is the wave number in free space, c is the light speed, $\hat{k} = \cos\varphi\hat{x} + sin\varphi\hat{y}$ is the incident wave unit vector and φ is the incident angle. $\hat{x}$, $\hat{y}$ and $\hat{z}$ are the unit vectors in the x, y and z directions respectively.

B. Direct problem

The electric field integral equations governing the scattering problem are:

$$E^i(r) = \sum_{j=1}^{K} \frac{\omega\mu_0}{4} \oint_{C_j} J_j(r') H_0^{(2)}(k_0|r-r'|) dr', r \in \bigcup_{i=1}^{K} C_i,$$

$$E^s(r) = \sum_{j=1}^{K} -\frac{\omega\mu_0}{4} \oint_{C_j} J_j(r') H_0^{(2)}(k_0|r-r'|) dr', r \in \Omega,$$

where μ_0 is the permeability of free space, $J_j(\mathbf{r}')$ is the induced surface current intensity on the surface of the jth cylinder, $H_0^{(2)}(\cdot)$ is the second kind Hankel's function of zeroth order.

C. Inverse problem

The inverse problem is to locate the objects of interest and reconstruct their shape, given the measured scattered electric fields $\mathbf{E}^{sm}$ which is an $N_f \times N_a \times N_r$-dimensional vector $\left[E_1^{sm}, \cdots, E_j^{sm}, \cdots, E_{N_f\times N_a\times N_r}^{sm}\right]$, N_f, N_a, and N_r are the total number of frequencies, incident angles and receivers respectively. It is cast into an unconstrained functional minimization problem whose optimization parameters are:

$$\mathbf{x} = \bigcup_{i=1}^{K}\left[d_i, \psi_i, F_i(\theta_i)\right] = \left[\mathbf{x}_1, \cdots, \mathbf{x}_K\right],$$

and objective function is:

$$f(\mathbf{x}) = \frac{\left\|\mathbf{E}^{sm} - \mathbf{E}^s(\mathbf{x})\right\|}{\left\|\mathbf{E}^{sm}\right\|},$$

where $\mathbf{x}_i = [d_i, \psi_i, F_i(\theta_i)]$, $\mathbf{E}^s(\mathbf{x})$ is an $N_f \times N_a \times N_r$-dimensional vector of scattered fields corresponding to the profile $\bigcup_{i=1}^{K}[d_i, \psi_i, F_i(\theta_i)]$,

$$\left\|\mathbf{E}^{sm}\right\| = \sqrt{\sum_{j=1}^{N_f \times N_a \times N_r} \left|E_j^{sm}\right|^2},$$

$$\left\|\mathbf{E}^{sm} - \mathbf{E}^{s}\right\| = \sqrt{\sum_{j=1}^{N_f \times N_a \times N_r} \left|E_j^{sm} - E_j^{s}\right|^2}.$$

One of the distinctive mathematical features of functional $f(\mathbf{x})$ lies with its non-uniqueness, i.e., there are infinite minima corresponding to the infinite sets of (O_i, F_i) representing the same contours C_i. The electromagnetic equivalence principle makes the non-uniqueness issue more complicated.

Approximating $F_i(\theta_i)$ by $F_i^B(\theta_i)$ simplifies the unconstrained functional minimization problem into an unconstrained parameter minimization problem whose objective function is intact but the optimization parameters are:

$$\mathbf{x} = \left[\mathbf{x}_1, \cdots, \mathbf{x}_K\right],$$

where $\mathbf{x}_i = \left[d_i, \psi_i, C_{i0}, \cdots, C_{i(N-1)}\right]$.

III. DIFFERENTIAL EVOLUTION

A. General framework

As seen from Fig. 4, CDE and DDE share initialization, differential mutation with parental selection embedded, crossover, evaluation, and survival selection, but differ in evolution. Initial population $\mathbf{P}^0$ is generated through initialization. In CDE, population evolves generation by generation until at least one of the termination conditions is met, hopefully the objective is met. On the other hand, although index n is still in use in DDE for better clarity, it does not bear the same meaning as generation index in CDE. More importantly, the population continuously updates itself.

<table>
<tr><td>initialization</td><td colspan="2">$n = 0$
do $i = 1, N_p$
  do $j = 1, N$
$$x_j^{0,i} = a_j + r_j^{0,i}\left(b_j - a_j\right)$$
  end do
end do</td></tr>
<tr><td rowspan="2">evolution</td><td>CDE</td><td>DDE</td></tr>
<tr><td>do while (termination conditions not satisfied)
  $n = n + 1$
  do $i = 1, N_p$
    differential evolution to get $\mathbf{v}^{n+1, i}$
    crossover to get $\mathbf{c}^{n+1, i}$
    evaluation of $\mathbf{c}^{n+1, i}$
    survival selection to get $\mathbf{p}^{n+1\ i}$
  end do
end do</td><td>do while (termination conditions not satisfied)
  $i = 1$
  differential evolution to get $\mathbf{v}^{n+1, i}$
  crossover to get $\mathbf{c}^{n+1, i}$
  evaluation of $\mathbf{c}^{n+1, i}$
  survival selection to get $\mathbf{p}^{n+1\ i}$
  if (i .eq. N_p) then
    $n = n + 1$
    $i = 1$
  else
    $i = i + 1$
  end if
end do</td></tr>
</table>

Fig. 4. Pseudocode of differential evolution.

B. System of parametric study

1) Differential evolution strategies

Strategies in the system are classified into four categories according to their evolution and learning mechanism: CDE, DDE, CDE with opposition-based learning (OCDE) and DDE with opposition-based learning (ODDE) [29],[30]. An expanded notation system, i/x/(y,d)/z/s, is implemented to represent the specific operations in each strategy where x, y, and z bear the same meaning as in standard DE notation, i stands for initialization, d stands for donor selection, and s stands for survival selection. Therefore, the concerned DE strategies are:

(a) CDE/i/x/(y,d)/z/s,
(b) DDE/i/x/(y,d)/z/s,
(c) OCDE/i/x/(y,d)/z/s,
(d) ODDE/i/x/(y,d)/z/s.

It has to be pointed out that even the above expanded notation system cannot cover all DE strategies reported in literatures.

2) Intrinsic control parameters

DE is population-based. Therefore, all DE strategies share the same intrinsic control parameter, population size N_p. Two sets of population sizes, {8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 120, 160, 200, 400, 800, 1600} and

{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 150, 200, 250, 500, 1000, 2000} are implemented in the parametric study according to the dimension of the test problem. The first set is applied for the benchmark electromagnetic inverse scattering problem re-visited in this paper.

It is well accepted that differential mutation is one of the key innovations leading to the success of DE. From this point of view, differential mutation has to be present in all DE strategies. In accordance, all DE strategies share at least one more intrinsic control parameter, mutation intensity F, or mutation intensities F_y if there are more than one pair of donors. The tested cases of mutation intensity form a set as $\{F|F = j\Delta_m, 1 \leq j \leq (F^U - F^L)/\Delta_m\}$ where F^L and F^U are the lower and upper bounds. The step size Δ_m is usually adjusted according to the computational cost of the test problem. $F^L = 0$, $F^U = 1$ and $\Delta_m = 0.1$ are applied for the benchmark electromagnetic inverse scattering problem re-visited in this paper.

Traditionally, crossover is present in almost all applications of DE. In this regard, crossover probability p_c, the companion intrinsic control parameter to crossover, is also essential and shared. Similarly, the tested cases of crossover probability form a set as $\{p_c|p_c = j\Delta_c, 1 \leq j \leq (p_c^U - p_c^L)/\Delta_c\}$ where $0 \leq p_c^L < p_c^U \leq 1$. The step size Δ_c is also adjusted according to the computational cost of the test problem, usually in synchrony with Δ_m. $p_c^L = 0$, $p_c^U = 1$ and $\Delta_c = 0.1$ are applied for the benchmark electromagnetic inverse scattering problem re-visited in this paper.

Obviously, a specific DE strategy may have more companion intrinsic control parameters.

3) Test problems

Test problems in the system include toy functions and benchmark application problems. A literature survey on test problems was started at almost the same time as the literature survey on DE. As of Dec. 22, 2016, more than 500 toy functions and 100 application problems from different disciplines, including the benchmark electromagnetic inverse scattering problem re-visited in this paper, have been collected.

From the point of view of parametric study, landscape and mathematical features of test problems and their relationship with other components in the system for parametric study are more fundamental. Features such as decomposability, modality, continuity, differentiability, dimensionality, uniqueness, and many more are under careful scrutiny.

One of the control parameters companion to test problems is the search space of their optimization parameters. Whenever possible, a wider search space is more welcome because it imposes less requirement on *a priori* knowledge, especially for practical engineering optimization problems. The standard setting of $0 < d_i \leq 1\lambda$, $0 \leq \psi_i \leq 2\pi$, and $0 \leq C_{ij} \leq 1\lambda$, for the benchmark electromagnetic inverse scattering problem, is implemented in this paper.

4) Termination conditions

When optimum (minimum by default) of the optimization problem under test is known, it is very natural to terminate the search when the objective is met. It is mathematically formulated as:

$$f(\mathbf{x}) - f(\mathbf{x}^*) \leq \varepsilon,$$

where $f(\mathbf{x}^*)$ is the known minimum, and ε is the threshold value to reach. For most engineering optimization problem, $\varepsilon = 0.01$.

The optimum of some toy functions and most benchmark application problems is yet to determine. In this case, the second termination condition, time limit, is introduced straightforwardly. Due to its difficulty to implement across platforms, limit of number of objective function evaluations has been proposed as an equivalent alternate. In this paper, it is set as 2000 times the problem dimension.

5) Performance indicators

Three performance indicators, reliability, efficiency, and robustness are defined in descending order of priority to quantify performance of tested DE strategies. Reliability refers the capability to find the optimum of the concerned optimization problem, efficiency refers to the number of objective function evaluations before the optimum is successfully located, while robustness refers to the sensitivity of reliability and efficiency with respect to intrinsic control parameters of concerned optimization algorithms and/or other control parameters in the parametric study system. Due to page limitation, explicit and physically meaningful quantitative definition of the three performance indicators will be presented in a new book by this author, hopefully published by John Wiley & Sons and/or IEEE Press.

IV. NUMERICAL RESULTS AND INSIGHTS

A. Profile reconstruction

All profiles considered in [4] were re-simulated here. Due to non-uniqueness of the problem and stochastic nature of DE, special attention is given to consistency of successfully reconstructed profiles to address the effect of non-uniqueness and randomness.

Although the final optimization parameters **x** and the corresponding objective function value $f(\mathbf{x})$ obtained by successful searches are diverse, in terms of the final reconstructed profiles, all participating DE strategies perform perfectly. In all noiseless synthetic reconstruction, true profiles are successfully reconstructed to the acceptable engineering accuracy ε if DE converges. In real reconstruction, the objective function value cannot go below ε because of noise in the measured scattered electric fields. As a matter of fact, $f(\mathbf{x}^*) = h > \varepsilon$ where $\mathbf{x}^*$ is the true profile. Fortunately, the reconstructed profiles $\mathbf{x}$ agree quite well with those given in [4] when $f(\mathbf{x}) \le h + \varepsilon$.

B. Parametric study

Our attention in this paper is focused on evaluating DE instead of solving the benchmark electromagnetic inverse scattering problem. Therefore, besides the reconstructed profiles, we are more interested in the statistical characteristics of the reconstruction process.

The representative reliability and efficiency of DE is shown in Fig. 5. For consistency with [9], reliability here is represented by the percentage of total number of successful searches (TNSS) among all searches and the percentage of number of successful trials (NST) among all trials. Similarly, efficiency in Fig. 5 is defined as the minimal average number of objective function evaluations (MANOFE) of all successful trials. The presented results show DDE significantly outperforms CDE in terms of both reliability and efficiency.

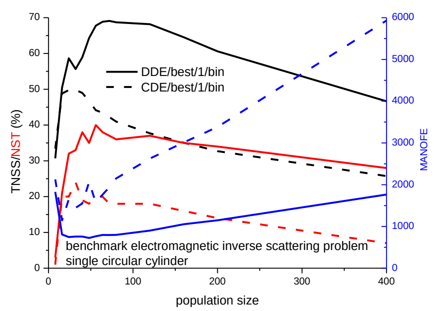

Fig. 5. Reliability and efficiency of differential evolution solving benchmark electromagnetic inverse scattering of a perfectly conducting circular cylinder.

It is also observed from obtained numerical results that:

(a) DE strategies with best differential mutation base show better performance than those with random differential mutation base;

(b) DE strategies with exponential crossover perform better than those with binomial crossover. The aforementioned statements about crossover are inconsistent with numerical results.

Due to page limitation, full set of performance has been scheduled to be published in this author's next monograph.

1) Optimal intrinsic control parameters

It is observed from Fig. 5 that the reliability of DE presents a shape of bell with respect to population size. Thus, although a larger population may be more diverse, extra-large population does not necessarily mean better performance. 20-80 seems acceptable for DDE while the optimal population size of CDE belongs to a much narrower range due to its much stronger sensitivity of efficiency with respect to population size. In another word, DDE is more robust.

The optimal mutation intensity and crossover probability corresponding to the most efficient successful trial is shown in Fig. 6. DDE prefers remarkably weaker mutation which is reasonable because dynamic evolution introduces more diversity into the population. In addition, both CDE and DDE prefer full or nearly full crossover which implies an important role of crossover in DE. The stronger and more inconsistent change of optimal mutation intensity and crossover probability of CDE in the vicinity of its optimal population size further demonstrates DDE's robustness.

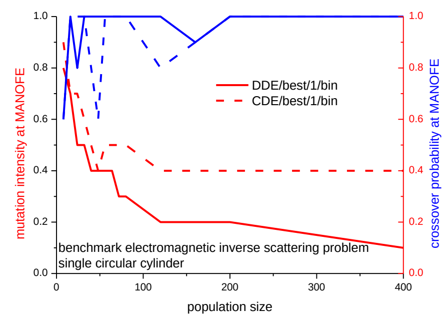

Fig. 6. Optimal mutation intensity and crossover probability.

2) Parametric crime

Robustness, or sensitivity of performance of DE with respect to its intrinsic control parameters at the vicinity of its optimal values has long been known an important issue. Intuitively, DDE can be observed from Fig. 5 more robust than CDE.

Essentially, the parametric study here involves two important aspects: identifying optimal values of intrinsic control parameter and quantifying robustness of DE at its vicinity. By now, this author is still working on a quantitative performance indicator to represent robustness. A promising candidate might be the safe zone borrowed from the idea of beam width or bandwidth in electromagnetics.

Although experiences show that DE is more robust, or less sensitive with intrinsic control parameters, underestimating or even ignoring it is by no means wise and might result in false and misleading conclusions. It implies flat performance throughout viable range of intrinsic control parameters which is absolutely not true. Such a misconduct is termed as parametric crime as sketched in Fig. 7 in analogy to inverse crimes in inverse problems.

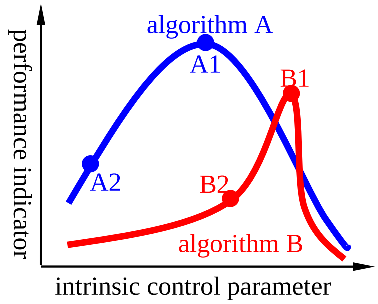

Fig. 7. Parametric crime.

Assume that algorithms A and B are compared with each other. Both of them involve an intrinsic control parameter. Obviously, algorithm A is better than B. However, if algorithm B at B1 is compared with algorithm A at A2, a false conclusion that algorithm B outperforms algorithm A will be drawn.

In Fig. 5, the efficiency of CDE for $N_p < 100$ is observed better than that of DDE for $N_p = 400$. Obviously, it is not true that CDE is better than DDE.

3) Stochastic crime

As a member of stochastic optimization algorithms, the process of a single run of DE is hardly reproducible. One might converge while the other fails although all settings are exactly the same. Likewise, two successful runs are very likely to have completely different converging process. In this regard, it would be a serious misleading conduct to define the performance of any stochastic optimization algorithm as the one of a single run. Similarly, such a misconduct is termed as stochastic crime.

Stochastic crime might also lead to reversed false conclusion about competing optimization algorithms. For example, in our practice, CDE only takes 720 objective function evaluations to solve the benchmark electromagnetic inverse scattering problem of a single perfectly conducting circular cylinder in the most efficient run, while one of the successful DDE runs takes 1280 objective function evaluations to solve the same problem. One might accordingly claim false advantage of CDE against DDE. In fact, the average number of objective function evaluations of CDE and DDE is 1248.8 and 625.07 respectively. The claim of advantage of CDE against DDE is apparently false.

V. CONCLUSIONS

In this paper, the benchmark electromagnetic inverse scattering problem is re-visited from a big data perspective. It serves as the benchmark application problem in systematic parametric study of DE. Preliminary numerical results re-confirm the advantage of dynamic evolution. Best differential mutation base and exponential crossover are also observed more competitive.

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