A Source Signal Recovery Method for Underdetermined Blind Source Separation based on Shortest Path

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Abstract **─** A new shortest path source recovery algorithm is presented for source signal recovery issue in underdetermined blind source separation, by which the source signals can be recovered in case the observed signals are no less than two dimensions. In this algorithm, two adjacent observed signals are taken everytime among *m* observed signals, marked as the i^{th} and j^{th} signals, and form a two-dimensional observed signal combination, $i=1,2,\ldots,m-1, j=i+1$. The first and m^{th} signals are used to form another two-dimensional observed signal combination, then *m* two-dimensional observed signal combinations are obtained. The number of source signals is *n*, the *n* signals can be obtained respectively after signal recovery by using each two-dimensional observed signal combination A matrix (i.e., $\tilde{\boldsymbol{s}}(p,q,:)=\left[\hat{\boldsymbol{s}}(1,1,:),\hat{\boldsymbol{s}}(1,2,:),\cdots,\hat{\boldsymbol{s}}(1,n,:),\hat{\boldsymbol{s}}(2,1,:),\cdots,\hat{\boldsymbol{s}}(m,n,:)\right]^{\mathrm{T}}$) which is a *mn*-dimensional vector combination matrix can be obtained using each signal combination A *mn*×*mn*-dimensional square matrix can be gotten by calculating the vector angle between rows of $\tilde{\boldsymbol{s}}(p,q,:)$ matrix, for the first *n* rows×*mn* columns, position the vector where the matrix elements are between 0 and θ_0. The mean is calculated for the signal vector with angle smaller than θ_0 as the estimate of the source signals. Thus, the estimates $\hat{\boldsymbol{s}}(1,:),\hat{\boldsymbol{s}}(2,:),\cdots,\hat{\boldsymbol{s}}(n,:)$ of *n* source signals can be obtained eventually. The method presented provides a new option for solving underdetermined blind source signal recovery problem.

Index Terms **─** Shortest Path Method, Source Signal Recovery, Sparsity, Underdetermined Blind Source Separation.

I. INTRODUCTION

Underdetermined blind source separation (UBSS) is a kind of signal processing techniques to estimate the source signals only using observed signals when the source signal prior information and propagation channel parameters are unknown and the number of the observed signals is smaller than the source signals. Recently underdetermined blind source separation has become a research hotspot in international signal processing community, which is generally applicable to the fields of biomedicine, mobile communication, radar signal processing, underwater acoustic signal processing, image processing, voice signal processing, and mechanical fault diagnosis, etc. For example, in the field of communication, it can be used for code division multiple access multi-user detection, interference suppression, noise cancellation, etc. to improve the call quality; in the field of radar signal processing, it can be used for distributed radar interference suppression, signal sorting, etc. to improve echo Signal anti-interference ability, realize radar, communication signal sorting, etc. under the condition of lack of prior information, spectrum aliasing, and on the same frequency [1,2]. But it is also a difficulty issue due to the number of the observed signals is always smaller than the source signals. The way for solving this problem generally divided into two steps: estimate the mixing matrix using observed signals, then recovery the source signals employing the mixing matrix estimated and observed signals [1,2]. The inverse matrix cannot be solved directly to recovery source signals since the mixing matrix is underdetermined. Therefore, the recovery of source signal is a complex issue. The recovery result of the source signals directly relates to the signal blind separation processing. In short, source signal recovery algorithm research is of important theoretical and practical significance.

In [3-5], it is assumed the source signals have strict orthogonality or quasi-orthogonality in time-frequency domain, which means they are not coincident or almost not coincident at some points, and the source signal separation is realized by time-frequency mask. Degenerate unmixing estimation technique (DUET) presented by Yilmaz et al. [3] is a typical time-frequency masking method. The DUET proposed by Cobos et al. [4] improves the accuracy in signal separation. However, both the methods presented are only applicable to the situation when the observed

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signals are two dimensional. In [5], the hypothesis of orthogonality is relaxed, and it is only required that the number of source signals that exist concurrently at the time-frequency point is smaller than observed signals. The assumptions are further relaxed in [6], in which it is only required that the number of the source signals existed concurrently at the time-frequency point is no more than the sensors. The main problem of these methods is the requirements on strict orthogonality or quasi-orthogonality of signals in time-frequency domain are still too rigid.

Bofill et al. [7] presented a source signal sparsity-oriented shortest path source signal recovery algorithm which is simple and efficacious, yet it is only applicable to two-dimensional observed signal. Georgiev et al. [8] proved that, l_1 norm minimizing method is equivalent to the shortest path method. The main problem of the shortest path is only applicable to the two-dimensional observed signals.

Xiao et al. [9] presented a statistically sparse decomposition principle (SSDP). The source signal is estimated using correlation coefficient of minimized source signal within a fixed time interval in the algorithm. It is required the number of non-zero source signals within the interval no more than two, thence it is not applicable to the recovery of underdetermined source signals that are not sufficiently sparse. Zhao et al. [10] expanded SSDP algorithm and obtained the Statistically Non-Sparse Decomposition Principle aimed at two-dimensional observed signals.

Compressed sensing theory is also used for recovery of underdetermined blind source signal. When the estimate of the mixing matrix has been completed, the recovery of underdetermined blind source signal is similar to compressed sensing reconstruction model. Fu et al. compared the three methods (greedy algorithm, l_1 norm algorithm and smooth l_0 norm algorithm) and proposed SCMP algorithm and plane pursuit algorithm [11,12] which are more accurate and less time waste. In [13], a new algorithm based on artificial neural network was introduced. Compressed sensing theory is applied, with the main problem of requiring the source signals have high sparsity, large data sampling points, and high calculation load.

In this paper, we propose a modified shortest path algorithm for UBSS problem. By employing the proposed algorithm, underdetermined blind source signal recover can be realized when the antenna array is more than or equal to two dimensions. The proposed algorithm provides a new technical approach to solve the difficulty of source signal recovery for UBSS.

The remainder of the paper is organized as follows. Section II presents the model for underdetermined blind source separation problem. Section III gives the new source signal recovery method with the modified shortest path algorithm. Section IV describes simulation results that illustrates the effectiveness of the proposed method. Finally, the conclusions are drawn in Section V.

II. MODEL FOR UNDERDETERMINED BLIND SOURCE SEPARATION

The general goal of blind source separation is to recover source signals from observed signals. The aliasing of the source signals to observed signals may be linear instantaneous, convolutional or nonlinear. Research is carried out for linear instantaneous aliasing of source signals in this paper. It is assumed that, there are n source signals expressed as $\boldsymbol{s}(t)=[s_1(t),s_2(t),...,s_n(t)]^{\mathrm{T}}$, where the superscript T means transposition and it has the same meaning in the following. The number of signal sampling points is $t=1,2,\cdots,T_0$. Signal aliasing will occur during the propagation of n source signals and the reception by sensors. The m observed signals received by antenna array are expressed as $\boldsymbol{x}(t)=[x_1(t),x_2(t),...,x_m(t)]^{\mathrm{T}}$. Any observed signal $\boldsymbol{x}(t)$ is the aliasing of the source signals $\boldsymbol{s}(t)$. The mathematical model of linear instantaneous aliasing blind source separation is:

$$\boldsymbol{x}(t)=\boldsymbol{A}\boldsymbol{s}(t)+\boldsymbol{n}(t). \tag{1}$$

Where, the prior information of mixing matrix $\boldsymbol{A}$ and source signals $\boldsymbol{s}(t)$ cannot be measured, only $\boldsymbol{x}(t)$ can be measured. $\boldsymbol{n}(t)$ refers to the additive noise aliased in the process of signal reception.

When the number of observed signals m is smaller than the number of source signals n, the blind source separation is referred to as underdetermined blind source separation. The mixing matrix $\boldsymbol{A}$ is column dissatisfaction rank in underdetermined blind source separation, so $\boldsymbol{s}(t)$ cannot be obtained by the inverse matrix of $\boldsymbol{A}$. Therefore, source signal recovery issue is a huge challenge in underdetermined blind source separation.

III. NEW SOURCE SIGNAL RECOVERY METHOD WITH THE SHORTEST PATH ALGORITHM

A. Sparsity-based underdetermined blind source signal recovery model

Blind source separation based on sparse representation can be used to solve the problem of underdetermined blind source separation. For blind source separation of sparse signals, it can be represented as below to seek for optimal solution:

$$\min_{\boldsymbol{A},\boldsymbol{s}} \frac{1}{2\sigma^2}\left\|\boldsymbol{A}\boldsymbol{s}-\boldsymbol{x}\right\|^2+\sum_{i,t}\left|\boldsymbol{s}_i(t)\right|. \tag{2}$$

Where, σ^2 is the noise variance, $\left\|\boldsymbol{A}\boldsymbol{s}-\boldsymbol{x}\right\|^2$ is the sum of square of reconstructed error. The final item is the non-sparse penalty (assuming the source signals are independent of each other). Many variables require optimization, and the model can be simplified as the following when the mixing matrix $\boldsymbol{A}$ is known:

$$\min_{s(t)} \frac{1}{2\sigma^2}\left\|\boldsymbol{As}(t)-\boldsymbol{x}(t)\right\|^2+\sum_i^n\left|\boldsymbol{s}_i(t)\right|,\quad t=1,2,\cdots,T_0. \quad (3)$$

In the absence of noise, equation (3) can be simplified as:

$$\min_{s(t)} \sum_i^n\left|\boldsymbol{s}_i(t)\right|,\quad t=1,2,\cdots,T_0. \quad (4)$$

The total optimization for underdetermined source signal recovery can be broken down into T_0 sub-optimization items.

B. Source signal recovery algorithm with the shortest path method

The shortest path method is a simple and effective source signal recovery algorithm for underdetermined blind source separation. It is suitable for the recovery of the source signal which is sufficiently sparse and the observed signals is two-dimensional. When the source signals are not sufficiently sparse in the time domain, they can be sparsely represented adopting time-frequency transformation or wavelet transformation method, therefore the source signal recovery can then be realized adopting the shortest path method. The source signal recovery issue can be converted into the solving of the optimization described below according to sparse component analysis theory:

$$\begin{cases} \min\limits_{s(t)} \sum\limits_{i=1}^n |\boldsymbol{s}_i(t)|, \\ \boldsymbol{x}(t)=\boldsymbol{As}(t)=\sum\limits_{i=1}^n \boldsymbol{a}_i\boldsymbol{s}_i(t). \end{cases} \quad (5)$$

Where, $\boldsymbol{x}(t)$ is the observed signals and the number is m; $\boldsymbol{A}$ is the mixing matrix; the number of source signals is n; $\boldsymbol{a}_i$ is the i^{th} column of the mixing matrix and $\boldsymbol{s}_i(t)$ is the i^{th} source signal. To minimize $\sum_{i=1}^n \boldsymbol{s}_i(t)$ at the time is to carry out linear decomposition for the observed signals in the directions of two columns of the mixing matrix and to find out the shortest path from the origin to the observed signals. When there are two observed signals, the method to solve the question is shown in Fig. 1. For minimizing $\sum_{i=1}^n |\boldsymbol{s}_i(t)|$ at the time, it can be known from Fig. 1, the shortest path from the origin to observed signal $\boldsymbol{x}$ is the two vectors ($\boldsymbol{a}$ and $\boldsymbol{b}$), which are closest to the angle of $\boldsymbol{x}$.

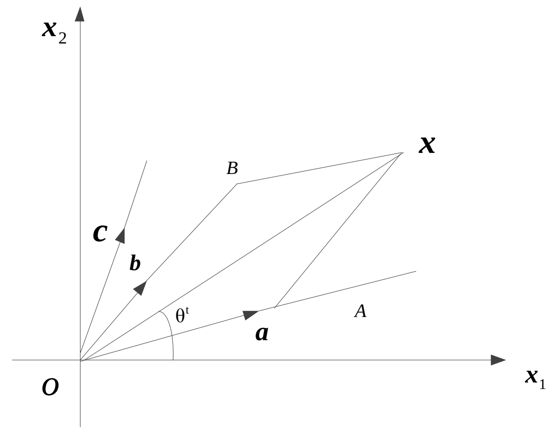


Fig. 1. Schematic for the shortest path method for the two-dimensional observed signals.

For the situation when there are more than two observed signals, the original shortest path method is improved in this paper, therefore it is applicable to the recovery of the source signals with two or more observed signals. The method is taking two adjacent observed signals every single time among m observed signals and express them as the i^{th} and j^{th} observed signals, where, $i=1,2,\cdots,m-1, j=i+1$, which means the observed signals processed each time is the combination of two adjacent observed signals, then $m-1$ combinations are obtained. The first and the m^{th} observed signals are taken to form another two-dimensional observed signal combination. Then m two-dimensional observed signal combinations can be obtained. For each combination, corresponding source signals can be recovered adopting the original shortest path method, then there are m groups of source signals recovered. As the number of source signals is n, then n signals can be obtained respectively by using each of the m two-dimensional observed signal combinations obtained from the original observed signals. Assuming that the signals obtained from separation of each combination is expressed as $\hat{\boldsymbol{s}}(i,k,:)$, where, $i=1,2,\cdots,m$ refers to the serial number of each two-dimensional observed signal combination; $k=1,2,\cdots,n$ is the serial number of the signal obtained from separation of each two-dimensional combination, : refers to the number of sampling points. A new matrix is gotten using the signals obtained from the separation of the m groups, and it can be expressed as:

$$\tilde{\boldsymbol{s}}(p,q,:)=[\hat{\boldsymbol{s}}(1,1,:),\hat{\boldsymbol{s}}(1,2,:),\cdots,\hat{\boldsymbol{s}}(1,n,:),\hat{\boldsymbol{s}}(2,1,:),\cdots,\hat{\boldsymbol{s}}(m,n,:)]^{\mathrm{T}}. \quad (6)$$

$\tilde{\boldsymbol{s}}(p,q,:)$ is a $mn\times T_0$ dimensional vector combination matrix. *mn* refers to the number of the separated signals. T_0 refers to the number of sampling points of the signal.

For matrix $\tilde{\boldsymbol{s}}(p,q,:)$, calculating the vector angles between its rows, then a *mn*×*mn*-dimensional square matrix *Q* can be obtained. For the first *n* rows×*mn* columns of the square matrix, check if the matrix element is between 0 and threshold value θ_0. If yes, it means that the intersection angle of the signals in matrix $\tilde{\boldsymbol{s}}(p,q,:)$ is smaller than θ_0, which reflects the strong similarity of the signals. The mean of the signal with angles less than θ_0 can be calculated respectively as the estimate of the source signals. Thus, the estimates $\bar{s}(1,:),\bar{s}(2,:),\cdots,\bar{s}(n,:)$ of *n* source signals (the source signals to be recovered) are gotten eventually.

The threshold θ_0 is a critical parameter in this algorithm. When determining θ_0, it should consider if the mixing matric can be estimated accurately. In this paper the threshold is determined based on the research of Zayyani et al. [14] and Cramér–Rao bound of mixing matrix. The threshold should be larger than the least angle that the mixing matrix can be estimated correctly. The extensive simulations show that the algorithm proposed in this paper has good robustness when the threshold θ_0 is larger. The simulation shows that, when the value of θ_0 is within $[15^\circ, 35^\circ]$, the difference of source signals recovery result is not obvious. A larger θ_0 is more favourable for signal recovery results since more signals can be integrated to obtain the estimates of source signals.

Specific steps of the algorithm presented in this paper are shown as follow:

Step 1: Pre-processing is carried out for the observed signals $\boldsymbol{x}(t)$, which can remove the column vectors with all components are zero. Then unification of direction is conducted.

Step 2: In *m* observed signals $\boldsymbol{x}(t)$ obtained from one measurement, $\boldsymbol{x}(t)=[\boldsymbol{x}_1(t),\boldsymbol{x}_2(t),\cdots,\boldsymbol{x}_m(t)]^{\mathrm{T}}$, the sampling time $t=1,2,\cdots,T_0$. Two adjacent observed signals $\boldsymbol{x}_i(t)$ and $\boldsymbol{x}_j(t)$ are selected each time, $i=1,2,\cdots,m-1$, $j=i+1$. The first and the *mth* observed signals are taken to form a two-dimensional observed signal combination. Then m two-dimensional observed signal combinations can be obtained in total, which are expressed as $\boldsymbol{x}_k(t)=[\boldsymbol{x}_i(t),\boldsymbol{x}_j(t)]^{\mathrm{T}}$ respectively, where, $k=1,2,\cdots,m$;

Step 3: Calculate the angle of each base vector of mixing matrix $\boldsymbol{A}$: the angle of base vector is defined as $\theta^{\boldsymbol{A}_j}=\tan^{-1}(\boldsymbol{A}_j^2/\boldsymbol{A}_j^1)$, $\boldsymbol{A}_j$ refers to the j^{th} column vector of the mixing matrix, $j=1,2,\cdots,n$; *n* is the number of source signals, the superscripts 2 and 1 mean the second and the first row of the column vector respectively;

Step 4: The angle of the observed signal vector $\boldsymbol{x}^t$ in each combination is calculated respectively adopting the original shortest path algorithm for the combination of m two-dimensional observed signals at each observed moment;

Step 5: Find out the two base vector angles closest to the observed signal vector angle θ^t at the moment, and record the two row vectors $\boldsymbol{a}^i$ and $\boldsymbol{b}^i$ in corresponding mixing matrix, where, $\boldsymbol{a}^i,\boldsymbol{b}^i\in\boldsymbol{A}$ and *i* refers to the serial number of the i^{th} combination in the *m* combinations, $i=1,2,\cdots,m$;

Step 6: Assume $\boldsymbol{A}_r=[\boldsymbol{a}^i\quad\boldsymbol{b}^i]$, $\boldsymbol{A}_r$ refers to a submatrix of 2×2 formed by two rows of $\boldsymbol{a}^i$ and $\boldsymbol{b}^i$ in mixing matrix $\boldsymbol{A}$. $\boldsymbol{a}^i$ and $\boldsymbol{b}^i$ are the two vectors closest to $\boldsymbol{x}^t$ at the moment of *t*, then make $\boldsymbol{W}_r=\boldsymbol{A}_r^{-1}$;

Step 7: The source signals at sampling moment *t* are recovered using the following equation:

$$\begin{aligned}&\boldsymbol{s}_r^t=\boldsymbol{W}_r\boldsymbol{x}^t,\\&\boldsymbol{s}_j^t=0\quad for\quad j\neq\boldsymbol{b}^i,\boldsymbol{a}^i.\end{aligned}\tag{7}$$

Where, $\boldsymbol{s}_r^t$ is the component of $\boldsymbol{x}$ in the directions of vector $\boldsymbol{a}^i$ and $\boldsymbol{b}^i$;

Step 8: For each two-dimensional observed signal combination $\boldsymbol{x}_k(t)=[\boldsymbol{x}_i(t),\boldsymbol{x}_j(t)]^{\mathrm{T}}$, $k=1,2,\cdots,m$, corresponding source signals are recovered adopting the original shortest path method, and there are m groups of source signals recovered. As the number of source signals is n, so n signals can be obtained after recovery by using each of m two-dimensional observed signal combinations. Assuming the signal obtained from separation of each combination is expressed as $\hat{\boldsymbol{s}}(i,k,:)$, where, $i=1,2,\cdots,m$ refers to each two-dimensional observed signal combination; $k=1,2,\cdots,n$ refers to the signal obtained from separation of each two-dimensional combination; : means the number of sampling points;

Step 9: A new matrix is formed using the signals obtained from the separation of *m* two-dimensional observed signal combinations, which can be expressed as $\tilde{\boldsymbol{s}}(p,q,:)=[\hat{\boldsymbol{s}}(1,1,:),\hat{\boldsymbol{s}}(1,2,:),\cdots,\hat{\boldsymbol{s}}(1,n,:),\hat{\boldsymbol{s}}(2,1,:),\cdots,\hat{\boldsymbol{s}}(m,n,:)]^{\mathrm{T}}$. $\tilde{\boldsymbol{s}}(p,q,:)$ is a $mn\times T_0$-dimensional vector combination matrix.

Step 10: A *mn*×*mn*-dimensional square matrix ***Q*** can be obtained by finding the angle between the row vectors of matrix $\tilde{\boldsymbol{s}}(p,q,:)$. For the first n rows×*mn* columns of the square matrix, check if the matrix element is larger than 0 but smaller than θ_0, meaning that the angle of the single vector in matrix $\tilde{\boldsymbol{s}}(p,q,:)$ is smaller than θ_0. The mean is calculated for the signal vectors with angle smaller than θ_0 as the estimate of the source signal. Thus, the estimates $\hat{\boldsymbol{s}}(1,:),\hat{\boldsymbol{s}}(2,:),\cdots,\hat{\boldsymbol{s}}(n,:)$ of n source signals (i.e., the source signals to be recovered) can be obtained eventually.

IV. SIMULATION AND ANALYSIS

The simulation platform is a DELL9020MT computer, Intel(R) Core(TM) i7-4770 CPU @3.40GHz, 64-bit Windows operating system. MATLAB software is used for the simulation experiment. The signal-to-noise ratio of the observed signals changes from 8 to 20dB and Monte Carlo simulation at each signal-to-noise ratio is carried out for 500 times. The performance indexes for evaluating the recovery effect of the source signal include the average separated signal to interference ratio $\overline{\text{SIR}} = mean(\text{SIR}_i)$ and the average similarity coefficient $\bar{\xi} = mean(\xi_{ij})$, $i, j = 1,2,\cdots,n$. Where, SIR and ξ are the separated signal to interference ratio and similarity coefficient respectively. The formulas for calculation of them are shown in (8) and (9), where, $\hat{\boldsymbol{s}}_i(t)$ and $\hat{\boldsymbol{s}}_j(t)$ are the i^{th} and j^{th} signals respectively. $\boldsymbol{s}_i(t)$ refers to the i^{th} source signal:

$$\text{SIR}_i = 10\lg\left(\frac{\sum_{t=1}^{T_0}\boldsymbol{s}_i^2(t)}{\left[\hat{\boldsymbol{s}}_i(t)-\boldsymbol{s}_i(t)\right]^2}\right), \quad (8)$$

$$\xi_{ij} = \frac{\left|\sum_{t=1}^{T_0}\boldsymbol{s}_i(t)\hat{\boldsymbol{s}}_j(t)\right|}{\sqrt{\sum_{t=1}^{T_0}\boldsymbol{s}_i^2(t)\sum_{t=1}^{T_0}\hat{\boldsymbol{s}}_j^2(t)}}. \quad (9)$$

The threshold value θ_0 for signal integration is set at 20^{o}.

Experiment 1:

There are 5 source signals. Assuming the source signals are sufficiently sparse in time domain, then the signal types and parameter settings are as follows:

s_1 is a conventional pulse signal, carrier frequency fc_1=5MHz, pulse width $t_{r1} = 10\mu s$, pulse repetition period $T_{r1} = 100\mu s$, pulse start time $t_{01} = 0$;

s_2 is a conventional pulse signal, carrier frequency fc_2=5MHz, pulse width $t_{r2} = 7\mu s$, pulse repetition period $T_{r2} = 100\mu s$, pulse start time $t_{02} = 10\mu s$;

s_3 is a linear FM signal, carrier frequency fc_3=5MHz, pulse width $t_{r3} = 10\mu s$, pulse repetition period $T_{r3} = 100\mu s$, pulse start time $t_{03} = 20\mu s$, instantaneous bandwidth is $B_3 = 10MHz$;

s_4 is a linear FM signal, carrier frequency fc_4=5MHz, pulse width $t_{r4} = 8\mu s$, pulse repetition period $T_{r4} = 100\mu s$, pulse start time t_{04}=30μs, intrapulse bandwidth $B_4 = 15MHz$;

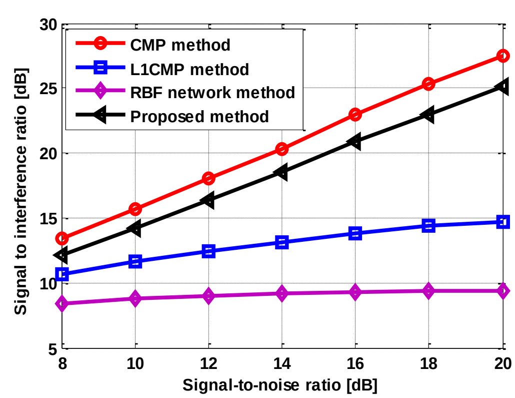
s_5 is a sinusoidal phase-modulated signal, carrier frequency $f_{c5} = 5MHz$, pulse width $t_{r5} = 8\mu s$, pulse repetition period $T_{r5} = 100\mu s$, pulse start time $t_{05} = 40\mu s$, modulated signal frequency $f_{a5} = 100kHz$, modulation index $a_5 = 5$.

The sampling frequency of the receiver is 50MHz; the number of sampling points is 10,000 and the dimensional of observed signals is 2. The mixing matrix is generated using rand function

$$\boldsymbol{A} = \begin{bmatrix} 0.3942 & 0.0162 & 0.9100 & 0.2152 & -0.9458 \\ -0.9190 & 0.9999 & -0.4146 & 0.9766 & -0.3246 \end{bmatrix}.$$

When $m = 2, n = 5$, there are two observed signals, the method proposed in this paper is actually the original shortest path method [7]. The source signals are recovered adopting the method mentioned in this paper and CMP method [15], L1CMP method [15] and RBF network method [16], and the results are shown in Figs. 2 (a)-(c).

Figure 2 shows that with the increase of signal-to-noise ratio of observed signals, the signal to interference ratio and similarity coefficient of the signals obtained from recovery using different signal recovery algorithms tend to increase, meaning that the signal separation result is acceptable The separation effect adopting the method presented in this paper is slightly lower than CMP method but better than L1CMP and RBF network methods. As for calculation efficiency, it is reflected in Fig. 2 (c) that the calculation time of the method presented in this paper is obviously less than other methods.



(a)

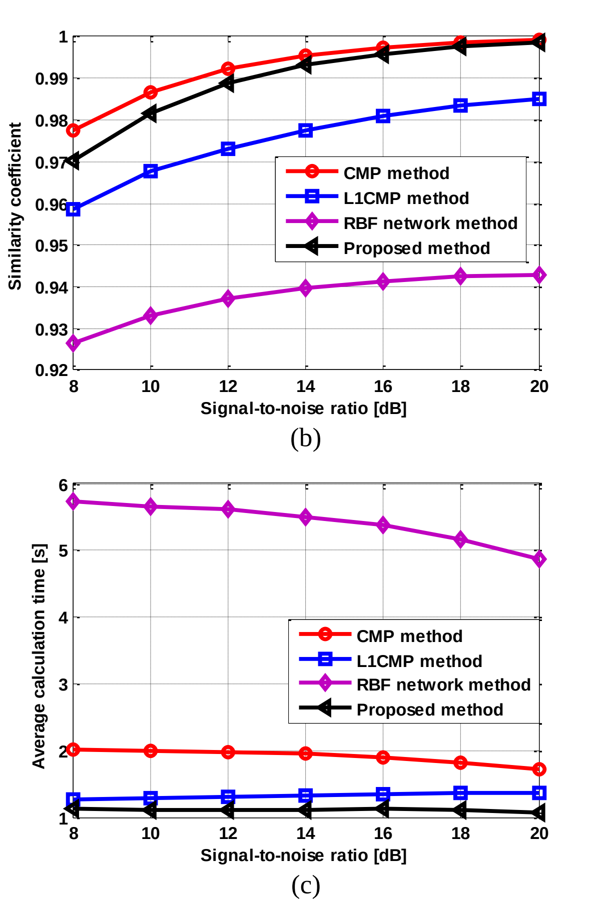


Fig. 2. Comparison of the performance and calculation time of different methods when m=2 and n=5. (a) The average signal to interference ratio of separated signals. (b) The average similarity coefficient of separated signals. (c) The average calculation time.

Experiment 2:

The parameters and number of source signals are the same as Experiment 1. The sampling frequency of the receiver is 50MHz and the number of signal sampling points is 10,000. The dimensional of observed signals is three and the mixing matrix is generated using rand function,

$$\boldsymbol{A}=\begin{bmatrix} -0.1605 & 0.8133 & 0.8729 & 0.3014 & 0.2312 \\ 0.8612 & -0.4460 & 0.4247 & 0.8280 & 0.9358 \\ -0.4823 & 0.3736 & 0.2401 & -0.4728 & -0.2662 \end{bmatrix}.$$

The recovery results of source signals by different methods are shown in Fig. 3.

Figure 3 shows that the separation effect adopting the method proposed in this paper is better than the other three methods. The computing efficiency of the method is higher.

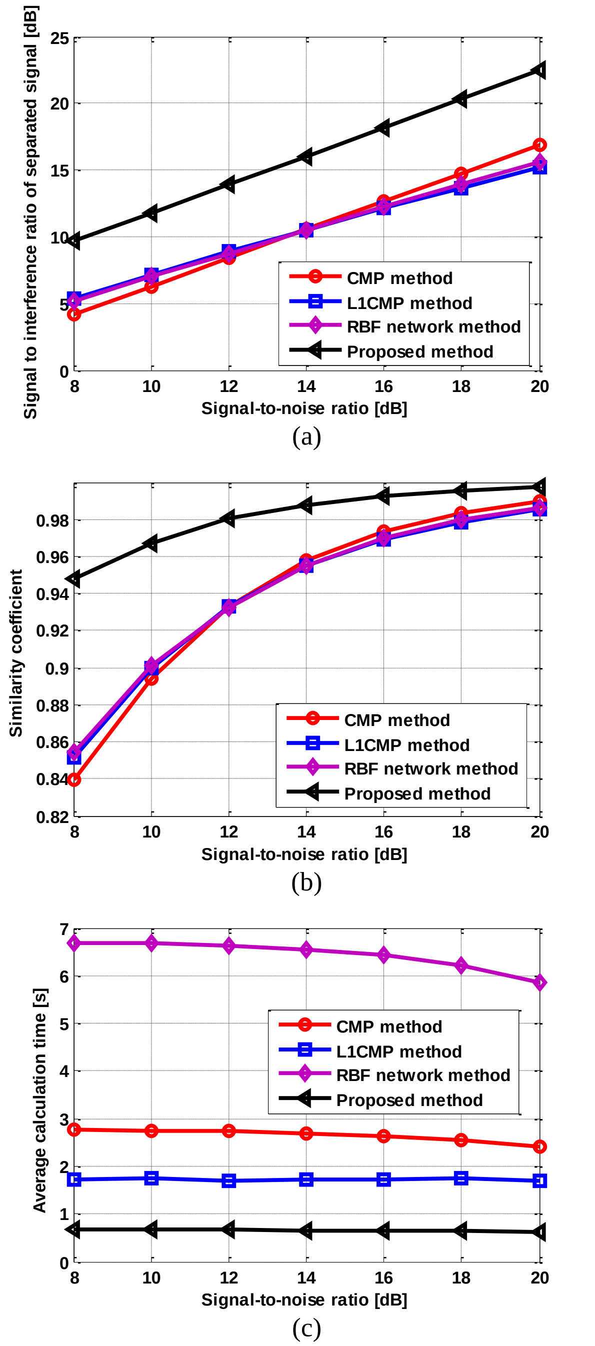


Fig. 3. Comparison of the performance and calculation time of different methods when m=3 and n=5. (a) The average signal to interference ratio of separated signals. (b) The average similarity coefficient of separated signals. (c) The average calculation time.

Experiment 3:

There are 7 source signals. Assuming the source signals are sufficiently sparse in time domain, and the signal types and parameter settings are as follows:

s_1 is a nonlinear FM signal, carrier frequency $f_{c1}=10MHz$, pulse width $t_{r1}=16\mu s$, pulse repetition period $T_{r1}=200\mu s$, instantaneous bandwidth $B_1=10MHz$, pulse start time $t_{01}=0$;

s_2 is a conventional pulse signal, carrier frequency $f_{c2}=8MHz$, pulse width $t_{r2}=15\mu s$, pulse repetition period $T_{r2}=180\mu s$, pulse start time $t_{02}=20\mu s$;

s_3 is a linear FM signal, carrier frequency $f_{c3}=5MHz$, pulse width $t_{r3}=15\mu s$, pulse repetition period $T_{r3}=180\mu s$, pulse start time $t_{03}=40\mu s$, intrapulse bandwidth $B_3=20MHz$;

s_4 is a linear FM signal, carrier frequency $f_{c4}=5MHz$, pulse width $t_{r4}=20\mu s$, pulse repetition period $T_{r4}=180\mu s$, pulse start time $t_{04}=60\mu s$, intrapulse bandwidth $B_4=15MHz$;

s_5 is a sinusoidal phase-modulated signal, carrier frequency $f_{c5}=5MHz$, pulse width $t_{r5}=20\mu s$, pulse repetition period $T_{r5}=200\mu s$, pulse start time $t_{05}=80\mu s$, modulation frequency $f_{a5}=200kHz$, modulation index $a_5=5$;

s_6 is a sinusoidal phase-modulated signal, carrier frequency $f_{c6}=5MHz$, pulse width $t_{r6}=15\mu s$, pulse repetition period $T_{r6}=200\mu s$, pulse start time $t_{06}=100\mu s$, modulation frequency $f_{a6}=200kHz$, modulation index $a_6=2$;

s_7 is a nonlinear FM signal, carrier frequency $f_{c7}=15MHz$, pulse width $t_{r7}=20\mu s$, pulse repetition period $T_{r7}=200\mu s$, intrapulse bandwidth $B_7=5MHz$, pulse start time $t_{07}=115\mu s$.

The sampling frequency of the receiver is 50MHz; the number of sampling points is 10,000 and the dimensional of observed signals is 4. The mixing matrix is generated using rand function,

$$\boldsymbol{A}=\begin{bmatrix} -0.5224 & -0.4859 & 0.1343 & -0.3511 & 0.4795 & 0.0934 & 0.2742 \\ 0.2835 & -0.5464 & -0.4342 & -0.1150 & -0.7562 & -0.6842 & -0.0333 \\ 0.5654 & 0.0297 & 0.7147 & 0.6674 & 0.4023 & 0.6325 & -0.6892 \\ -0.5719 & -0.6815 & 0.5316 & 0.6466 & 0.1906 & 0.3508 & 0.6698 \end{bmatrix}.$$

The recovery results of source signals by different methods are shown in Fig. 4.

Figure 4 shows that the separation effect adopting our method is slightly lower than the other three methods, but the signal to interference ratio of separation and similarity coefficient obtained using our algorithm are sufficient to accurately separate 7 source signals from 4 observed signals, with the law of the computing efficiency is generally the same as Experiment 2.

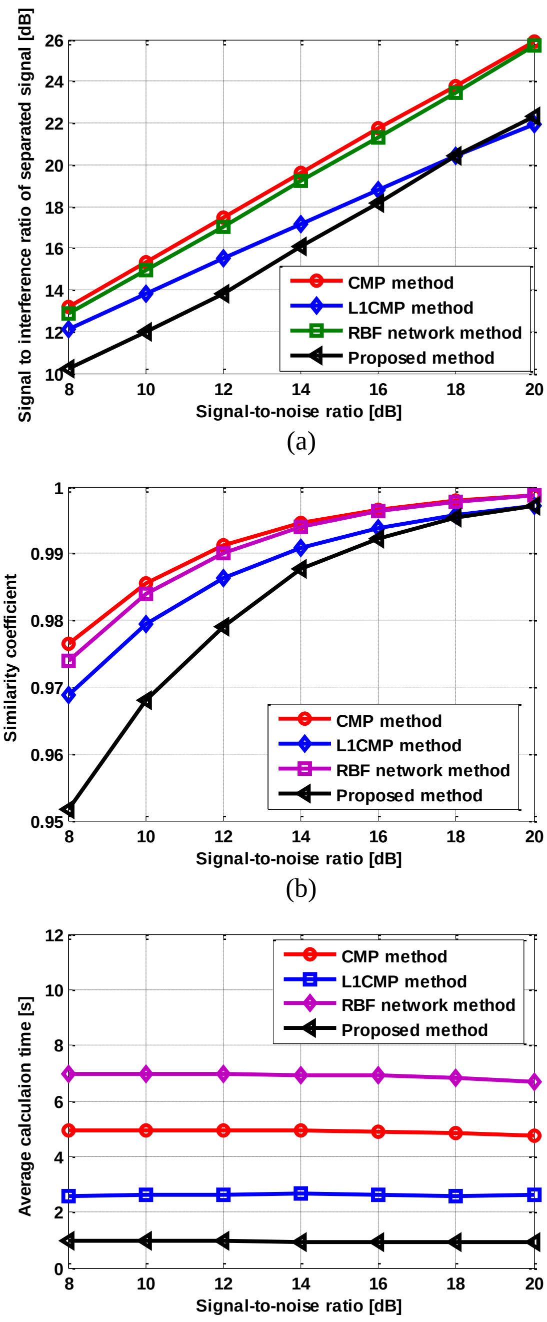


Fig. 4. Comparison of the performance and calculation time of different methods when m=4 and n=7. (a) The average signal to interference ratio of separated signals. (b) The average similarity coefficient of separated signals. (c) The average calculation time.

Experiment 4:

There are 5 source signals, and the source signals are not sufficiently sparse in time domain. Wavelet transform is used for sparse representation of source signals in this paper. The source signal types and parameter settings are as follows:

s_1 is a traditional pulse signal, carrier frequency $f_{c1} = 5MHz$, pulse width $t_{r1} = 100\mu s$, pulse repetition period $T_{r1} = 400\mu s$, pulse start time $t_{01} = 0$;

s_2 is a traditional pulse signal, carrier frequency $f_{c2} = 20MHz$, pulse width $t_{r2} = 100\mu s$, pulse repetition period $T_{r2} = 400\mu s$, pulse start time $t_{02} = 0$;

s_3 is a linear FM signal, carrier frequency $f_{c3} = 20MHz$, pulse width $t_{r3} = 50\mu s$, pulse repetition period $T_{r3} = 200\mu s$, pulse start time $t_{03} = 100\mu s$, intrapulse bandwidth $B_3 = 5MHz$;

s_4 is a linear FM signal, carrier frequency $f_{c4} = 60MHz$, pulse width $t_{r4} = 50\mu s$, pulse repetition period $T_{r4} = 200\mu s$, pulse start time $t_{04} = 150\mu s$, intrapulse bandwidth $B_4 = 5MHz$;

s_5 is a sinusoidal phase-modulated signal, carrier frequency $f_{c5} = 15MHz$, pulse width $t_{r5} = 50\mu s$, pulse repetition period $T_{r5} = 200\mu s$, pulse start time $t_{05} = 150\mu s$, modulation frequency $f_{a5} = 200kHz$, modulation index $a_5 = 1$.

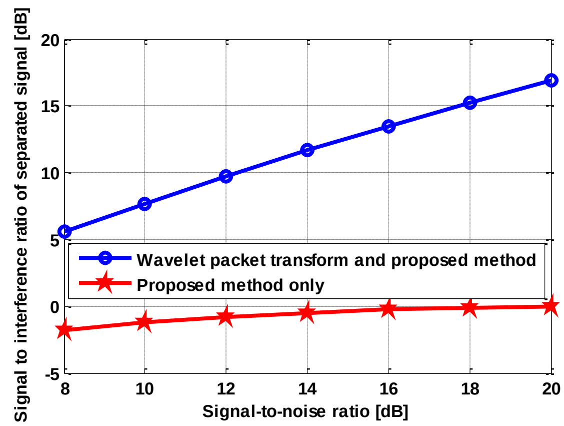
The sampling frequency of the receiver is 200MHz; the number of sampling points is 40,000. The values of two source signals at some moments are not zero, at some moments, only the value of one source signal is non-zero, while the values of other source signals are all zero, that is, the source signals are not sufficiently sparse in time domain. The mixing matrix is generated using rand function,

$$\boldsymbol{A} = \begin{bmatrix} 0.4638 & -0.6693 & -0.9474 & 0.3031 & -0.6117 \\ -0.5711 & -0.4033 & 0.0377 & -0.0828 & -0.7777 \\ 0.6773 & 0.6240 & -0.3177 & -0.9494 & 0.1454 \end{bmatrix}.$$

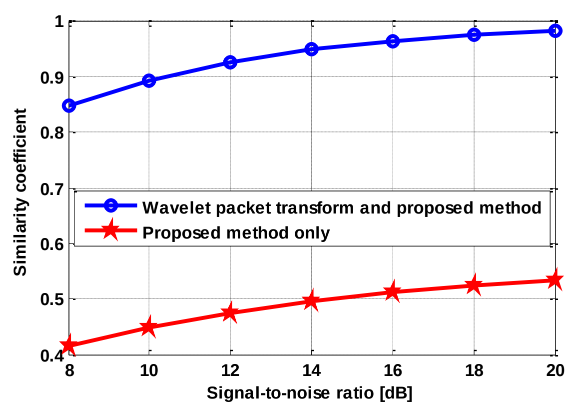
Figure 5 (a) and Fig. 5 (b) show the results by two processing methods, which are adopting the proposed shortest path method based on wavelet transformation and adopting the proposed shortest path method directly. The wavelet basis function is "dmey" and the wavelet is decomposed into 6 layers.

We can see that from Fig. 5, when the source signals are not sufficiently sparse in time domain, time domain separation signals can be obtained by seeking for sparse representation of the source signals using wavelet packet transformation. When compared with the source signal recovery result adopting our method directly in time domain, the recovery effect of the former processing method is obviously better.

The algorithm proposed can be used when there are two or more observed signals. In the shortest path method presented, ideal source signal recovery can be realized with high computing efficiency when the source signal is sufficiently or not sufficiently sparse in time domain. When the source signal is not sufficiently sparse, sparse representation can be used for the observed signals. When the source signal is sufficiently sparse in the transformation domain, the proposed algorithm can be used for source signal recovery.



(a)



(b)

Fig. 5. Performance comparison of different methods when m=3 and n=5. (a) The average signal to interference ratio. (b) The average similarity coefficient.

V. CONCLUSIONS

The shortest path method is a recovery algorithm for underdetermined blind source separation with good effect. It has the advantages of short computational time and high signal recovery accuracy. A new shortest path source signal recovery algorithm is presented based on the defect that the traditional shortest path method can only be used for the case with two observed signals. Underdetermined blind source signal recover can be realized when there are two or more observed signals by employing the proposed algorithm. The feasibility of the algorithm is validated by simulation experiments, and the proposed algorithm has high computing accuracy and low time complexity.

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