Adaptive Sparse Array Beamforming Using Correntropy Induced Metric Constrained Normalized LMS Algorithm

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Abstract **─**In order to further exploit the sparseness of antenna array and speed up the convergence of constrained normalized LMS (CNLMS) algorithm, maintaining good beam pattern performance and better output signal-to-interferences-plus-noise ratio (SINR), a new method with approximation l_0-norm constraint is proposed to improve CNLMS algorithm, and its derivation process is given in detail. In this newly proposed algorithm, the correntropy induced metric (CIM) is used to approximate the l_0-norm, which is considered construct a new cost function to fully exploit the sparsity of the antenna array and reduced the number of active array elements. Using the CIM penalty, the proposed CIM-based CNLMS (CIM-CNLMS) algorithm is derived in detail, where the Lagrange multiplier method is utilized to solve the cost function of the proposed CIM-CNLMS algorithm, and the steepest descent principle is considered to obtain the update equation. The computer simulation results demonstrate that compared with other CLMS algorithms, the new algorithm obtains better performance, which greatly reduces the proportion of active array elements in the thinned antenna array. Simultaneously, the new algorithm has excellent beam pattern performance and better SINR performance with faster convergence speed and more stable mean square error.

Index Terms **─** adaptive array beamforming, CNLMS algorithm, correntropy induced metric, l_0-norm constraint, sparse arrays.

I. INTRODUCTION

Adaptive beamforming, an essential and elementary problem in array signal processing, has attracted great attention in many applications, including sonar, radar, wireless and mobile communications, seismic sensing, biomedical engineering [1-2]. Moreover, the Linear Constrained Minimum Variance (LCMV) algorithm proposed by Frost [3] is a well-known algorithm, which gives a beam in the desired direction and forms a null in the direction of arrival (DOA) of the interfering signal. The LCMV algorithm minimizes the energy output in order to minimize the interference of the array output and the noise signal energy, maintaining a constant gain in the desired direction of the viewing direction. The adaptive beamforming algorithm adjusts the weight vector of the antenna array to match the desired signal as well as the interfering signal as a function of time. The CNLMS algorithm is considered as the normalization form of the LCMV algorithm, in which the array elements can be adjusted in real time [4].

In some applications, such as radar, sonar, in order to achieve the desired performance, we need to use many array elements to achieve the goal. However, if the array has too many array elements, the array requires a lot of operations and huge amounts of energy. Consequently, the application of many existing beamforming algorithms will cost a large amount of energy consumption in the antenna array, and the antenna array needs to provide a strong calculate ability, which make the antenna array increase the cooling equipment with superior performance, resulting in complex antenna equipment and high cost.

Till now, the existing algorithms solve the mentioned problems above. With the development of sparse signal processing technology, it becomes a new exploration direction that the algorithm of sparse signal processing is applied to design beamforming algorithms to make the weight coefficient of the array element toward sparse. In recent years, along with the development of Compressed Sensing [5], many works have been done in the field of sparse signal processing

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[6-15]. In these efforts, LASSO [16] and some new LMS-based algorithms are developed for sparse system identification [6-9]. For example, the zero-attractive least mean square (ZA-LMS) and re-weighted ZA-LMS (RZA-LMS) algorithms have been proposed for this purpose. The ZA-LMS algorithm uses a l_1-norm penalty on the LMS cost function, which gives a zero attractor in the iterations. The RZA-LMS algorithm utilizes a re-weighted zero attractor to further improve the ZA-LMS's performance.

Inspired by LASSO and the sparse LMS algorithms, the l_1-norm constrained LMS (l_1-CLMS) algorithm was proposed in [17]. After that, the new normalized version of l_1-CLMS (l_1-CNLMS) [18] algorithm and its re-weighted version (l_1-WCNLMS) [18] with superior performance have been proposed to make the antenna array element coefficients toward sparse, and the algorithm achieved good convergence performance. Recently, many re-weighted l_1-norm and l_p-norm penalties have been proposed and considered in [19-21]. And the l_0-CNLMS algorithm, which applies the penalty of approximate l_0-norm to adaptive beamforming, has been proposed in [22]. However, its computations are high because of the exponentiation operation.

To better exploit the sparse characteristics of the antenna array, and to fully utilize the advantages of CIM theory for calculating the number of non-zero entries in the array weight vector, a new CNLMS algorithm is proposed by utilizing the CIM theory to reduce the number of active array elements under the framework of adaptive beamforming. In our proposed algorithm, CIM theory is to construct a modified CNLMS cost function that implements a zero attractor in CNLMS's iterations with the help of Gaussian kernel theory, which is named as CIM-CNLMS. As a result, CIM acts as a l_0-norm to help to speed up convergence and reduce MSE of the CNLMS. The simulation results demonstrate that the proposed CIM-CNLMS algorithm can improve the sparseness of the antenna array and reduce the MSE compared with the l_1-WCNLMS algorithm. That is to say, the proposed algorithm has better superiority than the l_1-WCNLMS algorithm in terms of sparsity and MSE characteristics. The algorithms presented in this paper have potential applications in radar, sonar, and 5G antenna arrays.

II. THE ARRARY PROCESSING FUNDAMENTALS

As portrayed in Fig. 1, a planar antenna array model consisting of M omnidirectional antenna elements spaced by half-wavelength is utilized to discuss adaptive beamforming algorithms, where λ represents the wavelength of the electromagnetic wave actually used. Assuming (L+1) narrowband signals are received by the antenna elements, with one signal of interest (SOI) and L interference signals. We assume that the horizontal azimuth of the SOI is θ_{SOI} and the horizontal azimuth of the L interference signals is θ_p (p=1, 2, ..., L). Besides, the zenith of SOI is defined as $\emptyset_{SOI}$ and the zeniths of L interference signals are defined as $\emptyset_p$ (p=1, 2, …, L). The received signal at the n-th snapshot is expressed as:

$$\mathbf{x}(n) = \mathbf{a}_c\mathbf{s}(n) + \textstyle\sum_{l=1}^{L} \mathbf{a}_l\mathbf{i}_l(n) + \boldsymbol{\eta}(n), \tag{1}$$

where $\mathbf{a}$, $\mathbf{s}(n)$, $\mathbf{i}_l(n)$ and $\boldsymbol{\eta}(n)$ are the steering matrix related to the SOI and interference signal, the complex signal envelope vector as well as zero-mean Gaussian white noise vector, respectively. We assume that the SOI, interfering signals, and noise are statistically independent of each other.

Under these assumptions, the SINR of the beamformer is calculated using the following equation:

$$\mathbf{SINR}_{out} = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{R}_s\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{R}_{\xi}\mathbf{w}}, \tag{2}$$

where $\mathbf{w}$ represents the weight coefficient vector of a M×1 dimensional, and $\mathbf{R}_{\xi}$ represents the covariance matrix of the interfering signal plus noise, $\mathbf{R}_s$ represents the covariance matrix of the SOI. Then, we have:

$$\begin{cases} \mathbf{R}_s = \mathbf{a}_c\mathbf{s}(n)\mathbf{s}(n)^{\mathrm{H}}\mathbf{a}_c{}^{\mathrm{H}}, \\ \mathbf{R}_{\xi} = \mathbf{A}_k\mathbf{I}(n)\mathbf{I}(n)^{\mathrm{H}}\mathbf{A}_k{}^{\mathrm{H}} + \boldsymbol{\eta}(n)\boldsymbol{\eta}(n)^{\mathrm{H}}, \end{cases} \tag{3}$$

where $\mathbf{A}_k$ represents the steering matrix of L interfering signals, and $\mathbf{I}$ represents the interfering signal matrix composed of L interfering signals.

The output signal $\mathrm{y}(n)$ of the planar array at the nth snapshot can be expressed as:

$$\mathrm{y}(n) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(n). \tag{4}$$

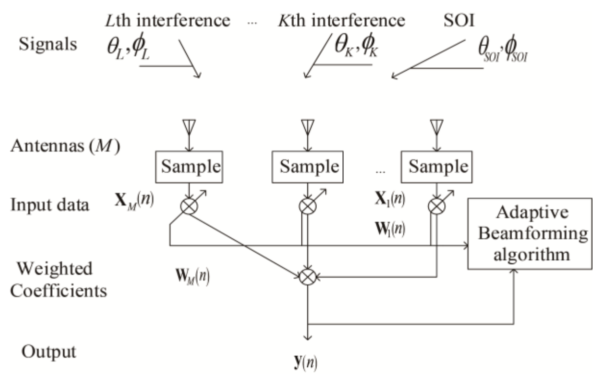


Fig. 1. Beamforming model.

III. THE CLMS ALGORITHM AND THE CNLMS ALGORITHM

A. The CLMS algorithm

The specific results of the LCMV algorithm are presented in [3], in which the weight vector representation of the LCMV algorithm is expressed as:

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{F}, \tag{5}$$

where $\mathbf{R}$ is the covariance matrix of the array input data and is defined as $E(\mathbf{XX}^{\mathrm{H}})$, $\mathbf{C}$ is the constraint matrix of the array, and $\mathbf{F}$ denotes the constraint vector whose elements are related to the signal and interference of interest, $(\cdot)^{\mathrm{H}}$ is the conjugate transpose operator.

The CLMS algorithm adopts an adaptive filtering technique, which can effectively increase the gain of the SOI and at the same time better attenuates the interference signals from other directions.

The following variables are defined as: $e_n \in \mathbb{C}$, $\mathbb{C}$ represents the set of complex numbers, e_n represents the estimation error of the adaptive filter, $\mathbf{w} \in \mathbb{C}^M$, $\mathbf{w}$ represents the vector of the coefficients, $\mathbf{x}_n \in \mathbb{C}^M$, $\mathbf{x}_n$ represents a vector composed of input signals, $d_n \in \mathbb{C}$, d_n represents the desired signal. Using the minimum mean square error criterion, the linear constraint minimum problem can be expressed mathematically as:

$$\min_{\mathbf{w}} E[|e_n{}^2|] \quad \text{s.t. } \mathbf{C}^{\mathrm{H}}\mathbf{w} = \mathbf{F}, \tag{6}$$

where $e_n = d_n - \mathbf{w}^{\mathrm{H}}\mathbf{x}_n$. $\mathbf{C}$ represents an $M \times (L+1)$ constraint matrix, and $\mathbf{F}$ represents a corresponding constraint vector containing (L+1) elements.

To solve this mathematical problem, the Lagrange multiplier is used to construct a cost function, which is given by:

$$\mathrm{G}(n) = E[|e_n|^2] + \boldsymbol{\gamma}^{\mathrm{H}}(\mathbf{C}^{\mathrm{H}}\mathbf{w} - \mathbf{F}), \tag{7}$$

where $\boldsymbol{\gamma}$ represents a Lagrange multiplier. To solve this cost function, the gradient descent principle is utilized for getting the optimal solution of the cost function.

Firstly, the two sides of the equation (7) are derived with respect to $\mathbf{w}$, and the following equation can be obtained:

$$\boldsymbol{\nabla}_{\mathbf{w}}\mathrm{G}(n) = -2E[e_n^*\mathbf{x}_n] + \mathbf{C}\boldsymbol{\gamma}, \tag{8}$$

where $\boldsymbol{\nabla}_{\mathbf{w}}\mathrm{G}(n)$ denotes the gradient vector.

In the real-time estimation, the equation (8) can be written as:

$$\boldsymbol{\nabla}_{\mathbf{w}}\mathrm{G}(n) = -2e_n^*\mathbf{x}_n + \mathbf{C}\boldsymbol{\gamma}. \tag{9}$$

Using the gradient descent principle, the update equation in the objective function is expressed as:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \frac{\mu}{2}\boldsymbol{\nabla}_{\mathbf{w}}\mathrm{G}(n). \tag{10}$$

Substituting (8) into (10), we can get the final update equation:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e_n^*\mathbf{x}_n - \frac{\mu}{2}\mathbf{C}\boldsymbol{\gamma}. \tag{11}$$

Multiply both sides of (11) by $\mathbf{C}^{\mathrm{H}}$, and combine (6) to find $\boldsymbol{\gamma}$:

$$\boldsymbol{\gamma} = (\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\left(\frac{2}{\mu}\mathbf{C}^{\mathrm{H}}\mathbf{w}_n + 2e_n^*\mathbf{C}^{\mathrm{H}}\mathbf{x}_n - \frac{2}{\mu}\mathbf{F}\right). \tag{12}$$

Re-substituting the obtained result (12) into (11) gives the updated equation:

$$\mathbf{w}_{n+1} = \mathbf{P}(\mathbf{w}_n + \mu e_n^*\mathbf{x}_n) + \mathbf{F}_{\mathbf{z}}. \tag{13}$$

In equation (13), we have:

$$\mathbf{P} = \mathbf{I}_{M\times M} - \mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}}, \tag{14}$$

$$\mathbf{F}_{\mathbf{z}} = \mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{F}. \tag{15}$$

B. The CNLMS algorithm

In the CLMS algorithm, the value of the step size μ is fixed. In order to speed up the convergence of the CLMS algorithm, the square of the posterior a posteriori error with respect to the step size at the snapshot point n can be minimized [4], [23]:

$$\frac{\partial\left[|e_{ap}(n)|^2\right]}{\partial u_n^*} = \frac{\partial\left[e_{ap}(n)e_{ap}^*(n)\right]}{\partial \mu_n^*} = 0, \tag{16}$$

where $e_{ap}(n) = d_n - \mathbf{w}_{n+1}^{\mathrm{H}}\mathbf{x}_n = e_n(1 - u_n\mathbf{x}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n)$.

Solving (16), the step-size at the nth snapshot point can be obtained:

$$\mu_n = \frac{\mu_0}{\mathbf{x}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n + \beta}, \tag{17}$$

where μ_0 is a fixed step size, which is the initial value of the algorithm's step size, and β is positive that is close to zero which is to prevent the denominator from being zero in (17).

After the previous calculations, the update equation of the CNLMS algorithm is as:

$$\mathbf{w}_{n+1} = \mathbf{P}\left[\mathbf{w}_n + \mu_0\frac{e_n^*\mathbf{x}_n}{\mathbf{x}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n + \beta}\right] + \mathbf{F}_{\mathbf{z}}. \tag{18}$$

IV. THE PROPOSED CIM-CNLMS ALGORITHM

In our proposed algorithm, the CIM is considered to approximate the l_0-norm to create a new cost function based on entropy theory [24-26]. As we know, the similarity between two vectors $\mathbf{X}$ and $\mathbf{Y}$ is measured using correntropy theory. The mathematical definition of the correlation entropy is presented as:

$$\mathrm{V}(\mathbf{X}, \mathbf{Y}) = \frac{1}{M}\sum_{i=1}^{M} k(x_i, y_i), \tag{19}$$

where $\mathbf{X} = [x_1, x_2, \dots, x_M]$, $\mathbf{Y} = [y_1, y_2, \dots, y_M]$,and $k(.)$ is a regenerative kernel function. In the equation, the Gaussian kernel function is given by:

$$k(x, y) = k(x - y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right), \tag{20}$$

where σ is the kernel width. The l_0-norm of an M-dimensional vector $\mathbf{w} = [\mathrm{w}_1, \mathrm{w}_2, \dots, \mathrm{w}_M]$ is defined mathematically as:

$$\|\mathbf{w}\|_0 = \mathrm{card}\{w_i : w_i \neq 0\}, \tag{21}$$

where the l_0-norm of the vector $\mathbf{w}$ is the number of non-zero entries in $\mathbf{w}$ and card is set the cardinality [27].

As we know, solving l_0-norm is an NP hard problem [27]. Hence, continuous function is often utilized to approximate l_0-norm [28]. In our proposed algorithm, the CIM theory is used to approximate the l_0-norm to further develop the sparsity of the CNLMS algorithm. Therefore, our approximation l_0-norm approximation:

$$\|\mathbf{w}\|_0 \sim \mathrm{CIM}(\mathbf{w}, 0) = \sqrt{\mathrm{k}(0) - \frac{1}{\mathrm{M}}\sum_{\mathrm{i}=1}^{\mathrm{M}}\mathrm{k}(\mathrm{w}_i, 0)}$$

$$= \sqrt{\frac{\mathrm{k}(0)}{\mathrm{M}}\sum_{i=1}^{\mathrm{M}}\left\{1 - \exp\left(-\frac{\mathrm{w}_i^2}{2\sigma^2}\right)\right\}}. \tag{22}$$

To simplify the expression in (22), the square root operation in (22) is removed, resulting in the approximation:

$$\|\mathbf{w}\|_0 \sim \mathrm{CIM}^2(\mathbf{w}, 0) = \frac{\mathrm{k}(0)}{\mathrm{M}}\sum_{i=1}^{\mathrm{M}}\left\{1 - \exp\left(-\frac{\mathrm{w}_i^2}{2\sigma^2}\right)\right\}, \tag{23}$$

with $k(0) = \frac{1}{\sqrt{2\pi}\sigma}$.

Then, a l_0-norm penalty on the filter coefficients is integrated in the cost function of the LMS algorithm to accelerate coefficient convergence. In our proposed algorithm, a new l_0-CNLMS algorithm is developed for adaptive beamforming control to make the coefficients toward sparseness:

$$\min_{\mathbf{w}} E[|e_n|^2] \ \text{s.t.} \begin{cases} \mathbf{C}^{\mathrm{H}}\mathbf{w} = \mathbf{F}, \\ \|\mathbf{w}\|_0 = t, \end{cases} \tag{24}$$

where $\|\mathbf{w}\|_0$ represents the number of non-zero entries in $\mathbf{w}$, and t represents the constraint value of the l_0-norm of $\mathbf{w}$.

The cost function with l_0-norm penalty is defined as:

$$\mathrm{G}_n^{l_0} = \mathrm{E}[|e_n|^2] + \boldsymbol{\gamma}^{\mathrm{H}}(\mathbf{C}^{\mathrm{H}}\mathbf{w} - \mathbf{F}) + \gamma_1(\|\mathbf{w}\|_0 - \mathrm{t}). \tag{25}$$

According to (23), we can rewrite (25) as:

$$\mathrm{G}_n^{l_0} = \mathrm{E}[|e_n|^2] + \boldsymbol{\gamma}^{\mathrm{H}}(\mathbf{C}^{\mathrm{H}}\mathbf{w} - \mathbf{F}) + \gamma_1 \left\{ \frac{1}{\mathrm{M}\sqrt{2\pi}\sigma} \sum_{i=1}^{\mathrm{M}} \left[1 - \exp\left(-\frac{\mathrm{w}_i^2}{2\sigma^2} \right) \right] - t \right\}. \tag{26}$$

Then, we have:

$$\begin{cases} \nabla_{\mathbf{w}} \mathrm{G}_n^{l_0} = -2e_n^*\mathbf{x}_n + \mathbf{C}\boldsymbol{\gamma} + \gamma_1 \mathbf{J}_n, \\ \mathbf{J}_n = \dfrac{1}{\mathrm{M}\sqrt{2\pi}\sigma^3} \left[\mathrm{w}_1 \exp\left(\dfrac{-\mathrm{w}_1{}^2}{2\sigma^2} \right), \dots, \mathrm{w}_M \exp\left(\dfrac{-\mathrm{w}_M{}^2}{2\sigma^2} \right) \right]^{\mathrm{H}}. \end{cases} \tag{27}$$

According to the gradient descent principle, the weight coefficient update equation of the CIM-CNLMS algorithm can be obtained:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \frac{\mu}{2}\{-2e_n^*\mathbf{x}_n + \mathbf{C}\boldsymbol{\gamma} + \gamma_1 \mathbf{J}_n\}. \tag{28}$$

Next, the coefficient $\boldsymbol{\gamma}$ is gotten by multiplying $\mathbf{C}^{\mathrm{H}}$ at both sides of (28). According to the constraint condition, $\mathbf{C}^{\mathrm{H}}\mathbf{w}_n = \mathbf{C}^{\mathrm{H}}\mathbf{w}_{n+1} = \mathbf{F}$, we get,

$$\boldsymbol{\gamma} = (\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}}(2e_n^*\mathbf{x}_n - \gamma_1\mathbf{J}_n). \tag{29}$$

Here, we assume that the update equation in the algorithm has tended to converge, i.e., $\mathbf{w}_n = \mathbf{w}_{n+1}$. The approximation condition $\mathbf{J}_n^{\mathrm{H}}\mathbf{w}_{n+1} = \mathrm{t}$ is proposed in [8] because $\mathbf{w}_n$ and $\mathbf{w}_{n+1}$ are in the same quadrant. So, we have the constraints:

$$\mathbf{J}_n^{\mathrm{H}}\mathbf{w}_{n+1} = \mathrm{t}, \quad \mathbf{J}_n^{\mathrm{H}}\mathbf{w}_n = \mathrm{t}_n. \tag{30}$$

By multiplying the left and right sides of the equation in (28) by $\mathbf{J}_{\mathrm{n}}^{\mathrm{H}}$, the equation is obtained:

$$\mathrm{t} = \mathrm{t}_n - \frac{\mu}{2}\{-2e_n^*\mathbf{J}_n^{\mathrm{H}}\mathbf{x}_n + \mathbf{J}_n^{\mathrm{H}}\mathbf{C}\boldsymbol{\gamma} + \gamma_1\mathbf{J}_n^{\mathrm{H}}\mathbf{J}_n\}. \tag{31}$$

Substituting the results in (29) into equation (31) and separating γ_1, we have:

$$\gamma_1 = -\frac{2}{\mu \mathrm{r}} e_0(n) + \frac{2e_n^*\mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n}{\mathrm{r}}, \tag{32}$$

where $\mathrm{r} = \mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{J}_n$ is a scalar, and $e_0(n) = \mathrm{t} - \mathrm{t}_n$.

Substituting (29) and (32) into (28), the final update equation is obtained:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \frac{e_0(n)}{\mathrm{r}}\mathbf{P}\mathbf{J}_n + \mu e_n^* \left(\mathbf{P}\mathbf{x}_n - \frac{\mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n\mathbf{P}\mathbf{J}_n}{\mathrm{r}} \right). \tag{33}$$

We set:

$$\begin{cases} \mathrm{r} = \mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{J}_n, \\ e_0(n) = \mathrm{t} - \mathbf{J}_n^{\mathrm{H}}\mathbf{w}_n, \\ \mathbf{F}_0(n) = \dfrac{e_0(n)}{\mathrm{r}}\mathbf{P}\mathbf{J}_n, \\ \mathrm{c} = \dfrac{\mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n}{\mathrm{r}}. \end{cases} \tag{34}$$

Then (33) can be written as:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e_n^*\mathbf{P}(\mathbf{x}_n - \mathrm{c}\mathbf{J}_n) + \mathbf{F}_0(n). \tag{35}$$

The previously derived normalized version of the CLMS algorithm is utilized in the l_0-CLMS algorithm and l_0-norm is approximated using CIM. Substituting (35) into $e_{ap}(n) = d_n - \mathbf{w}_{n+1}^{\mathrm{H}}\mathbf{x}_n$ gives the equation:

$$e_{ap}(n) = e_n[1 - \mu_n(\mathbf{x}_n^{\mathrm{H}} - \mathrm{c}^*\mathbf{J}_n^{\mathrm{H}})\mathbf{P}\mathbf{x}_n] - \mathbf{F}_0^{\mathrm{H}}(n)\mathbf{x}_n. \tag{36}$$

Based on the previous derivation of the CNLMS algorithm, the step size of the new CIM -CNLMS algorithm is realized and given by:

$$\mu_n = \frac{\mu_0[e_n - \mathbf{F}_0^{\mathrm{H}}(n)\mathbf{x}_n]}{e_n(\mathbf{x}_n^{\mathrm{H}} - \mathrm{c}^*\mathbf{J}_n^{\mathrm{H}})\mathbf{P}\mathbf{x}_n + \alpha}. \tag{37}$$

A fixed convergence control factor μ_0 is introduced to control the offset, and α is a positive parameter that is close to 0 to prevent the denominator from being 0 in the equation (37). By substituting (37) into (35), the updating equation of the new CIM-CNLMS is obtained and presented as:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu_n e_n^*\mathbf{P}(\mathbf{x}_n - \mathrm{c}\mathbf{J}_n) + \mathbf{F}_0(n), \tag{38}$$

where

$$\begin{cases} \mathbf{P} = \mathbf{I}_{M\times M} - \mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}}, \\ \mathrm{c} = \dfrac{\mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{x}_n}{\mathrm{r}}, \\ e_n = d_n - \mathbf{w}^{\mathrm{H}}\mathbf{x}_n, \\ \mu_n = \dfrac{\mu_0[e_n - \mathbf{F}_0^{\mathrm{H}}(n)\mathbf{x}_n]}{e_n(\mathbf{x}_n^{\mathrm{H}} - \mathrm{c}^*\mathbf{J}_n^{\mathrm{H}})\mathbf{P}\mathbf{x}_n + \alpha}, \\ \mathrm{r} = \mathbf{J}_n^{\mathrm{H}}\mathbf{P}\mathbf{J}_n, \\ \mathbf{F}_0(n) = \dfrac{e_0(n)}{\mathrm{r}}\mathbf{P}\mathbf{J}_n. \end{cases} \tag{39}$$

V. SIMULATION RESULTS

In this section, we constructed several simulation experiments to test the performance of the proposed algorithm, whose performance is compared with the algorithms mentioned in the previous references [18]. The signals used in the simulation experiment are QPSK signals, four interference signals and one SOI signal. The horizontal azimuth of the SOI signal is 90°, and the horizontal azimuth of the four interference signals are respectively 80°, 22°, 52°, 147°. The zeniths of all the signals are 45°. The signal-to-noise ratio (SNR) is set to 20 dB, and the interference-to-noise ratio (INR) is set to 40 dB. The frequency for all the experiments is 2 GHz. These signals are received by a regular hexagonal array (HA) with 91 elements. In the simulation, the initial steps μ_0 for the CIM-CNLMS algorithm, the CNLMS algorithm, and the l_1-WCNLMS algorithm are 8×10^{-2}, 5×10^{-3}, 5×10^{-2}, respectively. In the simulation experiment, the constraint factor t of the CIM-CNLMS algorithm is 0.4, and the constraint factor t of the l_1-WCNLMS algorithm is 1.07. In the simulation experiment, the number of iterations of the data is 12000. In the experiment, $\sigma = 0.0032$ is used in the CIM-CNLMS algorithm.

Figure 2 shows the beam pattern performance of the proposed algorithm, and compares the beam pattern performance of the proposed algorithm with other existing algorithms. Our proposed CIM-CNLMS algorithm forms a null at the horizontal incident angle of the interfering signal, providing almost identical main lobe at the incident direction of the SOI. In addition, we can clearly see in the figure that the proposed algorithm has lower side lobe level (SLL) compared to l_1-WCNLMS. Therefore, the proposed algorithm has better beam performance than the l_1-WCNLMS algorithm.

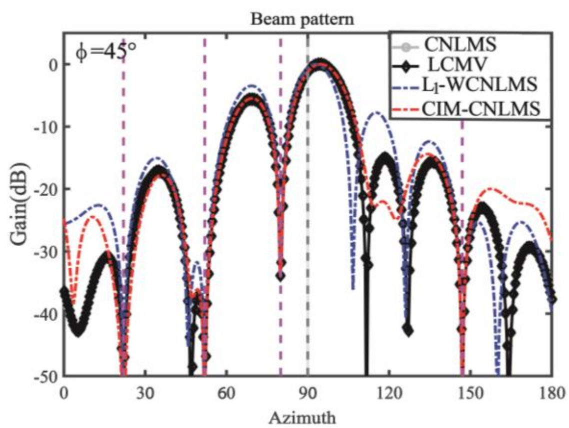
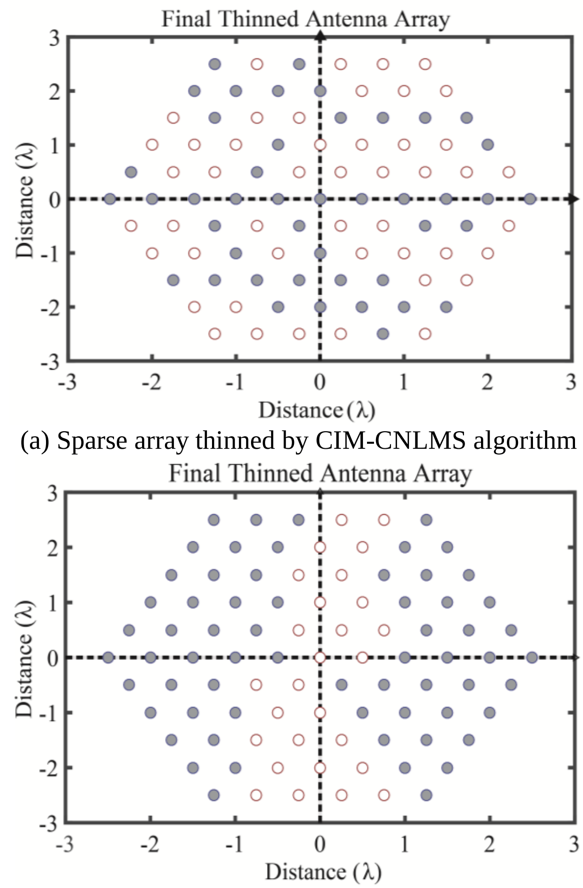


Fig. 2. Beam patterns comparison of CIM-CNLMS algorithm with the LCMV, CNLMS and l_1-WCNLMS algorithms. The pink dot lines represent the interferences, and the black dot line represents the SOI.

Figure 3 (a) is a sparse array generated by proposed CIM-CNLMS algorithm. The number of active array elements is 44 in an array with 91 elements, resulting in the sparsity of our proposed algorithm of 48.4%. Figure 3 (b) is a sparse array generated by the l_1-WCNLMS algorithm. The number of active array elements is 64, leading to the sparseness of the sparse array of 70.3%. Comparing the two previous algorithms, we can clearly see that our proposed CIM-CNLMS algorithm is far superior to the l_1-WCNLMS algorithm in terms of the finalized sparsity and the performance.

Figure 4 shows the MSE of the three algorithms, where the blue line represents the MSE of the CNLMS algorithm, the red line represents the MSE of the proposed CIM-CNLMS algorithm, and the yellow line denotes the MSE of the l_1-WCNLMS algorithm. From the figure, the proposed algorithm has the same MSE value as the l_1-WCNLMS algorithm after convergence, but our proposed algorithm converges at 1000th iteration, while the l_1-WCNLMS algorithm converges at 4000th iteration, and the CNLMS algorithm converges at the 8000th iteration. Therefore, the proposed algorithm converges the fastest. It is not difficult to conclude that the proposed algorithm has the best MSE performance among these algorithms.



(a) Sparse array thinned by CIM-CNLMS algorithm

(b) Sparse array thinned by l_1-WCNLMS algorithm

Fig. 3. The thinned sparse arrays with black dots of active array elements and white dots of inactive array elements.

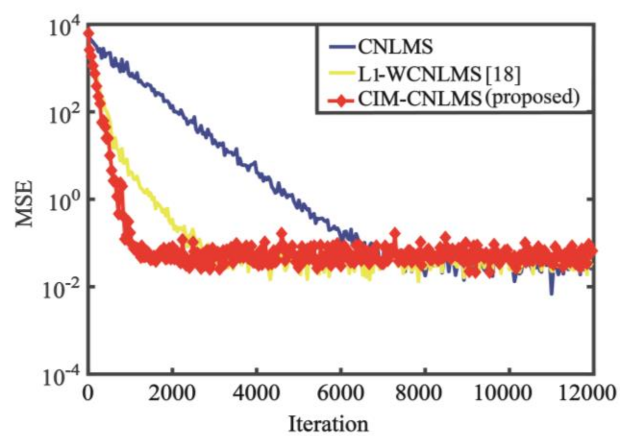


Fig. 4. Convergence of the proposed CIM-CNLMS algorithm.

The SINR results of the CIM-CNLMS are presented in Fig.5. From Fig.5, we can see that the SINR of the proposed CIM-CNLMS algorithm is better than the l_1-WCNLMS algorithm. However, it should be further improved to get a nearly same SINR with the optimal one. The proposed algorithm can also be used for sparse DOA applications like [29] and uses the block norm in [30-33]. In addition, the proposed techniques can also be used in MIMO arrays [34-36] and UAV systems [37].

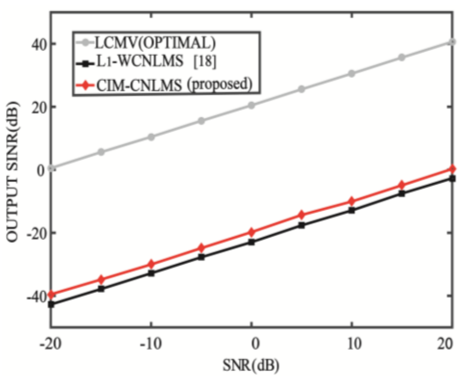


Fig. 5. SINR of the proposed CIM-CNLMS algorithm.

VI. CONCLUSION

In this paper, an improved adaptive beamforming algorithm, which is named as correntropy induced metric based constrained normalized least mean square (CIM-CNLMS), has been proposed and investigated for thinning arrays to reduce the computations and exploiting the sparsity. The CIM-CNLMS algorithm remained main lobe in the direction of SOI, and suppressed the interferences using nulls in the direction of the interferences. The simulation results demonstrated that the proposed CIM-CNLMS algorithm reduces the number of active array elements for achieving the desired performance like the l_1-WCNLMS algorithm. Additionally, the proposed algorithm has good beam pattern performance and better MSE performance in comparison with the popular algorithms mentioned in this paper. In addition, the output SINR of the proposed algorithm is better than the l_1-WCNLMS algorithm.

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