Integrated Simulation and Analysis of Super Large Slotted Waveguide Array

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Abstract ─ Aiming at the simulation problem of the super large slotted waveguide array antenna, the parallel higher-order method of moment is used and the coupling effect between each slot element is taken into account to perform the integrated and accurate simulation. In order to ensure that the algorithm is efficient and stable in the parallel process, the BDPLU strategy is introduced to reduce the communication pressure and eliminates the redundant communication of the equation solving when pivoting, which speeds up the process of matrix equation solving. According to different types of waveguide port forms, the computation of rectangular wave port and coaxial wave port is studied, and a new parallel matrix filling technique of wave port is used to accelerate the matrix filling process. Numerical examples calculated at "Tianhe-2" supercomputer show that the algorithm can efficiently and accurately handle the simulation analysis of most types of complex slotted waveguide array.

Index Terms ─ BDPLU algorithm, higher-order MoM, parallel algorithm, slotted waveguide array antenna, wave port.

I. INTRODUCTION

Slotted waveguide array antennas are widely used in microwave communication, military radar and other fields because of its low side-lobe and high gain characteristics [1,2,15]. In the military field, slotted waveguide array antennas are often used as receiver and transmitter in large radar systems. Such as the airborne radars for various types of aircraft, the guidance radars in weapon guidance systems and missile defense systems. In the civilian field, the high-speed development of 5G communication has increasingly higher performance requirements for base station antennas. As a kind of antenna with low ohmic loss in high-frequency, slotted waveguide array antennas are gradually being used as a new generation of 5G antennas. Therefore, with the development of computer technology, accurate and rapid simulation can effectively shorten the antenna design cycle and improve the antenna design efficiency. As one of the important method of antenna design, numerical simulation has been paid more attention in the design of slotted waveguide array antennas. For the simulation of slotted waveguide array antenna, the mainstream research methods are divided into two categories: high frequency and low frequency. High-frequency methods include Physical Optics (PO), Uniform Theory of Diffraction (UTD), Shooting and Bouncing Ray (SBR), etc. The advantages of those methods are faster calculation and low resource consumption. But the disadvantage is that those methods cannot calculate the coupling effect between the units, so the accuracy of those methods are lower and the error between simulation and practice is larger. Low frequency methods include Method of Moments (MoM), Finite Element Method (FEM), Finite-Difference Time-Domain (FDTD), etc. The advantage of those methods is that the calculation is accurate, but the disadvantages are that they need long calculation time and high resource consumption. Therefore, the paper [3, 4] proposed a hybrid method of high frequency method and low frequency method for antenna simulation, which can increase the scale of the problem and improve the efficiency of the solution while losing some accuracy. The paper [5] uses the parallel domain decomposition method to analyze the slotted waveguide array antennas, and achieves the accuracy requirements by combining the finite element method, the boundary element method and the fast multiple method (FMM). The paper [13] combines MoM with the method of generalized equivalent circuit to carry out a computational study of broadband waveguide. However, so far in the published paper, there are still few mentions of accurately integrated simulation of super large slotted waveguide array antennas with more than 4000 units.

Traditional simulation algorithms use the fast approximation algorithms such as the iterative algorithms (like the multilevel fast multipole algorithm) and the high-low frequency hybrid algorithms (like PO-MoM and PO-FEM). But when dealing with such super large slotted waveguide array antennas, the convergence of the algorithm cannot be guaranteed and

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the accuracy of engineering applications is difficult to achieve. Therefore, in order to solve this problem, this paper uses the previously completed parallel higher-order MoM kernels [6-8], which uses the higher-order basis functions instead of the traditional RWG (Rao-Wilton-Glisson) basis functions that can greatly reduce the unknowns of MoM. At the same time we introduce parallel computing technology to expand the solution scale of MoM and break the frequency limit. So the MoM can solve high-frequency complex electromagnetic simulation problems. In order to ensure the efficiency and stability of the solution of the matrix equation of the large-scale electromagnetic simulation problems, a new matrix equation solving algorithm BDPLU (Block Diagonal Pivoting LU Decomposition) is proposed by studying the matrix characteristics of the higher-order MoM. This algorithm can alleviate the problem of communication congestion when solving very large matrices. The BDPLU algorithm changes the traditional principal component selection strategy, completely eliminating the communication of the process of the principal component selection in the LU decomposition process, speeding up the process of the matrix equation solution, and avoiding the unstable factors caused by dense and frequent decomposition matrix communication. The comparison results for a single waveguide slot antenna between this algorithm and the commercial software FEKO are given to prove the accuracy of this algorithm. With the help of the "Tianhe II" supercomputer, a wide-side waveguide slot array antenna and a narrow-side waveguide slot array antenna are simulated by this algorithm using 19200 CPU cores. The number of the units of the two slotted array antenna are both more than 4000 and the computing time of them are both about 4.5h. Numerical examples show that the algorithm used in this paper can stably and efficiently solve the simulation problems of different types of super large slotted waveguide array antenna. And this algorithm provides an effective and reliable guarantee for the future analysis and design of super large slotted waveguide array antenna.

II. THEORETICAL ANALYSIS

A. Higher-order moment of method

For the electrically large scale problem, the traditional RWG MoM will produce a huge complex dense matrix. Therefore, large resource consumption and long calculation time make MoM difficult to effectively deal with such problems. Compared with the traditional RWG basis functions to establish the current continuity equation on the surface of a pair of triangulars, higher-order MoM introduces a higher-order polynomial basis functions and the equation is established by using a bilinear surface. The increase of the basis functions' order can use fewer basis functions to simulate the current distribution, thereby greatly reducing the unknown quantity of the complex dense matrix produced by the higher-order MoM. The number of unknowns produced by the higher-order MoM is about one tenth of the traditional RWG MoM, and there is almost no loss of accuracy. The storage complexity and computation complexity of the LU decomposition of MoM matrix are $O(N^2)$ and $O(N^3)$ respectively, where N is the number of unknowns, and the trend of N is increasing exponentially. Therefore, under the premise of ensuring accurate simulation, the cost of storage and the computation time of the higher-order MoM compared with the traditional RWG MoM will be greatly reduced.

The bilinear surface is a non-planar surface quadrilateral that can be determined with four vertices, as shown in Fig. 1. Its parametric equation is:

$$\begin{aligned} \mathrm{r}(p,s) = \frac{1}{\Delta p \Delta s}[& r_{11}(p_2 - p)(s_2 - s) + r_{12}(p_2 - p)(s - s_1) \\ & + r_{21}(p - p_1)(s_2 - s) + r_{22}(p - p_1)(s - s_1)] \end{aligned} \quad ,(1)$$

$$\Delta p = p_2 - p_1,\ \Delta s = s_2 - s_1,\ p_1 \le p \le p_2,\ s_1 \le s \le s_2$$

where r_{11}, r_{12}, r_{21} and r_{22} are the position vectors of the four vertices, respectively; p and s represent local coordinate systems; p_1 and p_2 are the starting and ending coordinates along the p direction; s_1 and s_2 are the starting and ending coordinates along the s direction.

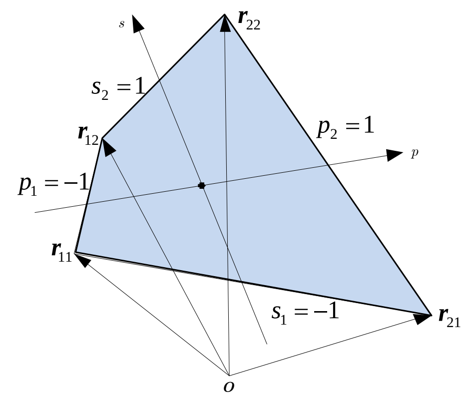


Fig. 1. Bilinear surface.

B. BDPLU algorithm

The commercial math library needed in the matrix solving process, such as Intel MKL [16] and the open source library ScaLapack [17]. However, these libraries fail or deteriorates on supercomputers with special architectures, such as Tianhe-2 system from Guangzhou in China. Therefore, our previous work developed a direct solver using the CALU algorithm to improve the performance of the panel factorization in parallel LU decomposition [18]. Due to reduced communication of CALU, it has better scalability than Intel MKL and ScaLapack. However, those parallel LU solvers are general-purpose solvers for solving matrix equations, without considering the specific features of the MoM impedance matrices. Under this situation, based on our previous studies, we introduce a new pivoting scheme based on diagonally dominant matrices of MoM, which named as Block Diagonal Pivoting LU.

The matrix generated by the higher-order MoM can be divided into two parts: self-impedance element and mutual impedance element. From the perspective of physical concepts, diagonal elements represent the self-action of the same basis functions, and non-diagonal elements represent the interactions between the various basis functions [10]. Generally speaking, the self-effect is greater than the mutual effect, so that the complex dense matrix produced by the higher-order MoM has the characteristic of diagonal dominance, and this characteristic has been maintained during the LU decomposition process, as shown in Fig. 2. Utilizing the diagonal-dominant property of MoM, the diagonal block of the matrix is always guaranteed during the parallel matrix filling process and the parallel matrix equation solving process. Therefore, the operation of pivoting during the LU decomposition process can be omitted, and the time of pivoting is saved. At the same time, the communication between processes in the LU decomposition process is avoided, which improves the efficiency of the algorithm and ensures the stability of the algorithm.

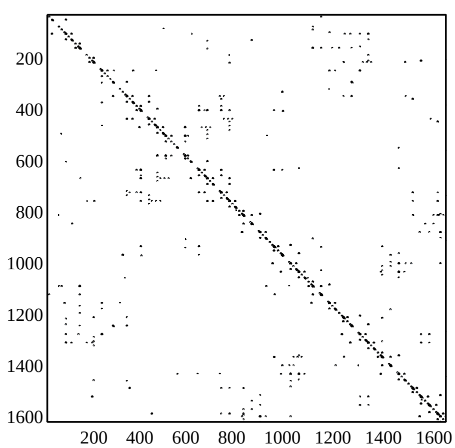


Fig. 2. Diagonally dominant of matrix.

The performance of the parallel LU decomposition solver can be further improved, with the diagonal dominance characteristic of impedance matrices taken into account. Compared with the traditional LU solver, the communication and computation time of the new pivoting scheme is analyzed.

Assume that the communication latency is α and the communication bandwidth is $1/\beta$. Thus, the communication time T taken to send a message of size L is:

$$T = \alpha + \beta L. \tag{2}$$

The process of pivoting in the BDPLU solver is given in Fig. 3. As we can see that there is no internodes communication during the kth panel column rotation. For every column in the panel, those should perform n_b size binary-exchange. The total communication time is:

$$\begin{aligned} T_{comm,BDPLU} &= \alpha \times \log_2 P_r + n_b \times (n_b \beta) \times \log_2 P_r \\ &= \alpha \log_2 P_r + \beta n_b^2 \log_2 P_r, \end{aligned} \tag{3}$$

where n_b is the number of columns in the panel, $log_2 P_r$ is complexity of binary-exchange.

The total communication time of the traditional LU is:

$$\begin{aligned} T_{comm,traditional\,lu} &= n_b \times (\alpha + 2 \times n_b \beta) \times \log_2 P_r \\ &= n_b \alpha \log_2 P_r + 2\beta n_b^2 \log_2 P_r. \end{aligned} \tag{4}$$

And the computation time of both is:

$$T_{comp,lu} = (m - \frac{n_b}{3}) n_b^{\,2} \gamma. \tag{5}$$

Comparing the formula (4) and (5), it can find that DBPLU requires less communication time than traditional LU.

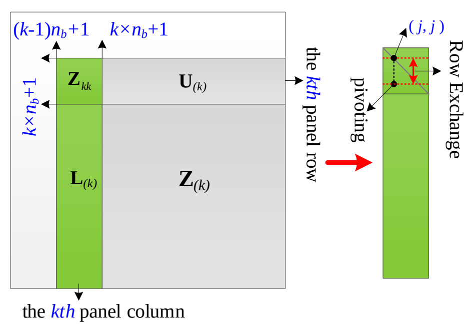


Fig. 3. Block diagonal pivoting LU scheme.

C. Wave port theory

The wave port theory of MoM is developed based on the pattern matching theory and the aperture coupling equivalent principle. Two types of the ports used in this paper are rectangular wave port and coaxial wave port. The normalized tangential vectors of the electric field of them are different, and the electromagnetic fields generated by the equivalence principle of LOVE are different.

For a rectangular waveguide, the normalized tangential vector of the electric field in the TE model is:

$$\boldsymbol{e}_i=\begin{cases} -\sqrt{\frac{2}{ab}}\sin(\frac{n\pi}{b}y)\hat{\boldsymbol{a}}_x & m=0,\ n\neq 0 \\ \sqrt{\frac{2}{ab}}\sin(\frac{m\pi}{a}x)\hat{\boldsymbol{a}}_y & m\neq 0,\ n=0 \\ \sqrt{\frac{2}{ab}}\sqrt{\frac{2}{(an)^2+(bm)^2}}\begin{bmatrix} bm\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)\hat{\boldsymbol{a}}_y \\ -an\cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)\hat{\boldsymbol{a}}_x \end{bmatrix} & m\neq 0,\ n\neq 0 \end{cases}, \quad (6)$$

where a and b are the length and width of the rectangular waveguide, respectively; $\hat{a}_x$ and $\hat{a}_y$ are the direction vectors in the local coordinate system on the port surface, as shown in Fig. 4 (a).

For a coaxial waveguide, its main model is the TEM model, and its normalized tangential vector of the electric field is:

$$\boldsymbol{e}_1=\frac{1}{\sqrt{2\pi\ln\left(\frac{b}{a}\right)}}\frac{1}{r}\cdot\hat{\boldsymbol{a}}_r, \quad (7)$$

where b is the outer diameter of the coaxial, a is the inner diameter of the coaxial, and r is the distance from any point on the port surface to the center of the coaxial, as shown in Fig. 4 (b).

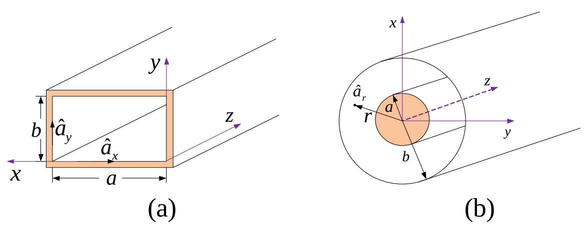


Fig. 4. Two port models: (a) rectangular model and (b) coaxial model.

D. Parallel port matrix filling

The MoM is a method that discretizes continuous equations into algebraic equations. The problem is discretized into an algebraic equation and then transformed into a matrix equation. Finally, the solution of the problem is obtained through the process of matrix inversion. Therefore, the size of the matrix generated by MoM will affect the calculation speed.

In order to speed up the port matrix filling process, this paper adopts a new parallel matrix filling strategy based on [9]. The strategy allocates the impedance matrix generated by the port to each process in parallel in the form of a circular block distribution, and introduces local index information. On the basis of retaining the feature of renumbering common edges, the new impedance matrix is filled into the existing impedance matrix in parallel according to the sequence of each process, which reduces the reordering time and improves the matrix filling efficiency. The filling strategy is as shown in Fig. 5.

```
Do k = 1,nel                        ! loop geometric elements for basis functions
  Do kp = 1,nep(k)                  ! loop p-direction subdivisions of kth geometric element
    Do ks = 1,nes(k)                ! loop s-direction subdivisions of kth geometric element
      Do l = 1,nel                  ! loop geometric elements for testing functions
        Do lp = 1,nep(l)            ! loop p-direction subdivisions of lth geometric element
          Do ls = 1,nes(l)          ! loop s-direction subdivisions of lth geometric element
            find_Zmn_index(k,kp,ks,l,lp,ls)
            flag(ls)=0
            If (m,n belongs to this process) then
              If (flag(ls)==0) then          ! if flag is false, then perform integration
                Compute_integral
                flag(ls)=1
              Endif
              Calculate the value of Z(m,n)
            Endif
            … inner loops end here …
          Enddo
        Enddo
      Enddo
    Enddo
  Enddo
Enddo
```

Fig. 5. HOMoM filling strategy.

III. INTRODUCTION TO COMPUTING PLATFORMS

The computing platform used in this paper is the Tianhe II supercomputer at the National Center for Supercomputing Guangzhou, with a peak computing speed of 54.9 PFlops (PetaFlops) and a double-precision floating-point continuous computing speed of 33.9 PFlops. And the platform won the top spot in the world's supercomputer rankings six times in a row.

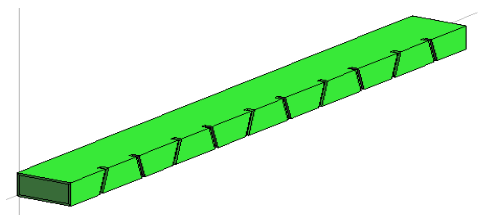
The Tianhe II supercomputer is composed of 16,000 computing nodes, each node contains 2 E5-2692 processors based on the Ivy Bridge architecture, with a total of 32,000 Ivy Bridge processors and 768,000 CPU cores. The operating system is Kirin operating system and Ubuntu Linux. Programming language environments include C, C ++, Fortran, Java, MPI, OpenMP. This paper uses a maximum of 19,200 cores with 800 nodes.

IV. NUMERICAL EXAMPLES

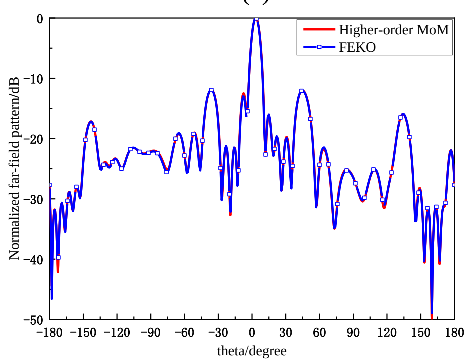
This paper uses the computing platform as the "Tianhe II" supercomputer platform, the higher-order MoM is used for waveguide slot array antenna simulation, and BDPLU algorithm is used to speed up the calculation. The algorithm reliability of parallel higher-order MoM has been verified in previous work [11,12,14]. The verification results of a single waveguide and a small waveguide array are given firstly. Secondly, the simulations are performed for K-band wide-side slotted waveguide array and Ka-band narrow-side slotted waveguide array, and the calculation results and resource consumption are given. It is proved that the method in this paper can efficiently, reliably and stably solve the simulations of the radiation problem of the super large slotted waveguide array antennas.

A. Correctness verification

To verify the correctness of the algorithm, a single narrow-side waveguide antenna is simulated. The working frequency of the waveguide is 9.375GH. There are 10 slot units, and the size of each unit is 22.86mm×10.16mm×266.58mm. The port is a rectangular wave port feed, and the other end is matched. The antenna model is shown in Fig. 6 (a). This method is compared with the RWG MoM of the commercial software FEKO, and the result is shown in Fig. 6 (b).



(a)

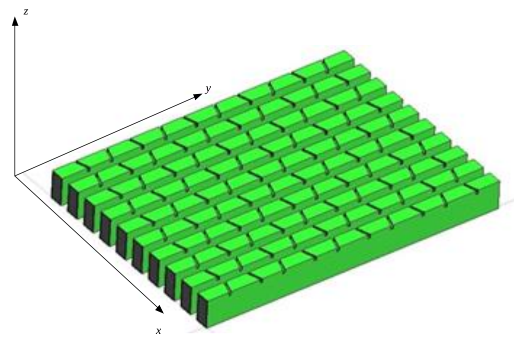


(b)

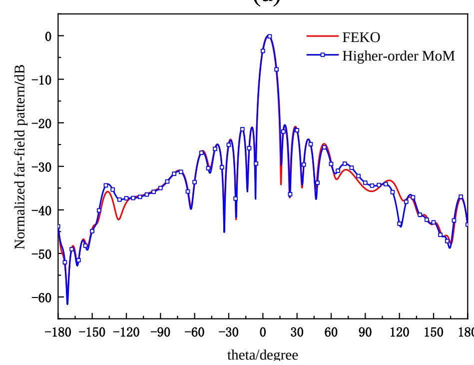
Fig. 6. (a) Model of single narrow-sided waveguide, and (b) result comparison.

The red solid line in Fig. 6 (b) is the result of this method, and the blue dashed line is the FEKO simulation result. It can be seen that the results of the two methods are in good agreement, with slight differences in some angles. The maximum difference does not exceed 0.5dB, which means the accuracy of this method meeting the requirements of engineering applications.

A small narrow slotted waveguide array is used to verify the accuracy of the coupling between units. This array is composed of 10 single waveguides described above. The antenna model is shown in Fig. 7 (a). The radiation pattern of the antenna array is calculated by using higher-order MoM and FEKO-RWG MoM respectively. For HOMoM, the unknown is 26.090, which requires about 10.9GB of memory; for the FEKO-RWG MoM, the unknown is 135,367, which requires about 293.1GB of memory. The HOMoM unknown is 1/5.19 of the FEKO-RWG MoM. The result comparison is shown in Fig. 7 (b). It can be seen that the two methods basically agree. The results show that the method is effective and feasible in dealing with the coupling between waveguides.



(a)



(b)

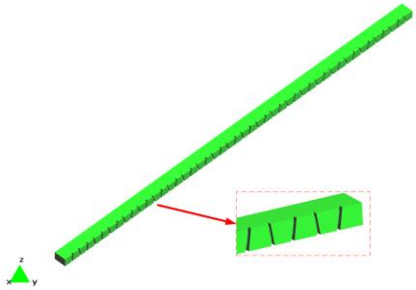
Fig. 7. (a) Model of narrow-sided waveguide array, and (b) result comparison.

B. Narrow-sided waveguide array

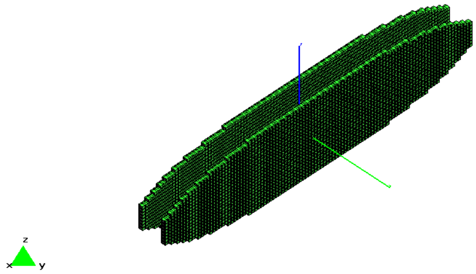
Narrow-side slotted waveguide array is a two-sided array composed of two waveguide arrays. Each array contains 2068 slot units, a total of 4136 slot units, and its model is shown in Fig. 8. The working frequency of the antenna is in Ka-band, and the port is fed by the rectangular port. The number of unknowns generated by the higher-order MoM is 1,167,436, and the required memory is 19.83TB. A total of 19200 CPU cores of 800 nodes are used for simulation. The results are shown in Fig. 8, and the resource consumption is shown in Table 1.

Table 1: Resource consumption of narrow-side waveguide array

Unknowns	Number of CPU Cores	Process Grid	Matrix Filling /s	Matrix Solving /s
1,167,436	19200	168×100	322.17	14749.02

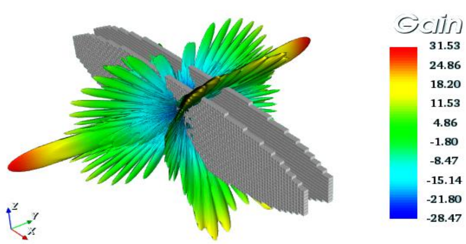


(a)

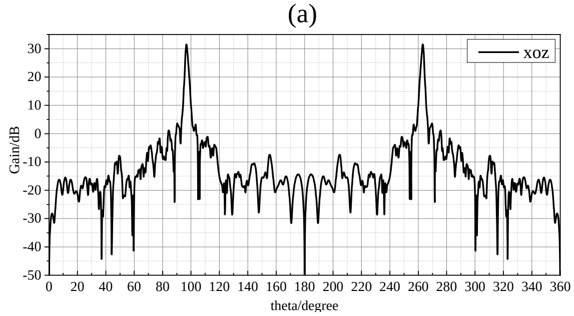


(b)

Fig. 8. Model of narrow-sided waveguide array: (a) single and (b) total.



(a)

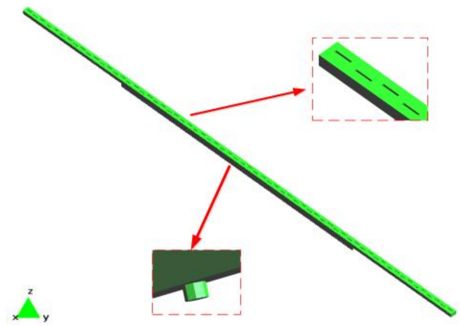


(b)

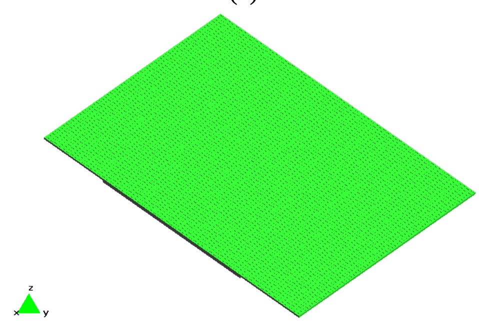
Fig. 9. Radiation pattern of narrow-sided waveguide array: (a) 3D and (b) 2D.

C. Wide-side waveguide array

The wide-edge slotted waveguide array consists of 64 waveguides, each of waveguide has 64 slots, a total of 4096 unit, and the model is shown in Fig. 10. The working frequency of the antenna is in K-band, and the port is fed by the coaxial port. Taylor synthesis is used to feed the port. The number of unknowns generated by the higher-order MoM is 1,190,950, and the required memory is 20.64TB. The calculation results are shown in Fig. 11, and the resource consumption is shown in Table 2.

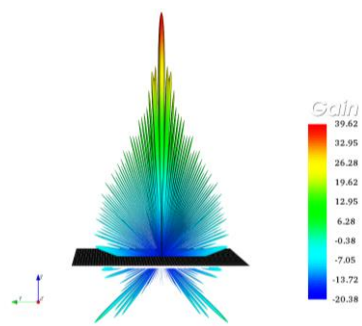


(a)

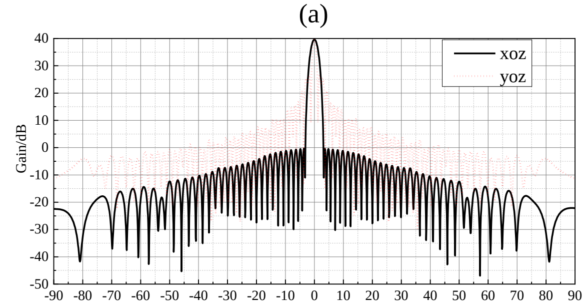


(b)

Fig. 10. Model of wide-sided waveguide array: (a) single and (b) total.



(a)



(b)

Fig. 11. Radiation pattern of wide-sided waveguide array: (a) 3D and (b) 2D.

Table 2: Resource consumption of wide-side waveguide array

Unknowns	Number of CPU Cores	Process Grid	Matrix Filling /s	Matrix Solving /s
1,190,950	19200	168×100	344.01	15582.85

V. CONCLUSION

This paper focuses on the accurate simulation of super large slotted waveguide arrays. The parallel higher-order MoM is used to solve this problem, and the BDPLU algorithm is used to accelerate the matrix equation solving to ensure the computation process is efficient and stable. Meanwhile, a new parallel matrix filling strategy of wave port is proposed to accelerate the process of matrix filling by studying the characteristics of the wave port matrix. Finally, two kinds of typical super large slotted waveguide array antennas are simulated with the help of "Tianhe II" supercomputer platform. The numerical example proved that the method can completely finish the accurate simulation and analysis in 4.5h for any type of waveguide slot array antenna within 4000 units in the high frequency band (about K-band). For the design of the next generation of UAV slotted waveguide array antenna, the simulation method proposed in this paper can reasonably provide simulation within the acceptable range of engineering.

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REFERENCES

[1] Z. Yefeng, Z. Yongzhong, L. Mingfei, et al., “A low-profile half-mode substrate integrated wave-guide circularly polarized antenna,” [J]. *Journal of Xidian University (Natural Science Edition)*, vol. 44, no. 6, pp. 169-174, 2017.

[2] Hu Wei, “Research on waveguide slot array antenna and printed slot unit antenna,” [D]. *Xidian University*, 2013.

[3] W. Zhiwei, “Research on fast hybrid algorithm of antenna radiation characteristics of complex electrical platform,” [D]. *Southeast University*, 2016.

[4] M. Yi, “Research and application of high and low frequency hybrid algorithm,” [D]. *Xidian University*, 2013.

[5] S. Xumin, Y. Minglin, and S. Xinqing, “Integration and efficient and accurate analysis of large waveguide slot array and radome,” [J]. *Journal of Radio Science*, vol. 31, no. 4, 2016.

[6] Y. Wang, Y. Li, X. Zhao, et al., “Analysis of electrically large slotted waveguide array using high-order MoM,” [C] *IEEE International Symposium on Antennas and Propagation & Usnc/ursi National Radio Science Meeting. IEEE*, pp. 975-976, 2015.

[7] Y. Li, Y. Wang, and Y. Zhang, “Analysis of large waveguide slotted antenna array using symmetry techniques,” [C] *Radar Conference 2015, IET International. IET*, vol. 4, no. 4, 2016.

[8] Y. Li, S. Zuo, Y. Wang, et al., “Analysis of geometric structure and wave port symmetry in higher-order moment method,” *[J]. Journal of Xidian University (Natural Science Edition)*, vol. 43, no. 6, pp. 34-38, 2016.

[9] Y. Chen, S. Zuo, Y. Zhang, et al., “Large-scale parallel method of moments on CPU/MIC hetero-geneous clusters,” [J]. *IEEE Transactions on Antennas & Propagation*, vol. 65, no. 7, pp. 3782-3787, 2017.

[10] F. X. Canning, “Solution of impedance matrix localization form of moment method problems in five iterations,” *Radio Science*, vol. 30, no. 5, pp. 1371-1384, Sept.-Oct. 1995. doi: 10.1029/95RS01457.

[11] Q. Chang, Y. Wang, Y. Zhang, and X. Zhao, “Higher-order MoM combined waveport analysis of circular waveguide problems,” [C] *IET International Radar Conference*, Hangzhou, pp. 1-4, 2015.

[12] S. Zuo, Y. Li, and Y. Zhang, “Analysis of the perturbation characteristics of airborne antennas with parallel outer-core high-order moment method,” [J]. *Journal of Terahertz Science and Electronic Information*, vol. 6, 2016.

[13] A. Nouainia and T. Aguili, “Analysis of shielding of metallic rectangular waveguide using new implementation of the MoM-GEC method based on wave concept,” [C] *2017 13th International Wireless Communications and Mobile Computing Conference (IWCMC)*, Valencia, pp. 830-835, 2017.

[14] Y. Wang, X. Zhao, Y. Zhang, S. W. Ting, T. K. Sarkar, and C. H. Liang, “Higher order MoM analysis of traveling-wave waveguide antennas with matched waveports,” [J]. *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 8, pp. 3718-3721, Aug. 2015.

[15] Y. Kimura, F. Nonaka, S. Shimamori, and S. Saito, “Design of microstrip antenna arrays fed by slots on broad and narrow walls of the rectangular waveguide,” [C] *2018 IEEE International Workshop on Electromagnetics: Applications and Student Innovation Competition (iWEM)*, Nagoya, pp. 1-1, 2018.

[16] Intel Copyright (2015) Intel Math Kernel Library for Linux OS User's Guide, Intel Corporation. Available: https://software.intel.com/sites/default/files/anaged/df/1e/mkl_11.3_lnx_userguide.pdf

[17] L. S. Blackford, J. Choi, and A. Cleary, “ScaLAPACK: A portable linear algebra library for distributed memory computers - Design issues and performance,” *Proceedings of the 1996 ACM/IEEE Conference on Supercomputing, IEEE*, pp. 1-20, 1996.

[18] Y. Zhang, Y. Chen, G. Zhang, X. Zhao, Y. Wang, and Z. Lin, “A highly efficient communication avoiding LU algorithm for methods of moments,” *2015 IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting*, pp. 1672-1673, 2015.