A Novel PO Solver for Uncertainty EM Computation of Electrically Large Targets

Yong Chen [1,2], Zhao-Guo Hou [2], Hong-Cheng Yin [2], and Ru-Shan Chen [3]

[1] School of Physics, Beijing Institute of Technology, Beijing, 100081, China
yonche@163.com

[2] Science and Technology on Electromagnetic Scattering Laboratory, Beijing, 100854, China
houzg@139.com, yinhc207@126.com

[3] Nanjing University of Science and Technology, Nanjing, 210094, China
eerschen@njust.edu.cn

Abstract **─** A novel PO method is proposed to analyze the uncertain scattering problems. The algorithm starts with modeling the target with a variable shape by using the non-uniform rational B-spline (NURBS) scheme. Then the scattering far-field is expressed in terms of the variable parameters in NURBS. It should be noted that the perturbation approach is applied to describe the uncertainty of the varying shapes. Compared with the traditional Monte Carlo (MC) method, only a few matrix equations are needed to be solved, so the efficiency will increase greatly. At last, several numerical examples are given to validate the accuracy and efficiency of the proposed method.

Index Terms **─** Electromagnetic scattering, perturbation approach, PO, varying geometric shape.

I. INTRODUCTION

In the field of computational electromagnetic, the methods to obtain EM scattering characteristics for certain targets have been well studied [1-6]. However, the uncertainty for modeling electromagnetic scattering of real targets also needs to be focused due to the manufacturing tolerance, environmental influence or insufficient information. Furthermore, the uncertainty of EM scattering characteristics is a key point of radar system design for target detection. In fact, the uncertainty of the target geometry is often difficult to describe. It is hard to get the EM scattering characteristic for the target with a variable shape. Therefore, it is significant to develop an efficient method for solving the scattering problems of targets with uncertain geometry structures.

A lot of works have been done in the past decades to analyze the uncertainty problems [7-22]. The Monte Carlo (MC) simulation is one of the most popular methods to evaluate the impact of uncertainty [15-16]. In this method, a series of samplings are chosen to describe the variation of the uncertain problems, thus an uncertain problem can be divided into several certain problems, which is easy and direct. Based on this, the computational efficiency of the MC method will become worse with the number of sampling points increasing [17]. Then a generalized polynomial chaos method [18] is proposed to further accelerate the convergence, in which the random variables can be expanded by a series of orthogonal polynomials. There are two common schemes in this method, namely Stochastic Galerkin (SG) approach [9-11,21-22] and stochastic collocation (SC) approach [14,19,20]. When the order of polynomial becomes higher, both the SG and SC method will result in a huge coupling system. In [26], a surrogate modeling technique for electromagnetic scattering analysis of objects with variable shapes is presented by using the method of moments, but this method is not easy to be realized due to the huge consumption of computational resources [27-32]. Therefore, it is urgent and necessary to develop an efficient tool to analyze the uncertain scattering problems for three-dimensional objects.

In this paper, the perturbation method is introduced into the physical optics (PO) method [27] to solve the uncertainty in scattering problems. Firstly, the varying shape on the surface of the target is modeled by using the non-uniform rational B-spline (NURBS) scheme [23-24]. In this way, the geometric uncertainty can be described in terms of several random variables. Then the scattering far-fields can be rewritten by the Taylor series, which is constructed by the random variables. As a result, the geometry can be easily changed by adjusting the variables. Numerical results are compared with the traditional MC method, which demonstrates the accuracy and efficiency of the proposed method.

The remainder of this paper is organized as follows. In Section 2, the theory and the formulations are given.

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Three numerical experiments are presented in Section 3 to show the efficiency of the proposed method. Section 4 concludes this paper.

II. THEORY AND FORMULATIONS

A. NURBS surface modeling

A plane with 0.74m*1.15m is considered. The number of control points in the u direction (i.e., the x-axis) is set to seven, and the number of control points in the v direction (i.e., the y-axis) is set to nine. All the control points are numbered. The first control point is labeled P_{00}, and the last control point is labeled P_{68}. So the NURBS surface can be redrawn by MATLAB as shown in Fig. 1.

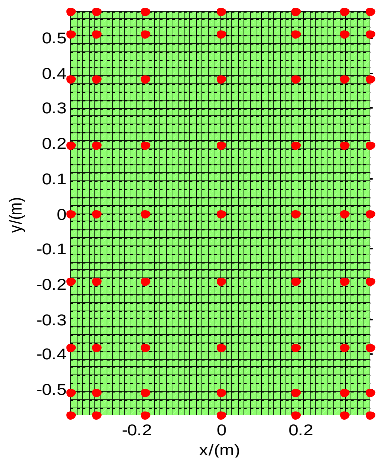


Fig. 1. NURBS surface with controlling points.

A new plane can be got by turning the z coordinate of $P_{3,2}$ to -0.4 and the z coordinate of $P_{3,6}$ to 0.4, which is shown in Fig. 2. It can be seen that as the two points of $P_{3,2}$ and $P_{3,6}$ changes, the surface shape closer to the two control points is bent. But the other parts far away from the two points on the plane are not deformed.

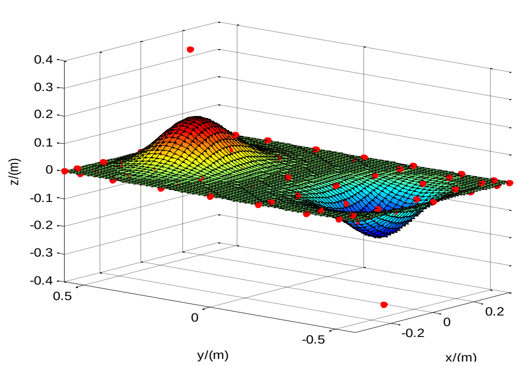


Fig. 2. Reconstructed surface with varying shapes.

Because of the influence of the external environment or the other factors, the geometrical shape of the target is uncertainty. The varying geometrical shape will directly cause the varying of target's EM scattering characteristics. As shown in Fig. 3, the side length of the cube model varies in the interval of $[l-\Delta l, l+\Delta l]$.

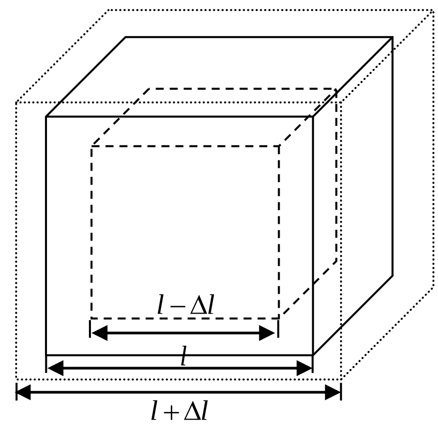


Fig. 3. The cube model with uncertainty geometrical shape.

B. Relationship between variables and equations

An object with a certain size of $\boldsymbol{\alpha}^c$ is considered. The largest varied size is assumed as $\Delta\alpha$. That is, the range of the size for the object is $\left[\boldsymbol{\alpha}^c-\nabla\alpha, \boldsymbol{\alpha}^c+\nabla\alpha\right]$. The far-field scattering field of a PEC object can be calculated as follows:

$$\mathbf{E}_s(\mathbf{r})=\frac{jk\eta}{4\pi R_0}e^{-jkR_0}\int_s \hat{k}_s\times\left[\hat{k}_s\times\left(2n\times H_i\right)\right]e^{jk\cdot\left(\hat{k}_s-\hat{k}_i\right)\mathbf{r}'}ds'. \quad (1)$$

The above formula can be written as:

$$\mathbf{E}\left(\boldsymbol{\alpha}^I\right)=\mathbf{b}\left(\boldsymbol{\alpha}^I\right), \quad (2)$$

where $\boldsymbol{\alpha}^I$ represents any point in $\left[\boldsymbol{\alpha}^c-\nabla\alpha, \boldsymbol{\alpha}^c+\nabla\alpha\right]$. And then the equation (3) can be obtained by using the first-order Taylor series to expand the equation (1) at the point $\boldsymbol{\alpha}^c$:

$$\begin{aligned}\mathbf{b}\left(\boldsymbol{\alpha}^I\right)&=\mathbf{b}\left(\boldsymbol{\alpha}^c\right)+\sum_{i=1}^{n}\frac{\partial\mathbf{b}\left(\boldsymbol{\alpha}^c\right)}{\partial\alpha_i}\Delta\alpha_i\,, \\ &=\mathbf{b}\left(\boldsymbol{\alpha}^c\right)+\Delta\mathbf{b}^I\end{aligned} \quad (3)$$

where n is the number of random variables and $\Delta\alpha_i$ is largest varied size in the i-th random variable, there is:

$$\mathbf{E}^c+\Delta\mathbf{E}^I=\mathbf{b}\left(\boldsymbol{\alpha}^c\right)+\Delta\mathbf{b}^I, \quad (4)$$

where $\Delta\mathbf{E}^I$ represents the change of the far-field scattering field. $\mathbf{E}^c$ represents the far-field scattering field of the model with the size of $\boldsymbol{\alpha}^c$, namely:

$$\mathbf{E}^c=\mathbf{b}\left(\boldsymbol{\alpha}^c\right). \quad (5)$$

Then the change of the far-field scattering field can be expressed as:

$$\Delta \mathbf{E}^{I} = \sum_{i=1}^{n} (\frac{\partial \mathbf{b}\left(\boldsymbol{\alpha}^{c}\right)}{\partial \alpha_i}) \Delta \alpha_i \,, \tag{6}$$

where $\partial \mathbf{b}\left(\boldsymbol{\alpha}^{c}\right) / \partial \alpha_i$ is always the same for different varied sizes, thus the system just needs to be solved once. Compared with the Monte Carlo method, much more time can be saved by the proposed method in this paper.

When an object is modeled with a NURBS surface, any point on the object can be represented by:

$$\mathbf{S}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} R_{i,j}(u,v) P_{ij}, \tag{7}$$

where $R_{i,j}(u,v)$ is a piecewise rational basis function. P_{ij} is a control point, and the x, y, and z coordinates of the control point are represented by $P_{ijx}, P_{ijy}, P_{ijz}$ respectively. Then the relationship between the coordinates of the point on the object and the coordinates of the control point is:

$$\begin{aligned} S_x &= \sum_{i=0}^{N} \sum_{j=0}^{M} R_{i,j} P_{ijx} \\ S_y &= \sum_{i=0}^{N} \sum_{j=0}^{M} R_{i,j} P_{ijy} \\ S_z &= \sum_{i=0}^{N} \sum_{j=0}^{M} R_{i,j} P_{ijz}, \end{aligned} \tag{8}$$

where S_x, S_y, S_z represent the x, y, z coordinates of the point on the object. And then the shape of the object can be controlled by adjusting the coordinates of the control points. All the x, y, and z coordinates of the control points are mutually independent. In the paper, the x, y, and z coordinates of the control point $P_{ijx}, P_{ijy}, P_{ijz}$ are can be seen as random variables. The derivative of the equation (1) can be written as:

$$\begin{aligned} \frac{\partial \mathbf{b}\left(\boldsymbol{\alpha}^{c}\right)}{\partial \alpha_i} &= jk\eta \frac{e^{-jkR_0}}{4\pi R_0} \frac{\partial \int_s \hat{k}_s \times \left[\hat{k}_s \times (2\hat{n} \times H_i)\right] e^{jk\left(\hat{k}_s - \hat{k}_i\right) \bullet \mathbf{r}'} ds'}{\partial \alpha_i} \\ &= jk\eta \frac{e^{-jkR_0}}{4\pi R_0} \frac{\hat{k}_s \times \left[\hat{k}_s \times (2\hat{n} \times H_i)\right] \partial \left[e^{jk\left(\hat{k}_s - \hat{k}_i\right) \bullet \mathbf{r}'} \bullet A\right]}{\partial \alpha_i} \\ &= jk\eta \frac{e^{-jkR_0}}{4\pi R_0} \hat{k}_s \times \left[\hat{k}_s \times (2\hat{n} \times H_i)\right] \left(\frac{\partial \left[e^{jk\left(\hat{k}_s - \hat{k}_i\right) \bullet \mathbf{r}'}\right]}{\partial \alpha_i} \bullet A + \frac{\partial A}{\partial \alpha_i} \bullet e^{jk\left(\hat{k}_s - \hat{k}_i\right) \bullet \mathbf{r}'} \right) \end{aligned} ,\tag{9}$$

where $\partial \alpha_i$ represents the random variable $P_{ijx}, P_{ijy}, P_{ijz}$, and A is the area of the triangle. The derivation of area $\frac{\partial A}{\partial \alpha_i}$ can be derived as:

$$\frac{\partial A}{\partial P_{ijx}} = \frac{1}{4} \cdot \frac{1}{2\sqrt{\left[4a^2b^2 - (a^2+b^2-c^2)^2\right]}} \cdot \begin{bmatrix} 4b^2 \cdot \frac{\partial a^2}{\partial P_{ijx}} + 4a^2 \cdot \frac{\partial b^2}{\partial P_{ijx}} \\ -2(a^2+b^2-c^2) \cdot (\frac{\partial a^2}{\partial P_{ijx}} + \frac{\partial b^2}{\partial P_{ijx}} - \frac{\partial c^2}{\partial P_{ijx}}) \end{bmatrix}, \tag{10}$$

$$\frac{\partial A}{\partial P_{ijy}} = \frac{1}{4} \cdot \frac{1}{2\sqrt{\left[4a^2b^2 - (a^2+b^2-c^2)^2\right]}} \cdot \begin{bmatrix} 4b^2 \cdot \frac{\partial a^2}{\partial P_{ijy}} + 4a^2 \cdot \frac{\partial b^2}{\partial P_{ijy}} \\ -2(a^2+b^2-c^2) \cdot (\frac{\partial a^2}{\partial P_{ijy}} + \frac{\partial b^2}{\partial P_{ijy}} - \frac{\partial c^2}{\partial P_{ijy}}) \end{bmatrix}, \tag{11}$$

$$\frac{\partial A}{\partial P_{ijy}} = \frac{1}{4} \cdot \frac{1}{2\sqrt{\left[4a^2b^2 - (a^2+b^2-c^2)^2\right]}} \cdot \begin{bmatrix} 4b^2 \cdot \frac{\partial a^2}{\partial P_{ijz}} + 4a^2 \cdot \frac{\partial b^2}{\partial P_{ijz}} \\ -2(a^2+b^2-c^2) \cdot (\frac{\partial a^2}{\partial P_{ijz}} + \frac{\partial b^2}{\partial P_{ijz}} - \frac{\partial c^2}{\partial P_{ijz}}) \end{bmatrix}, \tag{12}$$

where a,b,c is the side length of the triangle mesh. $\partial a^2 / \partial \alpha_i$, $\partial b^2 / \partial \alpha_i$ and $\partial c^2 / \partial \alpha_i$ are derived as:

$$\begin{aligned} \frac{\partial a^2}{\partial \alpha_i} &= 2(S_1 - S_2) \cdot (\frac{\partial S_1}{\partial \alpha_i} - \frac{\partial S_2}{\partial \alpha_i}) \\ &= 2(\sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^1 P_{ij} - \sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^2 P_{ij}) \cdot (\sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^1 - \sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^2) \end{aligned} ,\tag{13}$$

$$\begin{aligned} \frac{\partial b^2}{\partial \alpha_i} &= 2(S_2 - S_3) \cdot (\frac{\partial S_2}{\partial \alpha_i} - \frac{\partial S_3}{\partial \alpha_i}) \\ &= 2(\sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^2 P_{ij} - \sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^3 P_{ij}) \cdot (\sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^2 - \sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^3) \end{aligned} ,\tag{14}$$

$$\begin{aligned} \frac{\partial c^2}{\partial \alpha_i} &= 2(S_3 - S_1) \cdot (\frac{\partial S_3}{\partial \alpha_i} - \frac{\partial S_1}{\partial \alpha_i}) \\ &= 2(\sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^3 P_{ij} - \sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^1 P_{ij}) \cdot (\sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^3 - \sum_{i=0}^{N}\sum_{j=0}^{M} R_{i,j}^1) \end{aligned} ,\tag{15}$$

where S_1, S_2 and S_3 are the three vertices of triangle mesh. In this way, the deviation of the scattering field $\Delta \mathbf{E}$ can be obtained by:

$$\begin{aligned} \Delta \mathbf{E} &= \sum_{i=1}^{n} (\frac{\partial \mathbf{b}(\boldsymbol{\alpha}^{\mathbf{c}})}{\partial \alpha_i}) \Delta \alpha_i \\ &= \sum_{i=1}^{n} (jk\eta \frac{e^{-jkR_0}}{4\pi R_0} \frac{\partial \int_s \hat{k}_s \times [\hat{k}_s \times (2\hat{n} \times H_i)] e^{jk(\hat{k}_s - \hat{k}_i) \cdot \mathbf{r}'} ds'}{\partial \alpha_i}) \Delta \alpha_i \end{aligned} .\tag{16}$$

It should be noted the first-order Taylor series is used to expand the formula of far-field scattering field at the mean value. Therefore, the error will be introduced into the approximate calculation formula. More specifically, the error will increase with varying interval of the shape becomes bigger. The experience indicates that the interval should be less then 0.4λ. As shown in Eq. (4), the change of the far-field scattering field should be calculated only once for each $\Delta \alpha_i$. Therefore, the computation complexity has a linear relation to the number of random variables. In other words, the equation should be solved *n* (number of random variables) times totally for the uncertain problems. However, for the MC method, the computational efficiency will decrease as the number of sampling points increases. Generally, the number of sampling points in the MC method is far greater than the number of random variables in the proposed method. Therefore, the computational time can be saved a lot when compared with the traditional MC method.

III. NUMERICAL RESULTS

In this section, a series of examples are presented to demonstrate the efficiency of the proposed method.

A. The bistatic RCS for a PEC slab

Firstly, a slab model with uncertain side length is analyzed with the proposed method at the frequency of 1GHz. The side length of the slab model is set as [1.91m, 2.09m], as shown in Fig. 4. To verify the accuracy of the proposed method for uncertainty problems, the result simulated by the MC method with 1000 sampling points is used as a reference [39-40]. The incident angle of plane wave is set at $\theta_i = 0^\circ$, $\varphi_i = 180^\circ$. The bistatic RCS results are compared in Fig. 5 between the MC method and the proposed method. It can be seen that there is a good agreement between them. Moreover, the comparisons of CPU time cost between the proposed method and MC method with 1000 samples are listed in Table 1.

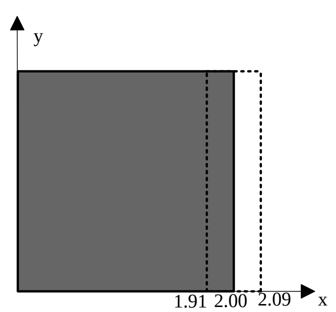


Fig. 4. The slab model with uncertain side length.

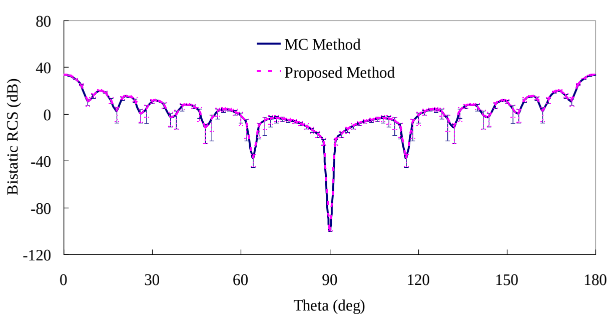


Fig. 5. Bistatic RCS of a slab model with uncertain side length.

Table 1: Comparisons of CPU Time between the Proposed Method and MC Method with 1000 Samples

Method	CPU Time (s)
Proposed method	41
Monte Carlo	1158

B. The monostatic RCS for a PEC aircraft

Secondly, the analysis of monostatic RCS is taken for a PEC aircraft at the frequency of 1.0 GHz. As shown in Fig. 6 (a), the nose of aircraft is along y axis. The varying length of wings is set as the uncertain scattering property of the aircraft model with the variation of $[-0.09m, 0.09m]$. It can be seen from Fig. 6 (b) that there are eight control points to describe the varying shape of this aircraft. The incident angle of plane wave is set at $\theta_i = 90^\circ$, $\varphi_i = 0-180^\circ$. As shown in Fig. 7, the monostatic RCS results are given and it can be found that there is a good agreement between the MC method and the proposed method. Moreover, the comparisons of CPU time cost between the proposed method and MC method with 1000 samples are listed in Table 2.

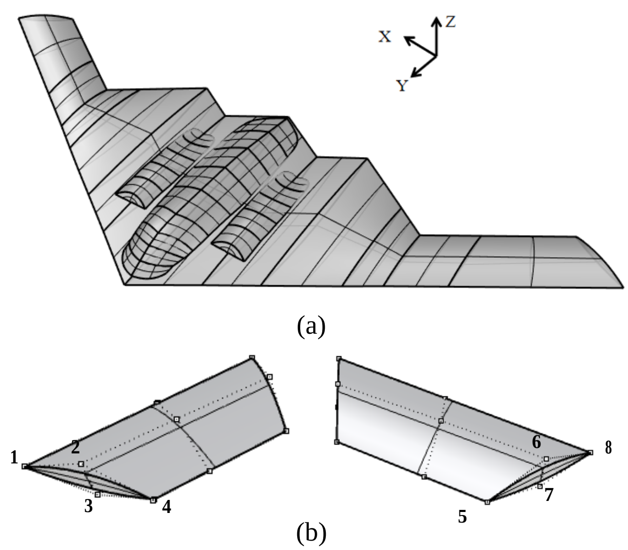


Fig. 6. (a)The aircraft model with varying length of wings. (b) The aircraft model constructed by NURBS Approach (Points 1-8 are used to control the varying of the wings length).

Table 2: Comparisons of CPU time between the proposed method and MC method with 1000 samples

Method	CPU Time (s)
Proposed method	2817
Monte Carlo	81095

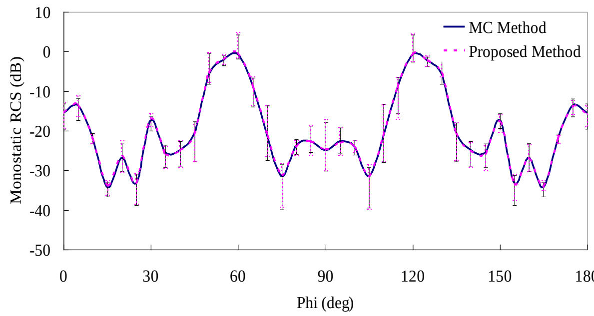
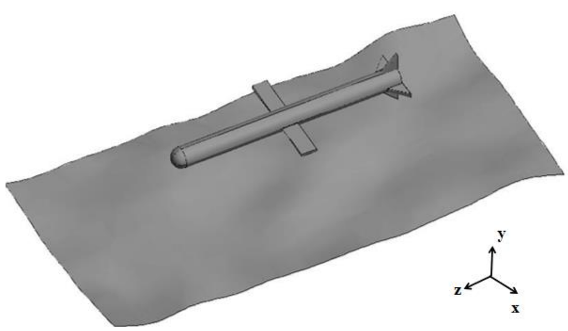


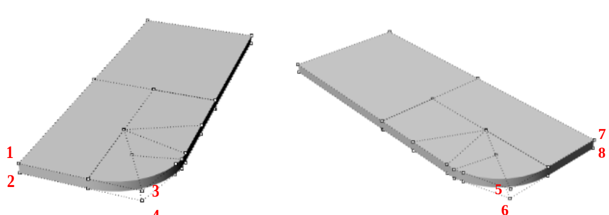
Fig. 7. Monostatic RCS of an aircraft model with varying length of wings.

C. The monostatic RCS for a PEC missile over a rough medium sea surface

At last, we consider the scattering from a missile over a rough medium sea surface at the frequency of 1 GHz. As shown in Fig. 8 (a), the nose of missile is along z axis. The varying length of wings is set as the uncertain scattering property of the missile model with the variation of [2.9m, 3.1m]. As shown in Fig. 8 (b), there are eight control points to describe the varying shape of this missile. The incident angle of plane wave is set at $\theta_i = -90^\circ \sim 90^\circ$, $\varphi_i = 180^\circ$. As shown in Fig. 9, the monostatic RCS results are given and it can be found that there is a good agreement between the MC method and the proposed method. Moreover, the comparisons of CPU time cost between the proposed method and MC method with 1000 samples are listed in Table 3.



(a)



(b)

Fig. 8. (a) The missile model with varying length of wings over a rough medium sea surface. (b) The aircraft model constructed by NURBS Approach (Points 1-8 are used to control the varying of the wings length).

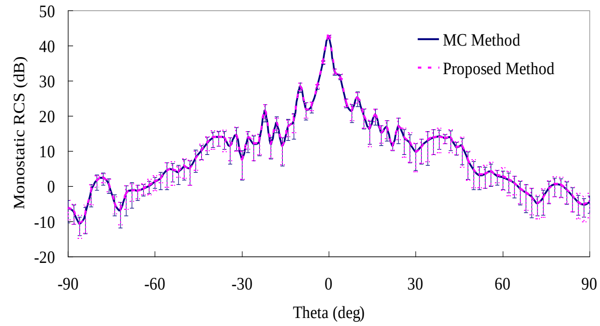


Fig. 9. Monostatic RCS of a missile model with varying length of wings.

Table 3: Comparisons of CPU time between the proposed method and MC method with 1000 samples

Method	CPU Time (s)
Proposed method	502
Monte Carlo	18459

IV. CONCLUSION

In this paper, the perturbation approach is used to analyze the uncertain scattering from electrically large targets. By using the non-uniform rational B-spline (NURBS) scheme, the varying geometrical shape can be modeled with several variables. In this way, the scattering far-fields can be calculated by the PO method. Less matrix equations are needed to be solved when compared with the traditional Monte Carlo method.

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Yong Chen was born in Tianmen, Hubei Province, China, in 1983. He received the B.S. degree from WuHan University in 2005, and the M.S. degree in Electromagnetic Field and Microwave Technology from the graduate school of Second Academy, China Aerospace Science &Industry Corporation in 2008. He is currently working towards the Ph.D. degree in Physics at Beijing Institute of Technology.

He is a Senior Engineer at Science and Technology on Electromagnetic Scattering Laboratory. His research interest includes the electromagnetic scattering modeling and radar signal simulation.

Zhaoguo Hou was born in Dingxi, Gansu Province, China, in 1983. He received the B.S. degree from Beijing Institute of Technology in 2005, the M.S. degree in condensed matter Physics from Beijing Institute of Technology in 2007, and the Ph.D. degree in Electromagnetic Field and Microwave Technology from Communication University of China in 2011.

He is a Senior Engineer at Science and Technology on Electromagnetic Scattering Laboratory. His research interests include computational electromagnetics and radar signal simulation.

Hong-Cheng Yin was born in Jiangxi, China. He received the B.S. degree from Northwest Telecommunication Engineering Institute, Xi'an, China, in 1986, the M.S. degree from Beijing Institute of Environmental Features (BIEF), Beijing, China, in 1989, and the Ph.D. degree from Southeast University, Nanjing, China, in 1993, all in Electromagnetic Field and Microwave Technique. He is currently a Researcher at the Science and Technology on Electromagnetic Scattering Laboratory, BIEF. His research interests include numerical methods in electromagnetic fields, electromagnetic scattering and inverse scattering, radar target recognition. Yin is a Fellow of Chinese Institute of Electronics.

Ru Shan Chen was born in Jiangsu, China. He received the B.Sc. and M.Sc. degrees from the Department of Radio Engineering, Southeast University, China, in 1987 and 1990, respectively, and the Ph.D. degree from the Department of Electronic Engineering, City University of Hong Kong, in 2001.

He joined the Department of Electrical Engineering, Nanjing University of Science and Technology (NJUST), China, where he became a Teaching Assistant in 1990 and a Lecturer in 1992. Since September 1996, he has been a Visiting Scholar with the Department of Electronic Engineering, City University of Hong Kong, first as Research Associate, then as a Senior Research

Associate in July 1997, a Research Fellow in April 1998, and a Senior Research Fellow in 1999. From June to September 1999, he was also a Visiting Scholar at Montreal University, Canada. In September 1999, he was promoted to Full Professor and Associate Director of the Microwave and Communication Research Center in NJUST, and in 2007, he was appointed as the Head of the Department of Communication Engineering, NJUST. He was appointed as the Dean in the School of Communication and Information Engineering, Nanjing Post and Communications University in 2009. And in 2011 he was appointed as Vice Dean of the School of Electrical Engineering and Optical Technique, NJUST. Currently, he is a principal investigator of more than 10 national projects. His research interests mainly include computational electromagnetics, microwave integrated circuit and nonlinear theory, smart antenna in communications and radar engineering, microwave material and measurement, RF-integrated circuits, etc. He has authored or coauthored more than 260 papers, including over 180 papers in international journals.