Improving the Efficiency of Hybrid Boundary Element Method for Electrostatic Problems Solving

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Abstract **─** This paper describes a modification of the Hybrid Boundary Element Method (HBEM) for electrostatic problems solving. Such improved method is applied for transmission lines analyses. By taking a quasi-static TEM approach, the Hybrid Boundary Element Method is applied to determine the effective relative permittivity and the characteristic impedance of different stripline structures. Comparisons with already published numerical results and software simulation have been also performed with an aim to test the validity of the proposed approach. A close results match can be noticed. The main novelties of the proposed HBEM modification are better accuracy and ability to determine the polarization charges distribution on the separating surface between a strip and a dielectric layer. This was not possible before, using the previous version of the method.

Index Terms **─** Characteristic impedance, effective relative permittivity, finite element methods, hybrid boundary element method, stripline.

I. INTRODUCTION

The Hybrid Boundary Element Method (HBEM) is a semi-numerical method, [1]. This method presents a combination of several other methods: the boundary element method (BEM) [2, 3], the equivalent electrodes method (EEM) [4] and the point-matching method (PMM) for the potential of the perfect electric conductor electrodes and for the normal component of the electric field at the boundary surface between any two dielectric layers.

Different electromagnetic problems have been solved using this method up to now. The electromagnetic field distribution in the vicinity of cable joints and terminations is determined in [5], the calculation of permanent magnets magnetic force is performed in [6] and characteristic parameters of transmission lines have been calculated in [7-9]. Also, in [10] a two-wire line with a magnetic material in its vicinity is analysed and the line inductance is calculated. The grounding systems are analysed in [11] using the HBEM.

The main idea of this method is that each conductor surface and the boundary surfaces between any two dielectric layers should be divided into a large number of segments as in the EEM's application, [1]. Each of these segments is replaced by the equivalent electrodes (EEs), placed at the segment's centres. In the case of single layer problems, the HBEM application is identical to the EEM. But, for the multilayered systems, the HBEM application comes to the fore. At the boundary surfaces between any two dielectric layers only polarization (bound) charges exist, [12]. According to the HBEM, the segments at these surfaces are replaced by discrete equivalent polarization charges placed in the air. The boundary surfaces between the perfect electric conductor and the dielectric layers consist of total charges, i.e., free and polarization charges. In the previous HBEM application, an approximation was done – only the free charges on those boundary surfaces, placed in the corresponding dielectric layer, were considered, [7-9]. Such approximation gave satisfactory results for majority of solved problems. The results verification shows that the error rate is less than 1%, [6-10]. Using this approximation, it is not possible to determine the polarization charges distribution at the conductor-dielectric layer separating surfaces, but only at the separating surfaces of two dielectric layers, [7-9]. This paper attempts to address this issue and improve existing method.

The advantages of the improved HBEM are illustrated by numerical results. The computation time is shown, as well as the results convergence. The characteristic impedance and effective relative permittivity of single and coupled striplines are analysed. The computed results will be verified with the corresponding results obtained by the finite element method (FEM), [13] and data available in the literature.

II. HBEM MODIFICATION AND APPLICATION

The geometry of a system which consists of conductors and arbitrary number of dielectric layers is depicted in Fig. 1.

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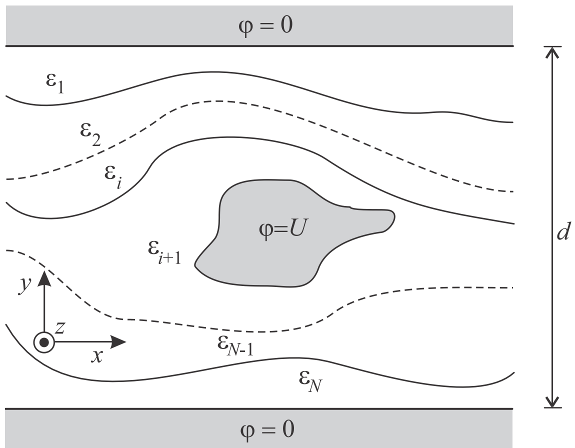

Fig. 1. Geometry of the problem.

The shapes of the conductor and layers are also arbitrary. The infinite parallel planes are on the zero potential and the conductor is on the potential U.

A similar system was already presented in [9] as well as corresponding HBEM model. Having the goal to improve the existing HBEM model in mind, as it is mentioned in the previous section, the total charges on the separating surface between the dielectric layer and conductor are taken into account now. They consist of free and polarized charges placed in the air. Only the polarized charges, placed also in the air, exist on the separating surfaces between dielectric layers.

That leads to the modification of the HBEM model, so the improved version is shown in Fig. 2.

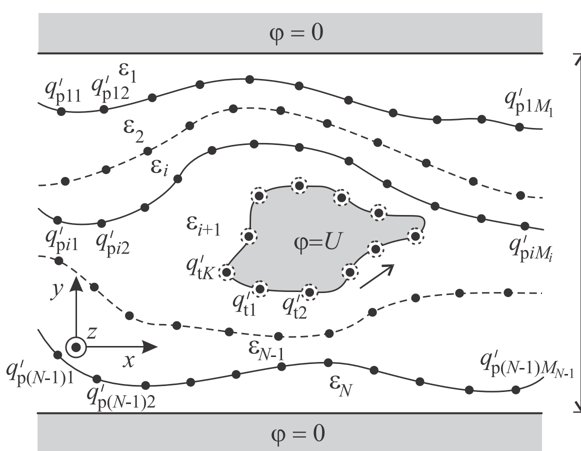

Fig. 2. Modified HBEM model.

In order to determine the potential at any point in the $x0y$ plane, it is necessary to use the Green's function for the line charge placed in dielectric of permittivity ε, between two infinite parallel grounded planes, at distance h from the bottom plane, given in [14]. The function is:

$$G(x,y)=\frac{1}{2\pi\varepsilon}\ln\sqrt{\frac{\cosh\left[\frac{\pi}{d}x\right]-\cos\left[\frac{\pi}{d}(y+h)\right]}{\cosh\left[\frac{\pi}{d}x\right]-\cos\left[\frac{\pi}{d}(y-h)\right]}}\,, \quad (1)$$

where d is a distance between two infinite parallel grounded planes. Therefore, it is possible to form an expression for the potential at any point of the system given in Fig. 2:

$$\varphi=\sum_{k=1}^{K}\frac{q'_{\mathrm{t}k}}{2\pi\varepsilon_0}\ln\sqrt{\frac{\cosh\left[\frac{\pi}{d}(x-x_{\mathrm{t}k})\right]-\cos\left[\frac{\pi}{d}(y+y_{\mathrm{t}k})\right]}{\cosh\left[\frac{\pi}{d}(x-x_{\mathrm{t}k})\right]-\cos\left[\frac{\pi}{d}(y-y_{\mathrm{t}k})\right]}}+$$

$$+\sum_{i=1}^{N-1}\sum_{m=1}^{M_i}\frac{q'_{\mathrm{p}im}}{2\pi\varepsilon_0}\ln\sqrt{\frac{\cosh\left[\frac{\pi}{d}(x-x_{\mathrm{p}im})\right]-\cos\left[\frac{\pi}{d}(y+y_{\mathrm{p}im})\right]}{\cosh\left[\frac{\pi}{d}(x-x_{\mathrm{p}im})\right]-\cos\left[\frac{\pi}{d}(y-y_{\mathrm{p}im})\right]}}\,,$$

$$N\geq 2\,. \quad (2)$$

With $q'_{\mathrm{t}k}=q'_{\mathrm{f}k}+q'_{\mathrm{p}k}$ $(k=1,\ldots,K)$ the total EEs charges, which consist of free (f) and polarized (p) line charges, placed at the conductor surface, are denoted. These charges are at the same positions $(x_{tk},y_{\mathrm{t}k})$. The polarized line charges, placed at the boundary surfaces between any two dielectric layers, are denoted with $q'_{\mathrm{p}im}$ $(m=1,\ldots,M_i,\ i=1,\ldots,N-1)$.

The total number of unknowns is:

$$N_{\mathrm{tot}}=2K+\sum_{i=1}^{N-1}M_i\,. \quad (3)$$

At the boundary surfaces, between two dielectrics, the boundary condition, should be satisfied, [12]:

$$\hat{\boldsymbol{n}}_{im}\cdot\boldsymbol{E}_{im}^{(0+)}=\frac{-\varepsilon_{i+1}}{\varepsilon_0(\varepsilon_i-\varepsilon_{i+1})}\eta_{\mathrm{p}im}\,, \quad (4)$$

while between the conductor and $(i+1)$-th dielectric, the boundary condition has a form:

$$\hat{\boldsymbol{n}}_{k}\cdot\boldsymbol{E}_{(i+1)k}^{(0+)}=\frac{-\varepsilon_0}{\varepsilon_0(\varepsilon_{i+1}-\varepsilon_0)}\eta_{\mathrm{p}k}\,. \quad (5)$$

$\boldsymbol{E}=-\mathrm{grad}(\varphi)$ represents the electric field strength; $\eta_{\mathrm{p}im}=q'_{\mathrm{p}im}/\Delta l_{im}$ and $\eta_{\mathrm{p}k}=q'_{\mathrm{p}k}/\Delta l_k$ are the surface charges on the segments of the lengths Δl_{im} and Δl_k, respectively, ($m=1,\ldots,M_i$, $i=1,\ldots,N-1$, $k=1,\ldots,K$); $\hat{\boldsymbol{n}}_{im}$ and $\hat{\boldsymbol{n}}_k$ are the unit normal vectors oriented from the layer ε_{i+1} towards the layer ε_i and from the conductor towards the layer ε_{i+1}, respectively.

In order to calculate the characteristic impedance and effective relative permittivity, the following procedure, described in detail in [9], should be applied:

a) Form the HBEM model (EE's positioning);
b) Form the system of linear equations matching

the potential of the conductor using (2) and boundary condition at the separating surfaces using (4) and (5);

c) Solve the formed system of linear equation;

d) Calculate the characteristic parameters.

III. NUMERICAL ANALYSIS

To validate and demonstrate the accuracy and efficiency of the proposed, improved version of HBEM, two types of striplines will be illustrated in this section. The analysis refers to the striplines, where the substrates are of infinite width, made of isotropic material. The parallel planes are also infinite with zero potential. These geometries are shown in Figs. 3 and 4.

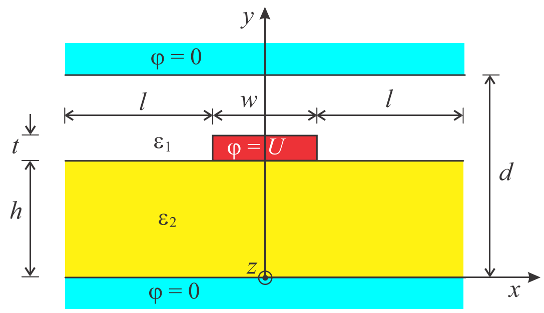

Fig. 3. Stripline.

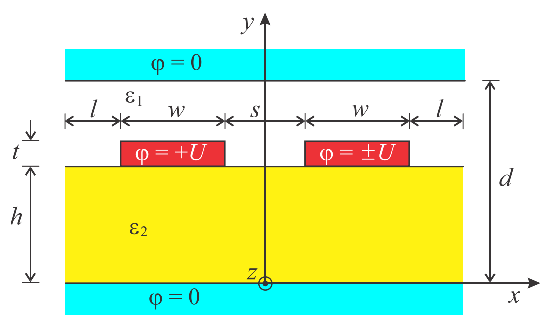

Fig. 4. Coupled stripline.

Therefore, the HBEM will be applied, unknown charges determined and the characteristic parameters calculated. Afterwards, we will compare the results with those obtained using the FEMM software, [13]. All calculations are performed on a 4 Core CPU (Intel i5) running at 3.1 GHz and 4GB RAM in double precision.

A. Results convergence and computation time

The influence of number of unknowns, N_{tot}, to the characteristic impedance results and computation time, is given in Fig. 5 for the problem from the Fig. 3. Increasing the number of EEs, a fast results convergence is obtained in a short period of time. For over 1000 unknowns the characteristic impedance value is almost constant.

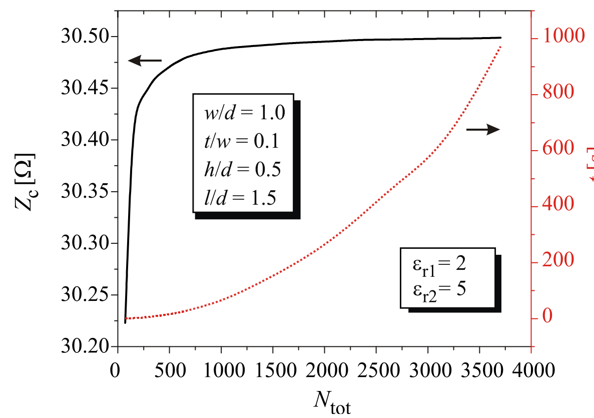

Fig. 5. Convergence of results and computation time.

In regards the computation time, (denoted with the red dotted line), it is visible that the time, necessary to place equivalent electrodes, then to form and solve the system of equations and to calculate characteristic impedance, significantly increases with the number of unknowns. For 1000 unknowns the computation time is less than 100 seconds. In all following calculations the number of unknowns will be 1500, so the computation time is about two minutes.

B. Influence of substrate width on characteristic impedance and effective relative permittivity

Considering that the substrates from the Fig. 3 and 4 are of the infinite width, it was necessary to analyse the influence of substrate width on characteristic impedance and effective relative permittivity values. What is the minimum substrate width above which the characteristic parameters do not change?

This analysis is performed for: $w/d=1.0$, $h/d=0.5$, $t/w=0.1$, $\varepsilon_{r1}=2$, $\varepsilon_{r2}=5$ (for the stripline from Fig. 3) and $w/d=1.0$, $h/d=0.5$, $s/d=0.5$, $t/w=0.1$, $\varepsilon_{r1}=2$, $\varepsilon_{r2}=5$ (for both modes of the coupled stripline from Fig. 4), and shown in Figs. 6 and 7.

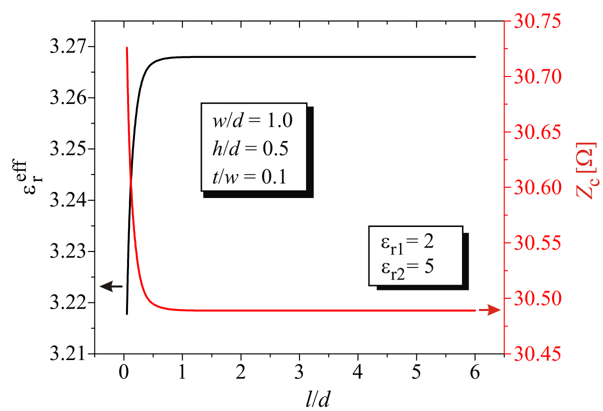

Fig. 6. Influence of parameter l/d on characteristic impedance and effective relative permittivity of stripline.

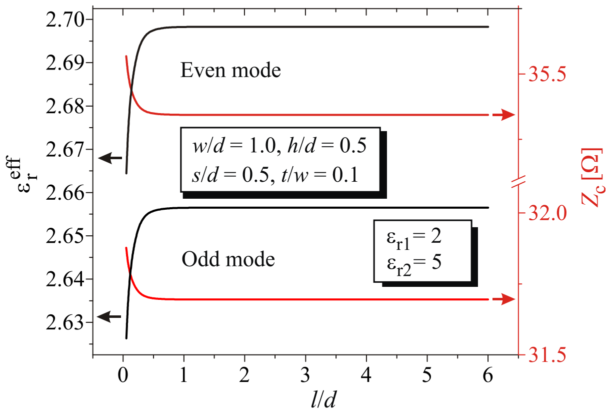

Fig. 7. Influence of parameter l/d on characteristic impedance and effective relative permittivity of coupled stripline.

As it can be noticed, for $l/d > 1.0$ the values for both characteristic parameters are constant. These led us to conclude that, in all following calculations, the value $l/d = 1.5$ can be used.

IV. RESULTS AND DISCUSSION

The analysed problems from Figs. 3 and 4 are on the left and right sides unbounded, i.e., "open". That means that electromagnetic fields should extend towards infinity. In order to simulate considered striplines with the finite element method solver such as FEMM [13] it was necessary to close the simulation domain "far enough". In that way, the influence of the terminating boundary conditions at the far end becomes negligible. This is the common and easiest used approach as it is mentioned in [15] and [16].

FEMM simulation will be done with about 400.000 finite elements. So, the verification of the results can be made.

In Tables 1 and 2 the effective relative permittivity and characteristic impedance versus the strip thickness, calculated using HBEM as well as using FEMM simulation, are given for the stripline from Fig. 3. The dimensions of the analysed stripline are: $w/d = 1.0$, $h/d = 0.5$, $l/d = 1.5$, $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 5$.

In those tables, the values obtained using the "old" version of HBEM (oHBEM) are also presented.

Table 1: Effective relative permittivity distribution versus parameter t/w

t/w	HBEM	oHBEM	FEMM
0.1	3.2680	3.1737	3.2777
0.05	3.3692	3.2867	3.3801
0.02	3.4314	3.3714	3.4466
0.01	3.4555	3.4080	3.4716

Table 2: Characteristic impedance distribution versus parameter t/w

t/w	HBEM	oHBEM	FEMM
0.1	30.489	30.955	30.437
0.05	32.549	32.969	32.481
0.02	33.907	34.219	33.809
0.01	34.405	34.653	34.291

It is visible that the results of the new and improved HBEM are close to the FEMM results. Also, it can be concluded that, when the strip thickness decreases, the analysed characteristic parameters increase.

Figure 8 presents a distribution of characteristic impedance versus parameter w/h, incorporating the effect of strip thickness. The stripline is without dielectric, placed in the centre of the system at equal distance from the parallel planes. In this figure the results from [17] are also shown. They are obtained using formulas given in [17] and [18] and the good results match is obtained. Also, it is evident that increasing the strip width, characteristic impedance decreases.

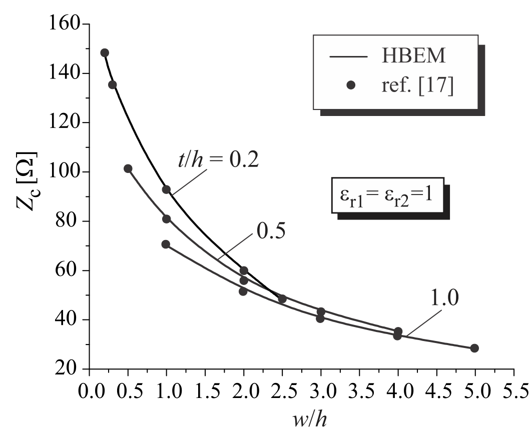

Fig. 8. Characteristic impedance of a stripline without dielectric.

The main advantage of the improved version of HBEM, besides the better accuracy, is a possibility to determine the polarized charges distribution on the separating conductor-dielectric surfaces. The attempt to apply this modified approach and show the normalized distribution is visible in Fig. 9 for stripline from Fig. 3, for parameters: $w/d = 1.0$, $h/d = 0.5$, $l/d = 1.5$, $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 5$. Considering that the strip is on the positive potential, those charges are negative.

The normalized polarized charges distribution if one of the layers is the air, $\varepsilon_{r1} = 1$, is presented in Fig. 10 for stripline from Fig. 3. Also in Figs. 11 and 12 the analyse was done for both modes of coupled stripline from Fig. 4 for parameters: $w/d = 1.0$ $h/d = 0.5$, $s/d = 0.5$,

$l/d = 1.5$, $t/w = 0.1$, $\varepsilon_{r1} = 1$, $\varepsilon_{r2} = 5$.

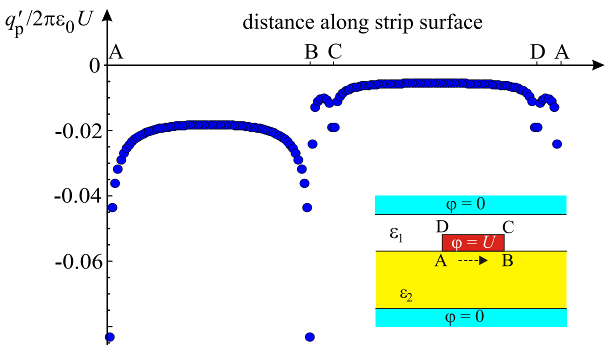

Fig. 9. Polarized charges distribution along strip surface.

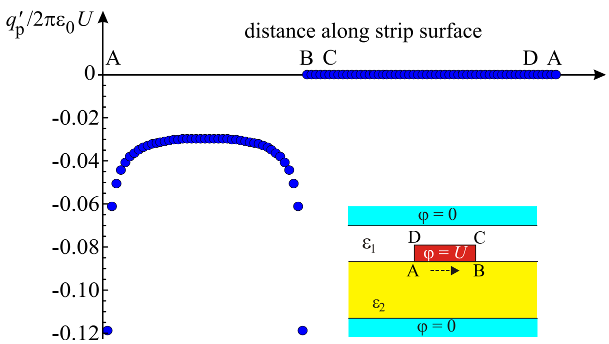

Fig. 10. Polarized charges distribution along strip surface of stripline for $\varepsilon_{r1} = 1$.

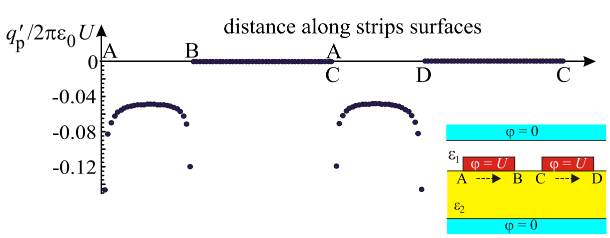

Fig. 11. Polarized charges distribution along strips surfaces of coupled stripline for $\varepsilon_{r1} = 1$ (even mode).

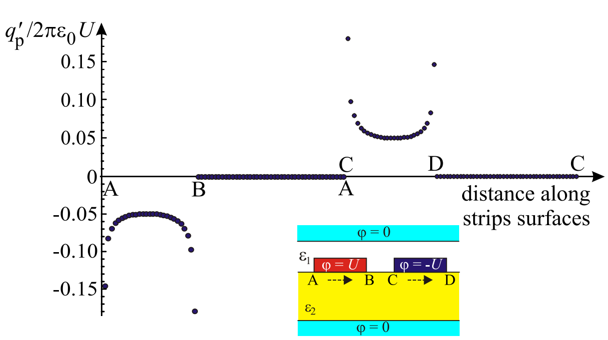

Fig. 12. Polarized charges distribution along strips surfaces of coupled stripline for $\varepsilon_{r1} = 1$ (odd mode).

As expected, the polarized charges do not exist on the air-strip boundary surfaces. Also, from those figures can be confirmed that the polarized charges are negative when the strips are on the positive potential and vice versa.

V. CONCLUSION

In this paper, an improved version of HBEM is applied to quasi-static analysis of different stripline structures. The presented numerical results demonstrate the accuracy and efficiency of the method. Applying the HBEM, the system matrix has the greatest elements on the main diagonal. This leads to a better conditioned system of linear equations and computation time is shorter. The method can be used to determine the polarized charges distribution on the separating surfaces of any two layers as well as conductor-dielectric separating surfaces. Also, using presented methodology, the strips of infinitely thin metallization thickness can be also modelled and analysed.

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