Comprehensively Efficient Analysis of Nonlinear Wire Scatterers Considering Lossy Ground and Multi-tone Excitations

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Abstract — In this paper based on intelligent water drops algorithm (IWD), comprehensively nonlinear analysis of nonlinearly loaded wire scatterers are carried out. The analyses involve two stages. First, the problem is modeled as a nonlinear multi-port equivalent circuit and it is then reformulated into an optimization problem which is solved by the IWD. The simulation results are compared with harmonic balance (HB), arithmetic operator method (AOM), approximate methods and experiment. Analysis of the problem under strongly nonlinear loads, presence of lossy ground, multi-port structures, and multi-tone excitations are included to cover all the complex aspects. In one hand, the proposed modeling approach is in excellent agreement with other conventional techniques. On the other hand, the run time is considerably reduced.

Index Terms — IWD, lossy ground, multi-tone excitation, nonlinear load, wire scatterers.

I. INTRODUCTION

As known, nonlinearly loaded wire scatterers in single and multi-port structures can be used in applications such as control of scattering response, microwave imaging and protecting against high-valued signals such as lightning return strokes [1-10].

In this paper, nonlinearly loaded wire scatterers as single-port and multi-port are investigated as shown in Fig. 1. Such structures are used to control scattering response at frequency harmonic of interest using tuning the spacing among antennas and respective nonlinear load. From now on, the mentioned scatterers are called nonlinear wire scatterer for simplicity. The analyses of such structures are carried out in the frequency domain [1-5], time domain [6, 7], and mixed time-frequency domain [8-10]. Although the frequency-domain methods are suitable for inclusion of frequency dependence of the lossy ground, they are limited to weakly nonlinear loads [1-3] and suffering from drawbacks of Newton Raphson algorithm [4, 5] which yields unacceptable results [5]. Time domain methods, on the other hand, are easily used to treat nonlinearity effect of arbitrary order, however, inclusion of frequency dependence effect of the lossy ground is difficult. To include both effects, the mixed time-frequency domain methods such as harmonic balance technique (HB) should be used [8-10]. In the last method, all the mentioned scatterers depending upon the number of ports are modelled as a nonlinear equivalent circuit as shown in Fig. 2. In this figure, the nonlinear circuits consists of two parts. The first part is the linear part representing Norton circuit viewed across the nonlinear load. This part includes $I_n$ as the short circuit current which is due to the applied excitation, $Y_{in}$ and $Y_n$ as the input admittance matrix viewed across the nonlinear load at mixing frequencies respectively for single-port and multi-port scatterers. The three mentioned quantities are conventionally computed using numerical methods such as method of moments (MoM). The second part in Fig. 2 is a nonlinear part which is used for modelling the nonlinearity effects of the device connected to the wire scatterer.

The HB technique is suitable for strongly nonlinear loads, but it is inefficient in analyzing multi-port wire scatterers under multi-tone excitations. Also, it suffers from initial guess and gradient operation in the Newton Raphson iteration algorithm. To remove these drawbacks, a number of optimization techniques [11-13] such as genetic algorithm (GA) [11], and particle swarm optimization (PSO) [12] for analysis of nonlinear wire scatterers under single tone excitation were proposed. Such approaches, although yield near global solutions, they are time-consuming especially for multi-port structures under multi-tone excitations. In addition, the mentioned Norton circuits are computed via MoM which is time consuming. To remove this complexity, Ostadzadeh et al. proposed a model based on fuzzy inference (MoF) to compute Norton circuit very efficiently [14-16]. The rest challenge is that to compute the induced voltage across the nonlinear load efficiently.
In our previously published literature [17], the IWD algorithm has been used for computing scattering response from Gun diode-loaded antenna array under single-tone excitation but its efficiency in the presence of lossy ground, strongly nonlinear loads and multi-tone excitations was not addressed. In this study, comprehensive analysis of nonlinear wire scatterers based on IWD algorithm considering the mentioned aspects is carried out. The simulation results show that the IWD-based results are in excellent agreement with the HB, AOM, experiment and approximate methods while the run-time is considerably reduced.

This paper is organized as follows. Section II is focused on formulation principles of the nonlinear wire scatterers. IWD algorithm is briefly explained in Section III. Section IV applies the IWD on the multi-port nonlinear wire scatterers considering different complex aspects. Finally conclusion is given in Section V.

II. ANALYSIS OF NONLINEAR WIRE SCATTERERS

As known, the main problem in analyzing nonlinear wire scatterers is computation of the induced voltage across the nonlinear load. To this end, the following cost function should be zero [8]:

$$\mathbf{c} = \mathbf{Y}_{sc} \mathbf{V}_s - \mathbf{I}_{sc} + \mathbf{D} \left( \overline{\mathbf{Y}} \mathbf{V}_s \right) \rightarrow \mathbf{0}. \tag{1}$$

All quantities in (1) are defined as below:

- $\mathbf{I}_{sc}$ is a vector of short-circuit currents due to the excitations which is computed only at $M$ excitation frequencies, that is,

$$\mathbf{I}_{sc} = [I_0 \ I_1 \ I_2 \ \cdots \ I_M]_1.$$  \tag{2}

- $\overline{\mathbf{Y}}_{sc}$ is a matrix containing input admittance viewed across the nonlinear load at $N$ mixing frequencies ($N>M$), that is,

$$\overline{\mathbf{Y}}_{sc} = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 \\
0 & G_{in1} & B_{m1} & 0 & \cdots & 0 \\
0 & -B_{m1} & G_{in1} & \ddots & 0 & \vdots \\
\vdots & 0 & 0 & \ddots & 0 & \vdots \\
0 & \vdots & \vdots & \ddots & 0 & G_{inN} \\
0 & 0 & \cdots & 0 & -B_{inN} & G_{inN}
\end{bmatrix}. \tag{3}
$$

- $\mathbf{D}$ and $\overline{\mathbf{Y}}$ are respectively transformation matrix from the frequency domain to the time domain and vice versa. Also, $f(.)$ is the $(i-v)$ characteristic of the nonlinear load. All the above quantities are known, except $\mathbf{V}_s$, which is an unknown vector including induced voltages across nonlinear load at $N$ mixing harmonic frequencies as follows:

$$\mathbf{V}_sc = [V_0 \ V_{ig} \ V_{ih} \ \cdots \ V_N \ V_{Nv}]. \tag{4}$$

The vectors of $\overline{\mathbf{V}}_s$ and $\mathbf{I}_{sc}$ are Fourier coefficients of the time-domain signals $i_{nc}(t)$ and $v(t)$, i.e.,

$$i_{nc}(t) = I_0 + \sum_{j=1}^{M} [I_{j} \cos(\omega_{0} t) + I_{j} \sin(\omega_{0} t)], \tag{5}$$

$$v(t) = V_0 + \sum_{j=1}^{N} [V_{j} \cos(\omega_{0} t) + V_{j} \sin(\omega_{0} t)]. \tag{6}$$

In dealing with Multi-port nonlinear wire scatterers, the input admittance matrix $\overline{\mathbf{Y}}_{sc}$ is replaced with the admittance matrix $\overline{\mathbf{Y}}$ as (7) in which diagonal elements are self-admittances which are approximately the one in single-port nonlinear wire scatterers, whereas off-diagonal ones are mutual admittances between among scatterers.

To solve Eq. (1) with guaranteed convergence, we resort to the intelligent water drop algorithm (IWD):

$$\overline{\mathbf{Y}} = \begin{bmatrix}
Y_{11} & \cdots & Y_{1M} \\
\vdots & \ddots & \vdots \\
Y_{M1} & \cdots & Y_{MM}
\end{bmatrix}. \tag{7}$$

Fig. 1. Nonlinearly loaded wire scatterer as: (a) single port and (b) multi-port structures.
III. IWD-BASED ALGORITHM

In this section, a succinct but nevertheless comprehensive explanation of the IWD algorithm is provided. The IWD algorithm is a constructive-based method, where a set of water drops move from one node to the next until a complete population of the solution is reached. This algorithm is based on the observation of the flow of water in rivers. The water in rivers is seen as a collection of water drops that flow from point in high terrain (source) to point in low terrain (destination) with a certain velocity. The water drops also carry some amount of soil with them while flowing along a path. Therefore, water drops are able to transport an amount of soil from one place to another. The parameters involved in this algorithm are classified into two kinds of static and dynamic parameters. Static parameters should be initialized before the algorithm starts, while dynamic parameters, as their name suggests, will vary as the algorithm iterates. By taking these two properties of water drops into account, IWD algorithm has been developed.

In summary, the algorithm of IWD as step by step is as follows:

1. Initialize the static parameters. The graph \((V, E)\) of the problem is given to the algorithm. The value of the total-best solution \(T_{\text{Best}}\), is initially assigned to the worst value:

\[
q(T_{\text{Best}}) = -\infty. \quad (8)
\]

The value of \(i_{\text{max}}\) is specified by the user. The initial value of the iteration counter is set to zero. That is \(i_{\text{count}}=0\). The algorithm terminates when \(i_{\text{count}}=i_{\text{max}}\).

The value of \(N_{\text{IWD}}\) (number of water drops) is set to a positive integer value, which is normally set to the number of nodes \(N_C\) in the graph. The location vector of each water drop represents the induced voltage at scatterer terminal in our problem. For soil updating \(a_s, b_s\) and \(c_s\), for velocity updating, the parameters are \(a_v, b_v\) and \(c_v\). The local soil updating parameter \(\rho_s\), which is a small positive number less than one, is set as \(\rho_s = 0.99\). The global soil updating parameter \(\rho^{\text{IWD}}\), which is selected from \([0, 1]\), is set as \(\rho^{\text{IWD}} = 0.99\). Furthermore, the initial soil on each path (edge) is denoted by the constant InitialSoil such as the soil of the path within every two nodes \(i\) and \(j\), that is set by \(\text{soil}(i, j) = \text{InitialSoil}\). The initial velocity value of each IWD is set to InitialVel. Both parameters InitialSoil and InitialVel are users choose and they should be tuned experimentally for the application.

2. Initialize the dynamic parameters. Every IWD has a visited node list \(V_{i}(\text{IWD})\), which is initially empty: \(V_{i}(\text{IWD}) = \{\}\); Each IWD’s velocity is set to InitialVel.

3. Spread the intelligent water drops randomly on the nodes of the graph as their first visited nodes.

4. Update the visited node list for each intelligent water drop to include the nodes that just visited.

5. Repeat steps 5.1 to 5.4 for those IWDs with partial solutions. Worth mentioning that partial solutions are solutions with certain degree of undesirability. Although partial solutions are not local solutions and are merely acceptable, they are not the optimum solution.

5.1) Consider water drop \(k\) residing at the current node (node \(i\)) intends to move to the next node (node \(j\)) through an edge \(e(i,j)\). The edge selection is done through a probability function, determined by \(P_{i}^{\text{IWD}}(j)\), as defined in (2). Then, the water drop visits node \(j\) by adding to \(V_{i}(\text{IWD})\):

\[
P_{i}^{\text{IWD}}(j) = \frac{d(\text{soil}(i,j))}{\sum_{k \in V_{i}(\text{IWD})} \text{soil}(i,k)}, \quad (9)
\]

\[
d(i, j) = \frac{1}{\epsilon + g(\text{soil}(i, j))}, \quad (10)
\]

where \(\epsilon\) is a small positive number used to prevent the division by zero in function \(d(., .)\), and,

\[
g(\text{soil}(i, j)) = \begin{cases} 
\text{soil}(i, j) & \text{if } \min_{k \in V_{i}(\text{IWD})} \text{soil}(i, j) \geq 0 \\
\text{soil}(i, j) - \min_{k \in V_{i}(\text{IWD})} \text{soil}(i, h) & \text{else} 
\end{cases}
\]

(11)

where \(\text{soil}(i, j)\) refers to the amount of soil within the local path between nodes \(i\) and \(j\). Then, add the newly visited node \(j\) to the list \(V_{i}(\text{IWD})\).

5.2) For each IWD moving from node \(i\) to node \(j\), update...
its velocity $v_IWD(t)$ by:

$$v_IWD(t+1) = v_IWD(t) + \frac{a_v}{b_v + c_v \Delta v_IWD(i,j)} ,$$  \hspace{1cm} (12)

where $a_v$, $b_v$, and $c_v$ are the static parameters used to represent the nonlinear relationship between the velocity of water drop, i.e. velocity $v_IWD$, and velocity $v_IWD(t+1)$ is the updated velocity for the IWD.

5.3 For the intelligent water drop (IWD) moving on the path from node $i$ to $j$, compute the change of soil $\Delta soil(i,j)$ that the intelligent water drop (IWD) loads from the path by:

$$\Delta soil(i,j) = \frac{a_s}{b_s + c_s time(i,j;vel_IWD)} ,$$  \hspace{1cm} (13)

where, $a_s$, $b_s$, and $c_s$ are the static parameters used to represent the nonlinear relationship between $\Delta soil(i,j)$ and the inverse of $vel_IWD$.

Note that $time(i,j;vel_IWD)$ refers to the time needed for water drop to transit from node $i$ to node $j$ at time $t+1$. It is defined as follows:

$$time(i,j;Vel_IWD) = \frac{HUD_{max}(\varepsilon, Vel_IWD)}{vel_IWD} ,$$  \hspace{1cm} (14)

where $HUD(.)$ is a local heuristic function defined to measure the degree of the undesirability of IWD to move between nodes $i$ and $j$. In our application, it is chosen to be unity.

5.4 Update the soil $soil(i,j)$ of the path from node $i$ to $j$ traversed by that IWD and also update the soil that the IWD carries $soil_{IWD}$ by:

$$soil(i,j) = (1-\rho)soil(i,j) - \rho soil_{IWD}(i,j) ,$$  \hspace{1cm} (15)

$$soil_{IWD} = soil_{IWD} - \Delta soil(i,j) ,$$  \hspace{1cm} (16)

where $\rho$ is a small positive constant between zero and one.

6) Find the iteration best solution $T_{Best}$ from all the solutions $T_{IWD}$ found by the I WDs using:

$$T_{Best} = \arg\max_{q(T_{IWD})} q(T_{IWD}) .$$  \hspace{1cm} (17)

7) Update the soils on the paths that form the current iteration best solution $T_{Best}$ by:

$$soil(i,j) = (1-\rho)soil(i,j) + \rho \frac{2soil_{IWD}}{N_e(N_e-1)} \forall (i,j) \in T_m .$$  \hspace{1cm} (18)

Update the total best solution $T_{TBest}$ by the current iteration best solution $T_{Best}$ using:

$$T_{TBest} = \begin{cases} T_{Best} & \text{if } q(T_{TBest}) \geq q(T_{Best}) \\ T_{TBest} & \text{else} \end{cases} .$$  \hspace{1cm} (19)

8) Increment the iteration value by:

$$I_t_{count} = I_t_{count} + 1 ,$$  \hspace{1cm} (20)

Then, go to Step 2 if:

$$I_t_{count} < I_t_{Max} .$$  \hspace{1cm} (21)

9) The algorithm stops here with the total best solution $T_{TBest}$.

Here $T_{TBest}$ is the best value for harmonics voltages. $I_t_{count}$ is the number of iterations when the algorithm has converged. The static and dynamic parameters in this paper are listed in Table 1 in [17].

IV. NUMERICAL SIMULATION AND DISCUSSION

A. Comparison with AOM

A.1 Strongly nonlinear load

To show the capability of the algorithm on the order of load nonlinearity, a single-port wire scatterer having length to diameter ratio $L/2a=74.2$, and one meter length $(L=1$ m) is selected. Also, the scatterer is illuminated by an incident plane wave of magnitude $E_i=1$(V/m) and centrally loaded with a very strongly nonlinear device, which is characterized by (22):

$$i=10^3 \times v^{13} .$$  \hspace{1cm} (22)

Fig. 3. The induced voltage across the strongly nonlinear load at the fundamental harmony under single-tone excitation.

Prior to analysis, the Norton circuit in Fig. 2 is efficiently computed based on [14, 15]. The voltage magnitudes for various values of length to wavelength are shown in Fig. 3. A comparison of IWD results with those obtained by the AOM [4] substantiates the high accuracy of the algorithm while the run-time is considerably reduced which will be discussed in next sections.

A.2 Lossy ground

In the second example, the versatility of the method considering lossy ground is tested. Hence, a single–port nonlinear wire scatterer is situated 0.4 m above a lossy ground, while it is illuminated by a plane wave with the polar angle of incidence $\theta=60$ deg. The ground is characterized by a relative permittivity of $\varepsilon_r=10$ and
conductivity of \( \sigma = 0.003 \text{S/m} \). The nonlinear load has a 
\((i-v)\) characteristic as below:
\[
i = \frac{1}{75} v + 4v^3.
\] (23)

Fig. 4. Spectral content across the nonlinear load of 
single-port nonlinear wire scatterer over lossy ground.

At first, the Norton circuit is efficiently computed 
based on the MoF in [16] and the IWD is then applied to 
Eq. (1). The voltage magnitudes are depicted in Fig. 5. A comparison of our results with those obtained by AOM [4] corroborates the accuracy of the proposed method. It should be mentioned that in this examples, the suppression of even harmonics are due to the cubic nature of the load's nonlinearity. As shown in Fig. 4, this even harmonic suppression is better demonstrated by the proposed method with respect to AOM. The mean square errors (MSEs) by the IWD, for the two mentioned cases are also provided in figure 5 for the first 30 iterations with deactivated stopping criteria (1.E-8). It shows that iteration process larger than 30 gives excellent agreement in the two examples.

B. Comparison with HB and NC

B.1. Single-port

In this sub-section, capability of the IWD method in comparison with HB technique [8] is investigated. Hence, a single-port wire scatterer the same as previous sections is exposed to multiple plane waves of the same amplitude \((E_i = 1 \text{V/m})\) and different frequencies, i.e., 140MHz, 160MHz.

The scatterer is centrally loaded with a p-n junction 
diode with the following \((i-v)\) characteristic:
\[
i = I_s(e^{i \nu_T} - 1),
\] (24)
where \(I_s = 10 \text{nA}\) , and \(\nu_T = 26 \text{mV}\). The induced voltage across the diode at different harmonic frequencies using the IWD, HB, and NC methods are shown in Fig. 6. From this figure, IWD and HB-based results are in good agreement, whereas the NC-based results are considerably violated.

B.1. Multi-port

In this section to show capability of the IWD in 
analyzing multi-port wire scatterer under multi-ton 
excitation, an infinite planar array of nonlinear wire 
scatterers is chosen. The scatterers in the array are 
equally spaced in vertical and horizontal directions. The 
nonlinear wire scatterers in the array are the same as the 
previous sub-sections. The induced voltage across the 
p-n junction diode are computed for the vertical and 
horizontal spacing 1 m and shown in Fig. 7.

The HB and NC-based results are included in the 
same figure as well. From this figure, IWD-based results 
are in good agreement with HB-based ones [18]. In
addition, although the approximated method of NC is efficient, it is violated under strong nonlinearity.

Fig. 7. The induced voltage across the diode at different harmonics for multi-port nonlinear wire scatterers.

To show the efficiency of the proposed approach better, the MSEs for the single and multi-port nonlinear wire scatterers with very fast convergences are shown in Fig. 8. Finally, the relative error percentages of the proposed method for single and multi-port wire scatterers in comparison with the HB technique are listed in Table 1. From this table, the precise of the IWD can be observed.

![MSE Plot](image)

**Fig. 8. The MSEs for single and multi-port nonlinear wire scatterer.**

To verify the performance of the IWD in such complex scatterers, a vertical rod with length L=3.05m, and radius a=12.7mm and buried in a lossy ground with conductivity \( \sigma = 0.11 S/m \), relative permittivity \( \varepsilon_r = 10 \) and critical electric field \( E_c = 127 kV/m \) is selected [29]. The lightning stroke is expressed as a current source and shown in Fig. 12 (dotted line). Prior to analysis, such current source is represented as Fourier series, and the input admittance in Fig. 2 (a) is then computed by MoM or the efficient method based on fuzzy inference (MoF) [30, 31]. Finally, applying the IWD method to Fig. 2 (a), the lightning-induced voltage in time domain is

<table>
<thead>
<tr>
<th>Table 1: Relative error percentage of the IWD algorithm in comparison with the HB technique at different harmonic frequencies</th>
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<tbody>
<tr>
<td><strong>Relative Error Percentage of Terminal Voltage</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Single port</td>
</tr>
<tr>
<td>Multi port</td>
</tr>
<tr>
<td>f (MHz)</td>
</tr>
<tr>
<td>Single port</td>
</tr>
<tr>
<td>Multi port</td>
</tr>
</tbody>
</table>

C. Comparison with measurement

C.1. Single-port

To illustrate another capability of the proposed approach, it is compared with measurement. Figure 9 shows complex wire scatterer namely vertical rod as single-and multi-port structures which are buried in a lossy ground and subjected to the high-valued lightning stroke. In this figure, the nonlinear load represents the ionization phenomenon in the lossy ground. The (i-v) characteristic of the nonlinear load is conventionally is expressed as follows [19]:

\[
v = \frac{R}{\sqrt{1 + i/I_s}} - 1,
\]

where \( R \) is low-frequency resistance of the grounding rod, and \( I_s \) is computed as below:

\[
I_s = \frac{E_c}{(2\pi R^2 \sigma)},
\]

where \( E_c \) and \( \sigma \) are respectively critical electric field and conductivity of the lossy ground. Transient analysis of such scatterers has been carried out in time domain [20] where the input admittance is first computed in the frequency domain and then converted to time domain by vector fitting method [21]. Consequently, such analysis demands more run-time. Also, more recently, the ionization phenomenon has been modelled as gradually increasing radius of the rod by the MTL [22-26]. This approach, however, can not include the hysteresis effect in the ionization process, whereas, by the proposed model in Fig. 9, it can be easily included [27, 28].
It’s worth mentioning that the HB technique is evidently inefficient for nonlinear multi-port networks under multi-tone excitations [8]. Hence, the single-port nonlinear scatterer only has been analyzed by Shariatinasab et al. [28] and the multi-port one has not been addressed yet, whereas the proposed method can be easily applied.

C.2. Multi-port

The analysis of multi-port rod by the IWD is the same as single port, except that Fig. 2 (b) is used. Each vertical rod has length L=3.05m, and radius a=12.7mm and buried in a lossy ground with σ = 0.016 S/m, εᵣ = 10 and Eᵢ = 50kV/m [29]. The spacing between rods is D=3.09 m and the lightning current is the same as previous sub-section. The lightning-induced voltage by the IWD is shown in Fig. 11 which is in good agreement with the measurement [29] (Fig. 9 in [29]). Note that the validity of the IWD method with commercial packages has been investigated [32].

D. Comparison of run-times

As a final advantage of the IWD method, its run-time in analyzing single and multi-port nonlinear wire scatterers under multi-tone excitations is compared with the mentioned methods.

D.1. Single-port

In the case of single-port scatterer, the wire scatterer is the one in the sub-section IV-A, and the nonlinear load is as Eq. (23). The run-times of different approaches are listed in Table 2. From this table, when the number of exciting frequencies is increased, the run-time of the IWD method is slightly increased. Moreover, although the run-time of NC-based method is very short, it is restricted to weakly nonlinear loads (Figs. 6 and 7).

D.2. Multi-port

In the case of multi-port nonlinear wire scatterers, the nonlinear load and wire scatterer are the same as previous sub-section, but the different arrangements of wire scatterers are investigated. The run-times of the different arrangements under double-ton excitation are listed in Table 3. From this table, high efficiency of the IWD in comparison with the others is once more proven.

<table>
<thead>
<tr>
<th>Exciting Frequency</th>
<th>Run-time (sec)</th>
</tr>
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<tbody>
<tr>
<td>(MHz)</td>
<td>IWD</td>
</tr>
<tr>
<td>150</td>
<td>1</td>
</tr>
<tr>
<td>140,160</td>
<td>2</td>
</tr>
<tr>
<td>140,160,180</td>
<td>6</td>
</tr>
<tr>
<td>140,160,180,200</td>
<td>17</td>
</tr>
<tr>
<td>140,160,180,200,220</td>
<td>29</td>
</tr>
</tbody>
</table>
Table 3: Run-time of different methods for multi-port nonlinear wire scatterers under double-tone excitation

<table>
<thead>
<tr>
<th>Multi-port Scatter</th>
<th>Run-time (sec)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>IWD</td>
</tr>
<tr>
<td>1 by 1</td>
<td>2</td>
</tr>
<tr>
<td>2 by 1</td>
<td>20</td>
</tr>
<tr>
<td>2 by 2</td>
<td>30</td>
</tr>
<tr>
<td>3 by 3</td>
<td>53</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this study, an efficient hybrid method for analyzing nonlinear wire scatterers over lossy ground under multi-ton excitations was proposed. The proposed method consists of linear and nonlinear parts. The linear part can be used for inclusion of frequency dependence of the lossy ground, while the nonlinear one considers nonlinearity effects of the device connected to the scatterer so that both effects are considered. The model was validated using extensive examples. Comparative studies show excellent agreement with the existing methods, while run-time is considerably reduced.

It worth noting that although all the nonlinear loads in this study are expressed as analytical models, the (i-v) characteristics of some of devices are based on experimental measurements and are not smooth curves [33, 34] (Fig. 7 in [33] and Table 2 in [34]). In such cases, the HB method which needs initial guess and gradient operation in the iteration process, may yield violated solutions, whereas the artificial intelligent (AI) approaches especially IWD outperforms the HB method.

REFERENCES


Amir Bahrami was born in Esfahan, Iran, on June 22, 1994. He received B.Sc. in Electrical Engineering from Arak University, Arak, Iran, in 2016. His research interests include nonlinear microwaves, nonlinear RF circuits, nonlinear differential equations and nonlinear functional analysis.

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